A Judgment Set Similarity Measure Based on Prime Implicants

(Extended Abstract)

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ABSTRACT

Distances and scores are widely used to measure similarity between collections of information, such as preference profiles, belief sets, judgment sets, argument labelings, etc. Defining a function that quantifies the similarity between information sets of logically interrelated information is non-trivial, as witnessed by the shortage of such quantifiers in the literature. We propose a similarity measure for judgment sets that is "sensitive" to logic dependencies among the judgments.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent systems; I.2.4 [**Knowledge representation formalisms and methods**]

Keywords

Judgment aggregation, Belief Merging, Social choice theory

1. INTRODUCTION

The aggregation of sets of logically related information has been studied in several disciplines that have intersecting areas of interest with multiagent systems. One of these is judgment aggregation [6], which studies the aggregation of, predominantly binary, judgments on a given set of issues called an agenda. One way to construct information aggregation operators is by using functions that quantify the similarity among the aggregated information sets [2, 4, 7]. While the idea of using a similarity quantifier is simple and intuitive, the construction of an adequate quantifier is not.

When the information on some issues is not entailed by, or does not entail the informations on other issues, a similarity quantifier can be obtained by simply counting the number of issues on which the two collections disagree. This simple quantifier, frequently used in information aggregation, and in judgment aggregation as well [1, 2, 7], is the Hamming distance [3]. When logic relations among aggregated issues do exist, as is the standard assumption in judgment aggregation, it has been argued that the Hamming distance is not the best similarity quantifier [2, 5]. Regardless, very little alternatives to the Hamming distance exit, particularly not quantifiers that are sensitive to the the underlying semantics of the aggregated information. One exception is the metric proposed in [2]. We show that more "semantic sensitive" similarity quantifiers can be constructed.

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2. PRELIMINARIES

Judgment aggregation problems are typically represented using a logic, or a set $\mathcal{L}$ of well-formed propositional logic formulas, including $\top$ (tautology) and $\bot$ (contradiction). An *issue* is a pair of formulas $\{\varphi, \neg\varphi\} \subset \mathcal{L}$ where φ is neither a tautology nor a contradiction. For simplicity, we often abuse notation and write only the positive formula when we discuss issues. Two issues φ_i, φ_j are *logically unrelated* iff they do not have atomic sub-formulas in common, and logically related otherwise. An *agenda* $\mathcal{A}$ is a finite set of issues, $\mathcal{A} = \{\varphi_1, \neg\varphi_1, \ldots, \varphi_m, \neg\varphi_m\}$. A *preagenda* $[\mathcal{A}]$ for $\mathcal{A}$ is the set $[\mathcal{A}] = \{\varphi_1, \ldots, \varphi_m\}$. A *judgment* on $\varphi \in [\mathcal{A}]$ is one of φ or $\neg\varphi$. A *judgment set* J is a subset of $\mathcal{A}$. J is *complete* iff for each $\varphi \in [\mathcal{A}]$ either $\varphi \in J$ or $\neg\varphi \in J$ and *consistent* iff it is a consistent set of formulas, *i.e.*, $J \not\models \bot$.

Let the set of all consistent judgment sets for $\mathcal{A}$ be $\mathcal{D}(\mathcal{A})$ and let $\mathbb{D}(\mathcal{A}) \subset \mathcal{D}(\mathcal{A})$ be the set of all judgment sets that are also complete. A *profile* $P \subset \mathbb{D}^n(\mathcal{A})$ is a list $P = \langle J_1, \ldots, J_n \rangle$, of judgment sets for agents $i \in [1, n]$.

An (irresolute) judgment aggregation rule is a correspondence $F : \mathbb{D}^n \to 2^{\mathbb{D}(\mathcal{A})} \backslash \varnothing$. A distance d between two judgments sets is a function $d : \mathbb{D}(\mathcal{A}) \times \mathbb{D}(\mathcal{A}) \to \mathbb{R}$, defined for every agenda $\mathcal{A} \subset \mathcal{L}$. A distance-based judgment aggregation rule $F^{d,\Sigma}$ is defined as

$$F^{d,\Sigma}(P) = \arg\min_{J \in \mathbb{D}(\mathcal{A})} \sum_{J_i \in P} d(J_i, J) \tag{1}$$

There are three distances used in judgment aggregation. The *Hamming distance* d_H between two complete judgment sets J_1 and J_2 is defined as $d_H(J_1, J_2) = |J_2 \backslash J_1|$. The Drastic distance between two complete judgment sets J_1 and J_2 is 0 iff $|J_2 \backslash J_1| = 0$ and 1 otherwise. The Duddy-Piggins distance d_G introduced in [2], is defined as the number of edges in the shortest path in the graph $G(\mathcal{A}) = \langle \mathbb{D}(\mathcal{A}), E \rangle$ where $E \subseteq \mathbb{D}(\mathcal{A}) \times \mathbb{D}(\mathcal{A})$ is defined as $(J_1, J_2) \in E$ iff there exists no $J_3 \in \mathbb{D}(\mathcal{A})$ such that $J_1 \cap J_2 \subset J_3$. The set J_3 is said to be *in-between* J_1 and J_2.

3. SENSITIVITY TO LOGIC RELATIONS

The d_G distance, unlike the Hamming and Drastic distances, does capture the existence of semantic relations among the agenda issues: if in two judgment sets J_1 and J_2 the judgments on some issues cannot be changed without changing the judgements on other issues, then $d_G(J_1, J_2) < d_H(J_1, J_2)$. It can be shown that the d_G distance "recognises" when issues are logically equivalent. Namely, when an agenda contains logically equivalent issues, *e.g.*, $[\mathcal{A}] = \{p, q, p \rightarrow q, \neg p \vee q\}$, a disagreement on all logically equivalent issues is only counted as a disagreement on one issue. Thus, for example, a disagreement on both $p \rightarrow q, \neg p \vee q$ is counted as one disagreement by d_G.

The $d_G(J_1, J_2)$ is the number of minimal "hops" from judgment set to judgment set one needs to "traverse" to get from J_1 to J_2. We take this idea of counting only minimal steps further. We count not the minimal "hops" as a path from J_1 to J_2, but as the minimal number of judgments that need to be modified in J_1 to make J_1 the same as J_2. To this end, we introduce the concept of *prime implicants* in judgment aggregation and use it to define a similarity quantifier.

4. SIMILARITY VIA PRIME IMPLICANTS

A *prime implicant* [8] is a minimal subset, with respect to set inclusion, of a set of formulas, s.t. when the truth value of each formula in the prime implicant is chosen, the truth value of every other formula in the set is determined as well. Prime implicants have been used in many disciplines such as belief revision, automated reasoning, decision theory etc. We extend the concept of prime implicants to judgment sets.

DEFINITION 1. *Consider an agenda $\mathcal{A}$, a consistent, but possibly incomplete, set of judgements $J \subset \mathcal{D}(\mathcal{A})$, and an $I \subseteq J$. The set I is an implicant of J iff for every $\varphi \in J$ it holds that $(\bigwedge I) \rightarrow \varphi$. I is a* prime J-implicant *iff I is an implicant of J and there exists no $I' \subset I$ s.t. $(\bigwedge I') \rightarrow \varphi$ for every $\varphi \in J$. The minimal prime J-implicant is a prime J-implicant that, among all of the prime J-implicants, has the minimal cardinality. We denote the minimal prime J-implicant with $MPI(J)$.*

We now define a similarity quantifier d_{mpi} based on minimal prime implicants. Intuitively, $d_{mpi}(J_1, J_2)$ is the minimum number of judgments that need to be reversed in J_1 in order to transform J_1 into J_2.

DEFINITION 2. *The function $d_{mpi} : \mathbb{D}(\mathcal{A}) \times \mathbb{D}(\mathcal{A}) \rightarrow \mathbb{N}^0$ is defined, for every $\mathcal{A} \subset \mathcal{L}$, as $d_{mpi}(J_1, J_2) = |MPI(J_2 \backslash J_1)|$.*

It is easy to show that d_{mpi} is not symmetric and thus this function, unlike d_D, d_H, and d_G, is not a metric in the topological sense. However, we can still use it to build meaningful judgment aggregation operators. Using the d_{mpi} we obtain a unique judgment aggregation rule:

$$F_{mpi}(P) = F^{d_{mpi},\Sigma}(P) = \arg\min_{J \in \mathbb{D}(\mathcal{A})} \sum_{J_i \in P} d_{mpi}(J_i, J).$$

A collective judgment set can be seen as a consensual judgment set for the agents in the profile. This is our motivation for asking, not how much should the potential judgment set $J \in \mathbb{D}(\mathcal{A})$ change to match J_i, but how many of her positions would the agent i be actually forfeiting if J is the chosen consensus. Hence, J_i is the first argument in $d_{mpi}(J_i, J)$. Example 4.1 illustrates how F_{mpi} aggregates the well known profile from the doctrinal paradox, see *e.g.*, [6].

EXAMPLE 4.1. *Consider $[\mathcal{A}] = \{p, q, p \wedge q\}$, and the profile from the doctrinal paradox $P = \langle\{p, \neg q, \neg(p \wedge q)\}, \{\neg p, q, \neg(p \wedge q)\}, \{p, q, p \wedge q\}\rangle$. We obtain that $F_{mpi}(P) = \{\{p, q, p \wedge q\}\}$ with d_{mpi} distances 1, 1, and 0, respectively, from the judgment sets in P.*

It is not difficult to show, using counter examples, that F_{mpi} is different from the other judgment aggregation rules proposed in the literature, including the scoring rules based on minimal-entailment scoring in [1]. For instance, for the rules in [1], the doctrinal paradox suffices as a counter example for demonstrating the distinctness of F_{mpi}.

5. CONCLUSIONS

We introduced the concept of prime implicants in judgment aggregation and used it to construct a similarity quantifier for sets of judgments. The similarity quantifier $d_{mpi}(J_1, J_2)$ looks at the cardinality of the minimal prime implicant of $J_2 \backslash J_1$, instead of the whole $J_2 \backslash J_1$ like the Hamming and Drastic distances do. Examples of other quantifiers that can be defined using MPI are:

- $min(|MPI(J \backslash J_i)|, |MPI(J_i \backslash J)|)$,
- $max(|MPI(J \backslash J_i)|, |MPI(J_i \backslash J)|)$,
- $|MPI(J \backslash J_i)| + |MPI(J_i \backslash J)|$, etc.

Note that these functions are also symmetric and thus pseudometrics in the topological sense.

The definitions of the MPI-based quantifiers can be easily adjusted to accept any two sets of formulas, not only complete and consistent judgment sets, thus making these quantifiers applicable to other information aggregation problems like belief merging, merging of arguments etc. This is one of the directions of future work we aim to pursue, in addition to studying the properties of the judgment aggregation rules based on comparing minimal prime implicants.

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