Majority Bargaining for Resource Division

(Extended Abstract)

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ABSTRACT

We are concerned with the problem of how a collection of agents can decide to share a resource, represented as a unit sized pie. We investigate a simple and natural non-cooperative bargaining protocol for this problem, in which players take it in turns to make proposals on how the resource should be allocated, and the other players vote on whether or not to accept the allocation. Voting is modelled as a weighted voting game: each player is assigned a weight, and a proposal for allocation is implemented if the weight of players in favour of that proposal meets or exceeds a given quota. The agenda, (i.e., the order in which the players are called to make offers), is defined exogenously. Thus, the outcome is an offer that has majority support. We consider two variants of this protocol, provide subgame perfect equilibrium strategies for both, and investigate their properties. Finally, we give those conditions for which the non-cooperative equilibria for the two games is in the core of the weighted voting game and those for which it is not.

Categories and Subject Descriptors

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Keywords

Bargaining, Resource division, Voting games

1. INTRODUCTION

In this paper, we address the problem of how a group of agents can resolve conflicts that arise in the context of dividing a pie between them [2]. We take a noncooperative approach and consider two different bargaining protocols built mainly on Rubinstein's bilateral bargaining game [3, 1]. The agents take it in turns to propose a division of the pie. After a proposal has been made, the players vote on whether to accept or reject the proposal. Voting takes place using a *weighted voting game*, in which each player has a weight, and a proposal is accepted if the sum of the weights of those in favour of the proposal meets or exceeds a certain quota. If the weight of players in favour of the proposal meets or exceeds the quota, then that proposal is implemented; otherwise we turn to the next player to make a proposal. If no proposal is accepted after the final player proposes, then all players receive nothing.

We explore two variations of this protocol, which we refer to as G_1 and G_2. These games differ in terms of the subset of players whose vote is counted for determining majority support. In G_1, players who have made an unsuccessful proposal are not eligible to vote, while in G_2, all other players can vote. Both games have a finite horizon, time discounting, and perfect information. Thus, bargaining is guaranteed to end after a fixed number of rounds; players are impatient, preferring an early outcome to the game; and players are in possession of complete information about the game.

For both games we provide subgame perfect equilibrium strategies that result in an instant Pareto optimal and unique agreement. Then, we give those conditions for which the non-cooperative equilibria for the two games is in the *core* of the weighted voting game and those for which it is not.

2. THE MODEL

We assume there are p players, where a player represents either an individual or a group of individuals. There is a resource, modeled as a unit-sized pie, that must be allocated between the players. An *allocation* specifies how the pie is split between the players. It is represented as a vector $(x_1, \ldots, x_i, \ldots, x_p)$. The idea is that element x_i ($0 \leq x_i \leq 1$) denotes player i's allocation, i.e., the amount of the resource that player i receives. We will let $\mathcal{X}$ denote the set of all possible allocations. A player's utility from an allocation depends both on his share of the pie and the time at which he receives his allocation. Time is divided into discrete time periods numbered $1, 2, \ldots$. Player i's utility from an allocation x at time t is given by the following function:

$$u_i(x,t) = \begin{cases} \delta^{t-1} \times x_i & \text{if} \quad t \leq T \\ 0 & \text{otherwise} \end{cases}$$

where $0 < \delta \leq 1$ is the *discount factor*. Thus, at time t utility gets discounted by the factor δ^{t-1}. We assume that all the players have the same discount factor δ. Thus, a player's utility is increasing in his share of the pie and decreasing in time. Furthermore, a player derives benefit from receiving a share *only* if he is allocated that share before a given time period T: after this time, the pie becomes useless to him.

The players want to implement an allocation that has majority support. Each of the two bargaining games is comprised of T discrete time periods, i.e., bargaining must end within T time periods. In each time period, a chosen player makes an offer that specifies an allocation. The *outcome* of bargaining is an offer that has the support of a majority of players.

2.1 The Weighted Voting Game

A weighted voting game is a 3-tuple $G = (P, w, q)$ where $P = (1, \ldots, p)$ is the set of players, w is a vector of weights for the players with w_i denoting the *weight* for player $1 \leq i \leq p$, and $q \in \mathbb{R}$ is the *quota*. Its characteristic function $v : 2^P \rightarrow \{0, 1\}$ is

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given as follows:

$$v(C) = \begin{cases} 1 & \text{if } \sum_{i \in C} w_i \geq q \\ 0 & \text{otherwise} \end{cases}$$

The total weight of coalition C is $w(C) = \sum_{i \in C} w_i$. A coalition C is *winning* if $v(C) = 1$, otherwise it is *losing*. A player is a *veto player* if a winning coalition cannot be formed without him. Let $Z \subseteq P$ denote the set of all veto players with $|Z| = z$. Also, let S_W denote the set of all winning coalitions. The set of all losing coalitions will be denoted S_L.

2.2 The Noncooperative Bargaining Games

Both games proceed in a series of rounds. For both games, the bargaining deadline is $T = p$, i.e., an agreement must be reached within p rounds, otherwise all the players will get zero utility. For both games, we suppose that the p players are ordered *exogenously* as per an agenda A.

Rules of the game G_1: Bargaining begins at $t = 1$ when player A_1 proposes an offer $x^t = (x_1^t, \ldots, x_p^t)$ that specifies a split the pie. All the remaining players then respond to the offer by either *accepting* or *rejecting* it. Let C_A^t denote the set of players that accept the proposal x^t and C_R^t the set of players that reject it. If $\sum_{i \in C_A^1} w_i \geq q$, then the game ends, the pie is split as per the offer x^1, and the resulting coalition structure is (C_A^1, C_R^1).

On the other hand, if $\sum_{i \in C_A^1} w_i < q$, then time is incremented and bargaining proceeds to the second round when A_2 makes a proposal x^t. The players $A_3, \ldots, A_p$ are allowed to respond but not player A_1. If A_2 gets majority support from $A_3, \ldots, A_p$, then the pie is split as per x^2, the coalition structure is $(C_A, C_R \cup (A_1))$, and the game ends. Otherwise, the process repeats. If no winning coalition is formed within p time periods, then the game ends and all the players get zero utility. In general, at time t, the player A_t will propose an offer to which only the players $(A_{t+1}, \ldots, A_p)$ can respond. Thus, we have $C_A^t \subseteq (A_{t+1}, \ldots, A_p)$, $C_R^t \subseteq (A_{t+1}, \ldots, A_p)$, $C_A^t \cup C_R^t = (A_{t+1}, \ldots, A_p)$, and $C_A^t \cap C_R^t = \emptyset$. If the offer x^t gets majority support, the game ends, the pie is split as per the offer, and the resulting coalition structure will be $(C_A^t, C_R^t \cup (A_1, .., A_{t-1}))$.

Rules of the game G_2: The difference between this game and G_1 is that, unlike G_1, here, all the players can vote on a proposal.

3. EQUILIBRIUM ANALYSIS

For the game G_1 (G_2), the strategies given in Table 1 (2) form a subgame perfect equilibrium. For a voting game $G = (P, w, q)$ with $0 \leq v < p$ veto players, the equilibrium outcome for G_1 (G_2) is unique and Pareto optimal, and there is instant agreement.

4. NON-COOPERATIVE EQUILIBRIUM AND THE CORE

An allocation $x \in \mathcal{X}$ is *Pareto optimal* if $\sum_{i=1}^{p} x_i = 1$, and *individual rational* if $x_i \geq v(i)$. An allocation is in the *core* of a weighted voting game $G = (P, w, q)$ if it is Pareto optimal, individual rational, and for each $S \subset P$, $\sum_{i \in S} x_i \geq v(S)$.

Let $x(G_1)$ denote the equilibrium allocation for $t = 1$ for the bargaining game G_1 and $x(G_2)$ that for G_2. Given this, the conditions for the non-cooperative equilibrium for G_1 (G_2) to be in the core are given in Theorem 1 (Theorem 3). The conditions when the non-cooperative equilibrium for G_1 (G_2) is not in the core are given in Theorem 2 (Theorem 4).

Time	Equilibrium strategy
$1 \leq t \leq \tau$	**Offer**: Player A_t proposes an x^t that solves the following optimization problem: O_t : Maximize $u_{A_t}(x^t, t)$ s.t. $u_c(x^t, t) \geq u_c(x^{t+1}, t+1)$ for $c \in \bar{c} - (A_t)$ and $\bar{c} \in C_w^t$ **Response to** x^t: If $u_c(x^t, t) \geq u_c(x^{t+1}, t+1)$ then $c \in (A_{t+1}, \ldots, A_p)$ accepts, otherwise c rejects the offer.
$\tau < t \leq n$	**Offer** Player A_t proposes to keep a 100% of the pie. **Response to any offer**: Accept.

Table 1: Subgame perfect equilibrium strategies for G_1.

Time	Equilibrium strategy
$1 \leq t \leq \tau$	**Offer**: Player A_t proposes an x^t that solves the following optimization problem: O_t : Maximize $u_{A_t}(x^t, t)$ s.t. $u_c(x^t, t) \geq u_c(x^{t+1}, t+1)$ for $c \in \bar{c} - (A_t)$ and $\bar{c} \in C_w^t$ **Response to** x^t: If $u_c(x^t, t) \geq u_c(x^{t+1}, t+1)$ then player $c \in P - (A_t)$ accepts, otherwise he rejects the offer.
$\tau < t \leq n$	**Offer** Player A_t proposes to keep the whole pie. **Response to any offer**: All the players accept.

Table 2: Subgame perfect equilibrium strategies for G_2.

THEOREM 1. *For an agenda A, $x(G_1)$ is in the core of the weighted voting game (P, w, q) if the pie does not shrink with time (i.e., $\delta = 1$) and A_τ is a veto player, or if the pie shrinks with time (i.e., $0 < \delta < 1$) and the first mover is a veto player (i.e, $\tau = 1$).*

THEOREM 2. *For an agenda A, $x(G_1)$ is not in the core of the weighted voting game (P, w, q) if the pie shrinks with time (i.e., $0 < \delta < 1$) and $\tau > 1$, or if A_τ is not a veto player.*

THEOREM 3. *For an agenda A, $x(G_2)$ is in the core of the weighted voting game (P, w, q) with $0 < v < p$ veto players if the pie does not shrink with time (i.e., $\delta = 1$), or if the pie shrinks with time (i.e., $0 < \delta < 1$) and the first mover is a veto player.*

It is straightforward to see that, for $v = 0$, $x(G_2)$ will not be in the core.

THEOREM 4. *For an agenda A, $x(G_2)$ is not in the core of the weighted voting game (P, w, q) if the pie shrinks with time (i.e., $0 < \delta < 1$) and the first mover is a non-veto player.*

5. REFERENCES

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