Generalized Second Price Auctions with Value Externalities

(Extended Abstract)

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ABSTRACT

We consider a new setting of ad auctions with value externalities. Under such a setting, we perform theoretic analysis on two implementations of generalized second price auctions (GSP): GSP_S and GSP_V. Our analysis shows that both GSP_S and GSP_V admit at least one pure Nash equilibrium for the single-slot case, while pure Nash equilibrium may not exist for the multi-slot case. Furthermore, we prove that the price of anarchy of GSP_S does not have a constant upper bound, while for GSP_V one can achieve a constant upper bound under mild assumptions.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent Systems; J.4 [**Social and Behavioral Sciences**]: Economics

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Economics, Theory

Keywords

Generalized second price auction, price of anarchy, value externality, Nash equilibrium

1. INTRODUCTION

Sponsored search auctions are the major revenue source of search engines. In sponsored search auctions, advertisers bid for ad slots on search result pages, and the search engine determines the allocation of ad slots and the payment of the advertisers based on their bids. For example, in the widely used generalized second price auction (GSP), ad slots are assigned to advertisers in the descending order of their bids and an advertiser pays per click the minimum bid price that is necessary to keep his/her rank position.

In most previous works, it is assumed that each advertiser only benefits from his/her own ad. However, in reality, a single ad may benefit multiple different advertisers [3], e.g., an ad about Dell laptop may not only benefit Dell, but also Microsoft and Intel since they provide the OS software and CPU for the laptop. In other words, an ad may generate values for multiple advertisers. This phenomenon motivates us to consider the value externalities among ads when designing and analyzing auction mechanisms for sponsored search.

We study the existence and social efficiency of pure Nash equilibria (NE) for GSP auctions with value externalities. As we know, in the conventional setting without value externality, GSP admits at least one pure Nash equilibrium [2], and the price of anarchy (PoA) for GSP is small [4]. However, little is known about how GSP will perform when there exists value externality. In our study, we consider two implementations of GSP auctions. The first implementation is the standard GSP auction (GSP_S), which ranks ads according to their bids and charges advertisers according to the next-price rule. In the second implementation, the search engine estimates the value externality among ads (e.g., by certain machine learning technologies), and then uses both the estimates and advertises' bids to determine the ranking and pricing of the ads.

2. THE MODEL

In this section, we provide formal description of auctions with value externality. Suppose there are n advertisers $i = 1, \ldots, n$, bidding for $k \leq n$ ad slots. The click through rates (CTRs) of slots are $\alpha_1, \ldots, \alpha_k$, respectively, and they satisfy $\alpha_1 \geq \ldots \geq \alpha_k \geq 0$. We use A_i to represent the ad of advertiser (or player) i .

Unlike the traditional setting where the type of each advertiser is given by a one dimensional value v_i, the types of advertisers in our setting are multi-dimensional. Formally, advertiser i's type is specified by an n-dimensional value vector $v^i = (v^i_1, \ldots, v^i_n)$, where v^i_i can be thought of as the conventional "private value" of advertiser i, while each other element v^i_j can be interpreted as the value brought by ad A_j to advertiser i.

In this paper, we consider the positive externalities, i.e., $v^i_j \geq 0, \forall i, j$. In addition, we assume that $v^i_i > \max_{j \neq i}\{v^i_j\}$, $\forall i = 1, \ldots, n$, which means that advertisers get the maximum value from their own ads. This assumption is very natural, since if $v^i_i \leq v^i_j$ for some i and j, then there is no specific need for advertiser i to use his/her own ad (e.g., he/she can simply use ad A_j in order to increase his/her payoff). Furthermore, we define the *externality gap* δ as follows: $\delta = \min_{i,j} \frac{v^i_i - v^i_j}{v^i_i}$. Then, from the above assumption we have $\delta > 0$. Each ad A_i also corresponds to a vector $v(A_i) = (v^1_i, \ldots, v^n_i)$, which characterizes the values of A_i.

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Define $w_i = \sum_j v_i^j / v_i^i$. It is easy to check $w_i v_i^i$ is the total values that ad A_i brings to all the advertisers.

A main difference between our setting and the traditional setting is that the utility of advertiser i is $u_i = \alpha_i(v_i^i - p_i) + \sum_{j \neq i} \alpha_j v_j^i$, where p_i is the payment of player i and for simplicity we assume advertiser i obtains slot i for $i = 1, \ldots, n$.

3. TWO AUCTIONS

We consider in this paper two implementations of the GSP auctions: GSP_S and GSP_V. GSP_S is the standard GSP auction, while GSP_V is a variant of GSP_S that refines the ranking and pricing rules by estimating the value externality.

The GSP_S auction.

Each advertiser i submits a one-dimensional bid b_i. The search engine allocates slots in the decreasing order of bids.

The GSP_V auction.

Each advertiser i submits a one-dimensional bid b_i, then the search engine estimates w_i for each ad A_i, and allocates slots in the decreasing order of $s_i = w_i b_i$.

If advertisers are numbered so that advertiser i obtains slot i, then advertiser i's payment is $p_i = b_{i+1}$ in GSP_S and $p_i = s_{i+1}/w_i$ in GSP_V.

We say that a set of bids $b = (b_1, \ldots, b_n)$ is a *Pure Nash Equilibrium* (PNE) if no bidder can increase his utility by changing his bid unilaterally, i.e.: $u_i(b_i, b_{-i}) \geq u_i(b'_i, b_{-i}), \forall b'_i$. The *Price of Anarchy (PoA)* is defined as the ratio between the social welfare of an optimal allocation and that of a worst Nash equilibrium [4].

Note that under the traditional setting without value externality, GSP_S and GSP_V are actually identical. And as we know, they admit at least one pure Nash equilibrium [2], and their PoA is small [4]. However, little is known about the existence and social efficiency of pure NE of GSP_S and GSP_V when there exists value externality.

4. EXISTENCE OF PNE

In this section, we consider the existence of pure NE in GSP_S and GSP_V in the single-slot case and multi-slot case respectively.

Define $c_i \equiv \min_{j \neq i}\{v_j^i\}$, for $i = 1, \ldots, n$, i.e., c_i is the minimum value that advertiser i can obtain from other single ad. We say pure bid b_i for advertiser i is *conservative* if $b_i \leq v_i^i$. We can verify that a bid $b_i > v_i^i - c_i$ is dominated by $b'_i = v_i^i - c_i$. Therefore, we only consider conservative bids throughout the rest of this paper, and we have the following result.

THEOREM 1. *For the single-slot case, both GSP_S and GSP_V admit a pure NE. While for the multi-slot case, pure NE may not exist in GSP_S.*

The non-existence of pure NE in multi-slot case implied directly by the following example.

EXAMPLE 1. *There are four advertisers A, B, C and D competing for two slots whose CTRs are both 1. The types of players are $v^A = (8, 3.4, 1, 6)$, $v^B = (0, 7, 4, 0)$, $v^C = (3, 1, 6, 1.5)$ and $v^D = (2, 2, 0, 6)$, respectively.*

As for GSP_V, unfortunately, we do not have a clear conclusion yet whether pure Nash equilibrium exists or not.

5. PRICE OF ANARCHY

In this section we study the price of anarchy of GSP_S and GSP_V conditioned on the existence of PNE. To our knowledge, it is a common practice to study PoA conditioned on the existence of PNE (e.g., see [1]).

From the following example, it is easy to see that the PoA of GSP_S can not be upper bounded by a constant.

EXAMPLE 2. *There are n advertisers $1, \ldots, n$, and one slot. The types of advertisers are $v^1 = (1 + 2\epsilon, 0, \ldots, 0, 1)$, $v^2 = (0, 1{+}\epsilon, 0, \ldots, 0, 1)$, $v^3 = (0, 0, 1, 0, \ldots, 0, 1)$, $\ldots$, $v^{n-1} = (0, \ldots, 0, 1, 1)$, $v^n = (0, \ldots, 0, 1)$.*

Formally, for the PoA of GSP_S we have:

THEOREM 2. *For GSP_S, PoA$= n$ in the single-slot case. For the multi-slot case of GSP_S with at least one pure NE, $n \leq$PoA$< \frac{3n-1}{2}$.*

The PoA of GSP_V can be significantly better than that of GSP_S, and it is upper bounded by a constant when the externality gap is not close to 0. Assume $\delta \geq \Delta > 0$, where Δ is a constant, then we have:

THEOREM 3. *For GSP_V, PoA$\leq \frac{1}{\Delta}$ in the single-slot case. For the multi-slot case of GSP_V with at least one pure NE, PoA$\leq 2 + \frac{1}{\Delta}$.*

By comparing the theoretical properties of GSP_S and GSP_V, one can see that it is necessary to refine the standard GSP auction (e.g., by using GSP_V) when there are value externalities in order to significantly improve the social efficiency.

6. CONCLUSION

In this paper we studied the game-theoretic properties of two implementations of GSP auctions in the setting with value externalities. Our main results show that the GSP_S mechanism does not always admit a pure NE in this setting, and its PoA does not have a constant upper bound. In contrast, the GSP_V mechanism that refines the ranking and pricing rules by estimating the value externalities performs significantly better than GSP_S in terms of PoA bounds.

As for future work, we plan to work on the following aspects. First, we only consider the PoA of pure Nash equilibrium in this work. We will explore the mixed Nash equilibrium and the Bayesian setting in the future. Second, we will extend the study on social welfare to the revenue of the auctioneer (search engines).

7. REFERENCES

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