Controlling Elections with Bounded Single-Peaked Width

Yongjie Yang
Universität des Saarlandes
Campus E 1.7, D-66123 Saarbrücken, Germany
yyongjie@mmci.uni-saarland.de

Jiong Guo
Universität des Saarlandes
Campus E 1.7, D-66123 Saarbrücken, Germany
jguo@mmci.uni-saarland.de

ABSTRACT

The problems of controlling an election have been shown $\mathcal{NP}$-complete in general but polynomial-time solvable in single-peaked elections for many voting correspondences. To explore the complexity border, we consider these control problems by adding/deleting votes in elections with bounded single-peaked width k. Single-peaked elections have single-peaked width $k = 1$. We prove that the constructive control problems for Copeland$^\alpha$ with $0 \leq \alpha < 1$ turn out to be $\mathcal{NP}$-hard even with $k = 2$, while for Copeland^1 and Maximin, the constructive control problems remain polynomial-time solvable with $k = 2$ but become $\mathcal{NP}$-hard with $k = 3$. In contrast, we show that the constructive control problems for Condorcet and weak Condorcet and the destructive control problems for Maximin and Copeland$^\alpha$ with $0 \leq \alpha \leq 1$ are all polynomial-time solvable with k being a constant; more precisely, these problems are fixed-parameter tractable ($\mathcal{FPT}$) with k as parameter. A byproduct of our results is that the Young winner determination problem is $\mathcal{FPT}$ with respect to k. Finally, for the class of voting correspondences passing the Smith-IIA criterion we provide a general characterization to identify voting correspondences whose control problems are $\mathcal{FPT}$ with respect to k.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity; G.2.1 [**Combinatorics**]: Combinatorial algorithms; J.4 [**Computer Applications**]: Social Choice and Behavioral Sciences

Keywords

single-peaked width; Condorcet; Copeland; Maximin; Smith set; parameterized complexity

1. INTRODUCTION

Strategic behaviors play a central role in the study of computational social choice. A control behavior normally involves an external agent (e.g., the chairman in an election) who wants to make a distinguished candidate win or lose an election by doing some tricks. It is called a *constructive control*, if the objective is to make someone win, and a *destructive control*, if the objective is to make someone lose. For each of constructive and destructive control behaviors, eleven standard control problems have been formulated. In both settings, one might add some extra votes or candidates to the election or delete some votes or candidates from the election. We refer to [10, 14] for more details.

The study of the computational complexity of the constructive control problems for diverse voting correspondences has been initialized by Bartholdi et al. [14]. Completing the results of Bartholdi et al., Hemaspaandra et al. [12] studied the destructive control problems. Various voting correspondences have been shown to admit $\mathcal{NP}$-hard control behaviors, for instance, the constructive control by adding/deleting votes for Condorcet [14], the constructive/destructive control by adding/deleting votes for Copeland$^\alpha$ [10], and the constructive/destrutive control by adding/deleting votes for Maximin [8].

Recently, a special model of elections has been introduced to the study of control behaviors, the so-called single-peaked elections [2]. In a single-peaked election, one can order the candidates from left to right such that for each voter, his/her preferences of candidates first increase and then decrease along this ordering. Restricted to single-peaked elections, the control problems for many voting correspondences become polynomial-time solvable (see [3, 11] for examples).

Based on these $\mathcal{NP}$-hardness and polynomial-time solvability results, it seems natural to explore the complexity border of these control problems from the general case to the single-peaked case under various voting correspondences. To this end, we adopt a newly introduced generalization of single-peaked elections, the so-called elections with bounded single-peaked width [5]. Other nearly single-peakedness concepts like k-maverick, k-global swaps, and k-candidate deletion have also been considered to cope with voting problems [4, 7, 9]. Intuitively, in an election with single-peaked width k, the candidates can be grouped together, where the size of each group is bounded by k, and for each group, every voter has the same preferences over all candidates in this group compared to candidates not in the group. Moreover, if considering each group as a candidate, the election is single-peaked. Clearly, single-peaked elections have a width equal to one. Cornaz et al. [5] first introduced single-peaked width to the complexity study of voting problems. In particular, they considered a multi-winner determination problem (the proportional representation problem) and proved that this problem is fixed-parameter tractable with the single-peaked width as the parameter. Later, Cor-

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naz et al. [6] showed that the Kemeny winner determination is fixed-parameter tractable with the single-peaked width as the parameter. Recall that a parameterized problem consists of instances of the form (I, k), where I denotes the main part and k is an integer. A parameterized problem is called fixed-parameter tractable ($\mathcal{FPT}$ for short) if it can be solved in $O(f(k) \cdot |I|^{o(1)})$ time, where f is a computable function. Clearly, an $\mathcal{FPT}$ problem is polynomial-time solvable for every constant value of k. Recently, many voting problems have been considered from the viewpoint of parameterized complexity, see [1] for an overview.

We mainly focus on the control behaviors with adding or deleting the votes. First, we study three concrete voting correspondences, namely, (weak) Condorcet, Copeland$^{\alpha}$, and Maximin. Concerning the constructive control problems, we achieved $\mathcal{NP}$-completeness for Copeland$^{\alpha}$ with $0 \leq \alpha < 1$ even with single-peaked width $k = 2$, while for Copeland^1 and Maximin, we show polynomial-time solvability with $k = 2$ but $\mathcal{NP}$-completeness with $k = 3$. In contrast, the constructive control problems for (weak) Condorcet turn out to be polynomial-time solvable for every fixed k. More precisely, we prove that for (weak) Condorcet, the constructive control problems are $\mathcal{FPT}$ with respect to the single-peaked width k. In the destructive control case, both Copeland$^{\alpha}$ for all $0 \leq \alpha \leq 1$ and Maximin behave in the same way, that is, for both correspondences, the destructive control problems are $\mathcal{FPT}$ with respect to the single-peaked width, implying polynomial-time solvability with every fixed k. Note that the destructive control problems for (weak) Condorcet are polynomial-time solvable, even in general (i.e., with unbounded k) [14].

In addition to these concrete voting correspondences, we provide a general characterization for a broad class of voting correspondences to identify the ones for which the control problems are $\mathcal{FPT}$ with respect to the single-peaked width k. The considered class contains all correspondences passing the Smith-IIA criterion. The *Smith set* in an election is a subset S of candidates with minimum size, such that every candidate in S is preferred by more voters than every candidate outside S. Clearly, every election has a unique Smith set. A voting correspondence *passes the Smith-IIA criterion* ("IIA" stands for "independence of irrelevant alternatives"), if deleting any candidate outside the Smith set does not change the winners. Several voting correspondences have been found passing the Smith-IIA criterion, for instance, Ranked pairs, Schulze's, and Kemeny. The characterization considers elections with odd number of votes and states that, if a control problem for a correspondence in the above class is $\mathcal{FPT}$ with the number of candidates as parameter, then the same holds for the single-peaked width being the parameter. This characterization applies to both constructive and destructive cases. We remark that all our results in this paper work for both unique-winner and nonunique-winner models. Due to lack of space, several proofs are deferred to the full version.

Elections: An *election* is a tuple $\mathcal{E} = (\mathcal{C}, \mathcal{V})$, where $\mathcal{C}$ is a set of candidates and $\mathcal{V}$ is a multiset of votes, each of which is defined as a linear order $\succ$ (to represent a voter's preference) over $\mathcal{C}$. For two candidates c, c' and a vote $\succ$, we say c is *ranked above* c' if $c \succ c'$. We use $N_{\mathcal{E}}(c, c')$ to denote the number of votes (here, we abuse the terminologies of votes and voters) ranking c above c' in $\mathcal{E}$. We drop the index $\mathcal{E}$ when it is clear from the context. For two candidates c and c', we say c *beats* c' if $N(c, c') > N(c', c)$, and c ties c' if $N(c, c') = N(c', c)$. A *voting correspondence*[1] φ is a function that maps an election $\mathcal{E} = (\mathcal{C}, \mathcal{V})$ to a non-empty subset $\varphi(\mathcal{E})$ of $\mathcal{C}$. We call the elements in $\varphi(\mathcal{E})$ the *winners* of $\mathcal{E}$. If $\varphi(\mathcal{E})$ contains only one winner, we call it a *unique winner*; otherwise, we call them *nonunique winners*. For an election $\mathcal{E} = (\mathcal{C}, \mathcal{V})$ and a subset $C \subset \mathcal{C}$, we use $\mathcal{E}|_C$ to denote the election restricted to C. Precisely, the restricted election $\mathcal{E}|_C$ has C as the candidate set, and the votes of $\mathcal{E}|_C$ are obtained from $\mathcal{E}$ by replacing each vote $\succ$ of $\mathcal{E}$ by a new vote $\succ'$ such that for every two candidates $a, b \in C$, $a \succ' b$ whenever $a \succ b$.

(Weak) Condorcet Winner: A *Condorcet winner* is a candidate which beats every other candidate. An election has either no Condorcet winner or only one Condorcet winner. A *weak Condorcet winner* is a candidate which is not beat by any other candidate. A voting correspondence is said to be weakCondorcet-consistent, if on every input that has at least one weak Condorcet winner, the winners, according to the voting correspondence, are exactly the set of weak Condorcet winners [3].

Single-Peaked Width: An election $(\mathcal{C}, \mathcal{V})$ is *single-peaked* if there is an order $\mathcal{L}$ of $\mathcal{C}$, from left to right, such that for every vote $\succ$ and every three candidates $a, b, c \in \mathcal{C}$ with $a \mathcal{L} b \mathcal{L} c$ or $c \mathcal{L} b \mathcal{L} a$, $c \succ b$ implies $b \succ a$, where $a \mathcal{L} b$ means a lies on the left-side of b in $\mathcal{L}$. We call $\mathcal{L}$ a *harmonious order*. See Fig. 1 for an example.

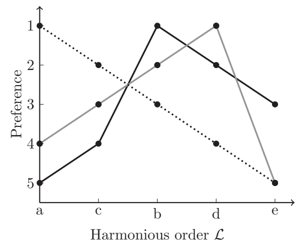

Figure 1: A single-peaked election with three votes $b \succ_u d \succ_u e \succ_u c \succ_u a$, $d \succ_v b \succ_v c \succ_v a \succ_v e$ and $a \succ_w c \succ_w b \succ_w d \succ_w e$. The votes $\succ_u$, $\succ_v$ and $\succ_w$ are illustrated by the dark line, the gray line, and the dotted line, respectively.

A subset $C \subseteq \mathcal{C}$ is called an *interval* if all candidates in C are ranked contiguously in every vote. For example, for the election with candidates $\{a, b, c, d, e\}$ and votes $\{a \succ_1 b \succ_1 c \succ_1 d \succ_1 e,\ d \succ_2 c \succ_2 b \succ_2 e \succ_2 a,\ a \succ_3 e \succ_3 b \succ_3 d \succ_3 c\}$, $\{b, c, d\}$ is an interval. *Contracting* an interval C is the operation that first adds a new candidate c' to the election such that $C \cup \{c'\}$ forms a new interval and the preference between any two candidates of $\mathcal{C}$ in each vote preserves the same as before, and then deletes all candidates in C. For example, after contracting the interval $\{b, c, d\}$ in the above example, we get the new election with candidates a, c', e and votes $\{a \succ_1 c' \succ_1 e,\ c' \succ_2 e \succ_2 a,\ a \succ_3 e \succ_3 c'\}$, where c' is the newly introduced candidate. Intuitively, contracting is to assign a new candidate to an interval which can represent the interval properly in the sense that the information of the preference between every candidate in the interval and every candidate outside the interval is preserved.

Let $P = (C_1, C_2, ..., C_\omega)$ be an ordered partition of $\mathcal{C}$ with each C_i being an interval. We say P is a *single-peaked partition* if contracting all intervals in P results in a single-peaked election with the harmonious order $(c_1, c_2, ..., c_\omega)$, where each c_i is the new candidate introduced for the in-

[1] A related terminology is *voting rule* which is defined as a function mapping an election to a single candidate. A voting correspondence can be easily modified to a voting rule using a certain tie-breaking method.

terval C_i. We say a vote *has its peak at C_i with respect to P* if the interval C_i is ranked above every other interval by the vote. The *width* of P is defined as $\max_{1 \le i \le \omega}\{|C_i|\}$. The *single-peaked width* of an election is the minimum width among all its single-peaked partitions.

Median Group: Let $P = (C_1, C_2, ..., C_\omega)$ be a single-peaked partition of the election $(\mathcal{C}, \mathcal{V})$, and let $(\succ_1, \succ_2, ..., \succ_n)$ be an order of $\mathcal{V}$ such that for $i < j$ the peak of $\succ_i$ does not lie on the right-side of the peak of $\succ_j$ in P. The set of all intervals lying between the peak C_i of $\succ_{\lceil n/2 \rceil}$ and the peak C_j of $\succ_{\lfloor n/2+1 \rfloor}$, together with C_i and C_j, denoted by $\mathcal{G}[C_i, C_j]$, is called the *median group*. If there is only one interval in the median group, we call it a *median interval*.

Voting Correspondences: We mainly study the following voting correspondences. For other voting correspondences mentioned in this paper, we refer to [1, 13].

Copeland$^\alpha$ ($0 \le \alpha \le 1$): For a candidate c, let $B(c)$ be the set of candidates who are beat by c and $T(c)$ the set of candidates who tie with c. The Copeland$^\alpha$ score of c is then defined as $|B(c)| + \alpha \cdot |T(c)|$. A Copeland$^\alpha$ winner is a candidate with the highest score.

Maximin The Maximin score of a candidate c is defined as $\min_{c' \in \mathcal{C} \setminus \{c\}} N(c, c')$. A Maximin winner is a candidate having the highest Maximin score.

Problem Definitions: Problems studied here are characterized by four factors, CC|DC specifying constructive or destructive control, AV|DV specifying adding or deleting votes, φ specifying the voting correspondence, and UNI|NON specifying the unique-winner or nonunique-winner models. For example, CCAV-φ-UNI denotes the problem of constructive control by adding votes for the unique-winner model under the voting correspondence φ. In the inputs of all these problems, we have a set $\mathcal{C}$ of candidates, a distinguished candidate p, and an integer $t \ge 0$. In the deleting votes case, there is only one multiset $\mathcal{V}_1$ of (registered) votes in the input, while the adding votes case distinguishes two multisets of votes, $\mathcal{V}_1$ the multiset of registered votes and $\mathcal{V}_2$ the multiset of unregistered votes. The goal here is to make p win (CC) or lose (DC) the election by adding at most t unregistered votes (AV) or deleting at most t votes (DV). Strictly speaking, (weak) Condorcet is not a voting correspondence, since the winner set could be empty. However, the control problems have been studied for (weak) Condorcet since the seminal paper by Bartholdi et al. [14]. We also include it here due to the importance of Condorcet elections.

All $\mathcal{NP}$-hardness reductions in this paper are from the following $\mathcal{NP}$-complete problem [15].

Exact 3 Set Cover (X3C)
Input: A universal set $U = \{c_1, c_2, ..., c_{3t}\}$ and a collection S of 3-subsets of U.
Question: Is there an $S' \subseteq S$ such that $|S'| = t$ and each $c_i \in U$ appears in exactly one set of S'?

2. CONDORCET AND WEAK CONDORCET

The constructive control problems by adding/deleting votes for (weak) Condorcet are $\mathcal{NP}$-hard in the general case [14] but turned out to be polynomial-time solvable when restricted to single-peaked elections [3]. On the other hand, the problems of destructive control by adding/deleting votes are polynomial-time solvable even in the general case [14]. In this section, we study constructive control by adding/deleting votes in Condorcet and weak Condorcet, restricted to elections with bounded single-peaked width. We prove that both problems are polynomial-time solvable if the single-peaked width is a constant. From the perspective of the parameterized complexity, our results indeed show that these problems are $\mathcal{FPT}$. The following observations are useful.

Observation 1. *Every two candidates from different intervals in the median group are tied.*

Proof. Let $(C_1, C_2, ..., C_\omega)$ be the single-peaked partition and $\mathcal{G}[C_l, C_r]$ be the median group. Let C_i and C_j be two arbitrary intervals in $\mathcal{G}[C_l, C_r]$ with $i < j$, and $c \in C_i, c' \in C_j$ be two candidates. Due to the definition of median group, all votes with peaks at C_l or on the left-side of C_l (let $\mathcal{V}_l$ denote the multiset of these votes) prefer c to c', and all votes with peaks at C_r or on the right-side of C_r (let $\mathcal{V}_r$ denote the multiset of these votes) prefer c' to c. Moreover, the size of $\mathcal{V}_l$ is equal to the size of $\mathcal{V}_r$. Therefore, c ties c'. □

Observation 2. *Every weak Condorcet winner is from the median group.*

Proof. This observation is correct since every candidate which is not in the median group is beat by at least one candidate in the median group. More precisely, suppose that c is a candidate contained in an interval lying on the right-side (resp. left-side) of the median group, then every candidate in C_r (resp. C_l) beats c, where C_l and C_r are the left boundary and the right boundary of the median group, respectively. □

Observation 3. *If an election $\mathcal{E}$ has a Condorcet winner, then the median group contains exactly one interval.*

Proof. Suppose that the median group $\mathcal{G}$ contains more than one interval. Due to Observation 1, every candidate in the median group ties at least one candidate in a different interval in the median group, and thus, the Condorcet winner cannot exist. □

In the following, "modifiable" votes refer to the registered votes in the case of control by deleting votes, and refer to the unregistered votes in the case of control by adding votes. For two subsets of candidates C and C' with $C \subseteq C'$, we say two votes $\succ_1$ *and* $\succ_2$ *are consistent with respect to* C and C' if they have the same preference over C and for every two candidates $c \in C$ and $c' \in C' \setminus C$, $c \succ_1 c'$ if and only if $c \succ_2 c'$.

Theorem 1. *The constructive control problems by adding or deleting votes for both Condorcet and weak Condorcet are $\mathcal{FPT}$ with respect to the single-peaked width k.*

Proof. We first consider for the Condorcet voting. We give only details for control by adding votes. The case of control by deleting votes is similar. Let $\mathcal{V}_1$ be the multiset of registered votes and $\mathcal{V}_2$ be the multiset of the unregistered votes. Let C_p be the interval containing p. Due to Observation 3, to make p the Condorcet winner we need to make the interval C_p the median interval and to make p beat all the other candidates in C_p. To this end, we first divide the unregistered votes $\mathcal{V}_2$ into three sets: X containing the votes with peaks on the left-side of C_p with respect to the single-peaked partition, Y the votes with peaks on the right-side of C_p, and Z the votes with peaks at C_p. Then, we further divide each of these three sets into at most 2^{k-1} subsets, each containing the votes which are pairwise consistent with respect to $\{p\}$ and C_p. By assigning to each subset a variable (indicating how many votes from this subset are in the solution), the problem is reduced to an ILP problem which can be solved in $\mathcal{FPT}$ time based on the Lenstra's theorem [16].

Let $\bar{x}, \bar{y}$ and $\bar{z}$ be the numbers of votes in $\mathcal{V}_1$ with peaks on the left-side of C_p with respect to the single-peaked partition, with peaks on the right-side of C_p, and with peaks at C_p, respectively. We will use x_β, y_β and z_β to denote the variables assigned to the subsets of X, Y and Z, respectively, where β is a subset of $C_p \setminus \{p\}$. Here, for each β, x_β (y_β, z_β) is assigned to the subset of X (Y, Z), which contains votes ranking every candidate of β above p and ranking every candidate not in β below p. Firstly, the ILP has the following constraints:

(1) $\bar{x} + \sum_\beta x_\beta - \bar{y} - \sum_\beta y_\beta = 0$

(2) $\sum_\beta z_\beta + \bar{z} > 0$

(3) $\sum_\beta (x_\beta + y_\beta + z_\beta) \leq t$

Here, (1) ensures that C_p is in the median group, (2) ensures that the median group contains only the interval C_p and (3) states that at most t votes are added. Then, for every $c \in C_p \setminus \{p\}$, there is a constraint:

$$N(p,c) + \sum_{c \notin \beta}(x_\beta + y_\beta + z_\beta) - N(c,p) - \sum_{c \in \beta}(x_\beta + y_\beta + z_\beta) > 0$$

where $N(.)$ is based on the registered votes $\mathcal{V}_1$.

These inequalities ensure that p beats every candidate in $C_p \setminus \{p\}$. Since we formulate the control problems as decision problems, there is no optimization function in the ILP.

Now we consider the weak Condorcet voting. Due to Observations 1 and 2, to make a candidate p a weak Condorcet winner, we have to make the interval C_p be included in the median group and to make p the weak Condorcet winner among the candidates in C_p. Here, again, we can use ILP to solve this problem by dividing the modifiable votes into different subsets, each containing "similar" votes, in the sense that all votes in each subset have their peaks either on the same side of C_p or at C_p and further, all votes in each subset are consistent with respect to $\{p\}$ and C_p. $\square$

Due to Theorem 1, we can directly get the following result for the Young winner determination problem which is $\mathcal{P}_{||}^{\mathcal{NP}}$-complete in general [19]. In an Young election, each candidate c has a *Young score* defined as the minimum number of votes to be deleted to make c the Condorcet winner. A *Young winner* is a candidate with the least Young score. The Young winner determination problem can be reduced to the problem of deciding whether a distinguished candidate can be made a Condorcet winner by deleting t votes, equivalent to the control problem by deleting votes for Condorcet.

Corollary 2. *Young winner determination is $\mathcal{FPT}$ with respect to the single-peaked width.*

3. COPELAND$^\alpha$

In this section, we study the control problems for Copeland$^\alpha$. Our results are summarized in Table 1. In particular, we prove that the constructive control problems by adding/deleting votes are $\mathcal{NP}$-complete for Copeland$^\alpha$ for every $0 \leq \alpha < 1$ but polynomial-time solvable for Copeland^1, when restricted to elections with single-peaked width 2. Moreover, we prove that the same problems become $\mathcal{NP}$-complete for Copeland^1 when restricted to elections with single-peaked width 3. In the contrast, the destructive control problems by adding/deleting votes for Copeland$^\alpha$ for all $0 \leq \alpha \leq 1$ turn out to be $\mathcal{FPT}$. Recall that the constructive/destructive control problems by adding/deleting votes are all $\mathcal{NP}$-hard for Copeland$^\alpha$ for all $0 \leq \alpha \leq 1$ [10].

	Single-peaked width		
	2	3	k
CCAV	$\mathcal{NP}$-c: $0 \leq \alpha < 1$ $\mathcal{P}$: $\alpha = 1$	$\mathcal{NP}$-c: $0 \leq \alpha \leq 1$	
CCDV			
DCAV	$\mathcal{P}$		$\mathcal{FPT}$
DCDV			

Table 1: Complexity of constructive/destructive control by adding/deleting votes in Copeland$^\alpha$. Here, "$\mathcal{NP}$-c" stands for $\mathcal{NP}$-complete and "$\mathcal{P}$" stands for polynomial-time solvable.

Theorem 3. *The constructive control problems by adding or deleting votes for Copeland$^\alpha$ for every $0 \leq \alpha < 1$ are $\mathcal{NP}$-complete when restricted to elections with single-peaked width 2, for both unique-winner model and nonunique-winner model.*

Proof. We provide only the proof for CCAV-Copeland$^\alpha$-UNI. The problem is clearly in $\mathcal{NP}$. In the following we prove the $\mathcal{NP}$-hardness. Let $\mathcal{E} = (U = \{c_1, c_2, ..., c_{3t}\}, S)$ be an instance of X3C. We construct an instance $\mathcal{E}'$ for CCAV-Copeland$^\alpha$-UNI restricted to elections with single-peaked width 2 as follows.

Candidates: There are totally $6t{+}2$ candidates. More specifically, for each $c_x \in U$ we create two corresponding candidates c'_x and c''_x which form an interval denoted by $I(c_x)$ in the election. In addition, we have two candidates p and p' which form an interval $I(p)$. The distinguished candidate is p.

Single-Peaked Partition: $(I(p), I(c_1), I(c_2), ..., I(c_{3t}))$.

Registered Votes: There are $t-1$ registered votes defined as $c'_{3t} \succ c''_{3t} \succ c'_{3t-1} \succ c''_{3t-1} \succ, ..., \succ p' \succ p$. In addition, there is one vote defined as $c''_{3t} \succ c'_{3t} \succ c''_{3t-1} \succ c'_{3t-1} \succ, ..., \succ p \succ p'$. Clearly, with the registered votes, p has Copeland$^\alpha$ score 0, p' has Copeland$^\alpha$ score 1, each c'_x has Copeland$^\alpha$ score $2x+1$, and each c''_x has Copeland$^\alpha$ score $2x$.

Unregistered Votes: The unregistered votes are created according to S. More precisely, for each $s = \{c_i, c_j, c_k\} \in S$, we create a vote $\succ_s$ defined as follows: the peak of the vote $\succ_s$ is at the interval $I(p)$ and $\succ_s$ prefers p to p'. For every two candidates $a \in I(c_x)$ and $b \in I(c_y)$ with $x < y$, we have $a \succ_s b$. Finally, in each interval $I(c_x)$, $\succ_s$ prefers c'_x to c''_x if $x \in \{i, j, k\}$ and prefers c''_x to c'_x otherwise.

In the following, we show that $\mathcal{E}$ has an exact 3-set cover if and only if we can add at most t unregistered votes to make p the unique winner.

($\Rightarrow$:) Let S' be an exact 3-set cover of $\mathcal{E}$. We claim that adding all votes corresponding to S', that is, the votes $\mathcal{V}' = \{\succ_s \mid s \in S'\}$, will make p the unique winner. It is clear that p beats p' after adding all votes in $\mathcal{V}'$ to the election. Since there are exactly t votes with peaks at $I(p)$ and exactly t votes with peaks at $I(c_{3t})$ after adding $\mathcal{V}'$ to the election, every two candidates which are in different intervals are tied. Therefore, p has Copeland$^\alpha$ score $6\alpha \cdot t + 1$ and p' has Copeland$^\alpha$ score $6\alpha \cdot t$ after adding all votes in $\mathcal{V}'$ to the election. We now analyze the Copeland$^\alpha$ score of other candidates. Let c'_x and c''_x be the two candidates in an interval $I(c_x)$ with $1 \leq x \leq 3t$. Since S' is an exact 3-set cover, due to the construction, there is exactly one vote in $\mathcal{V}'$ which prefers c'_x to c''_x and thus exactly $t-1$ votes in $\mathcal{V}'$ which prefer c''_x to c'_x. Together with the registered votes, c'_x ties c''_x. Since each of c'_x and c''_x ties all other candidates, as stated above, the Copeland$^\alpha$ score of c'_x and c''_x are both $\alpha \cdot (6t+1)$. Since $\alpha < 1$, p is the unique winner.

($\Leftarrow$:) Let $\mathcal{V}'$ be a solution for $\mathcal{E}'$ and S' be the subset of S corresponding to $\mathcal{V}'$, that is, $S' = \{s \mid \succ_s \in \mathcal{V}'\}$. It is easy to see that $\mathcal{V}'$ contains exactly t votes, since otherwise, one of c'_{3t} and c''_{3t} would beat all the other candidates and thus be a winner. Moreover, since all unregistered votes have their peaks at $I(p)$, every two candidates from different intervals are tied in the final election. Since all unregistered votes prefer p to p', the Copeland$^\alpha$ score of p is $6\alpha \cdot t + 1$. Since p is the unique winner after adding all votes in $\mathcal{V}'$ to the election, c'_x ties c''_x for all $1 \leq x \leq 3t$ (otherwise, at least one of c'_x and c''_x would have a Copeland$^\alpha$ score of $6\alpha \cdot t + 1$, contradicting that p is the unique winner). Then, according to the construction, for each c_x there is exactly one vote in $\mathcal{V}'$ preferring c'_x to c''_x. This implies that S' contains exactly one subset containing c_x; Thus, S' forms an exact 3-set cover of $\mathcal{E}$. $\square$

In the following, we study the control problems for Copeland^1. We first consider elections with single-peaked width 2. Observe that every election with single-peaked width 2 contains at least one weak Condorcet winner. More precisely, each interval in the median group contains at least one weak Condorcet winner. Note that every candidate in the median group beats or ties every candidate not in the median group. Furthermore, since Copeland^1 is weakCondorcet-consistent and the constructive control problems by adding/deleting votes are polynomial-time solvable for (weak) Condorcet when restricted to elections with single-peaked width 2, as implied by Theorem 1, the constructive control problems by adding/deleting votes for Copeland^1 are polynomial-time solvable in elections with single-peaked width 2. We remark that Copeland$^\alpha$ for every $0 \leq \alpha < 1$ is not weakCondorcet-consistent even when restricted to single-peaked elections [3], and thus, the following theorem does not work for $0 \leq \alpha < 1$.

Theorem 4. *The constructive control problems by adding or deleting votes for Copeland^1 are polynomial-time solvable when restricted to elections with single-peaked width 2, for both unique-winner and nonunique-winner models.*

Now we consider the problems restricted to elections with single-peaked width 3. In contrast to the polynomial-time solvability as stated in Theorem 4, we show that the constructive control problems become $\mathcal{NP}$-complete in elections with single-peaked width 3. We remark that, even though Copeland^1 is weakCondorcet-consistent, the argument for Theorem 4 does not hold in this case since there may not be a weak Condorcet winner in elections with single-peaked width 3.

Theorem 5. *The constructive control problems by adding or deleting votes for Copeland^1 are $\mathcal{NP}$-complete when restricted to elections with single-peaked width 3, for both unique-winner and nonunique-winner models.*

Proof. Clearly, the problems are in $\mathcal{NP}$. We prove $\mathcal{NP}$-hardness by reductions from X3C. We start with CCAV-Copeland^1-UNI.

Let $\mathcal{E} = (U = \{c_1, c_2, ..., c_{3t}\}, S)$ be an instance of X3C. We assume that $t \equiv 0 \bmod 6$. If $t \not\equiv 0 \bmod 6$, we can add some dummy elements to U and some 3-subsets to S which form an exact 3-set cover of the dummy elements. We construct an instance $\mathcal{E}'$ for CCAV-Copeland^1-UNI restricted to elections with single-peaked width 3 as follows.

Candidates: There are totally $9t+3$ candidates. More specifically, for each $c_x \in U$ we create three candidates c_x^1, c_x^2 and c_x^3 which form an interval denoted by $I(c_x)$. In addition, we have three candidates p, p' and p'' which form an interval $I(p)$. The distinguished candidate is p.

Single-Peaked Partition: $(I(p), I(c_1), I(c_2), ..., I(c_{3t}))$.

Registered Votes: There are $\frac{7}{3}t$ registered votes. In particular, we have

(1) $\frac{5}{6}t$ votes defined as

$$c_{3t}^1 \succ c_{3t}^2 \succ c_{3t}^3 \succ c_{3t-1}^1 \succ c_{3t-1}^2 \succ c_{3t-1}^3 \succ, ..., \succ p \succ p' \succ p''$$

(2) $\frac{5}{6}t$ votes defined as

$$c_{3t}^2 \succ c_{3t}^3 \succ c_{3t}^1 \succ c_{3t-1}^2 \succ c_{3t-1}^3 \succ c_{3t-1}^1 \succ, ..., \succ p \succ p' \succ p''$$

(3) $\frac{2}{3}t$ votes defined as

$$p \succ p' \succ p'' \succ c_1^3 \succ c_1^1 \succ c_1^2 \succ, ..., \succ c_{3t}^3 \succ c_{3t}^1 \succ c_{3t}^2$$

Clearly, with the registered votes, p has Copeland^1 score 2, p' has Copeland^1 score 1, p'' has Copeland^1 score 0, and each c_x^γ with $1 \leq x \leq 3t$ and $\gamma = 1, 2, 3$ has Copeland^1 score $3x + 1$.

Unregistered Votes: We create the unregistered votes according to S. Precisely, for each $s = \{c_i, c_j, c_k\} \in S$, we create a vote $\succ_s$ defined as follows: the peak of the vote $\succ_s$ is at $I(p)$ and $\succ_s$ prefers p to p' to p''. For every two candidates $a \in I(c_x)$ and $b \in I(c_y)$ with $x < y$, we have $a \succ_s b$. Finally, in each interval $I(c_x)$, we set $c_x^2 \succ c_x^3 \succ c_x^1$ if $x \in \{i, j, k\}$ and set $c_x^3 \succ c_x^1 \succ c_x^2$ otherwise.

The $\mathcal{NP}$-hardness reduction for CCAV-Copeland^1-NON can be modified from the above construction by deleting the candidate p'' from the election.

In the following, we show the $\mathcal{NP}$-hardness reduction of CCDV-Copeland^1-UNI from X3C. For each $c \in U$, let $o(c)$ be the number of sets in S which contain c and let $\bar{o}(c)$ be the number of sets in S which do not contain c. We assume that $o(c) \geq 3$ and $\bar{o}(c) \geq t-1$ for all $c \in U$ and $|S| \geq t+2$. For a given instance $\mathcal{E} = (U = \{c_1, c_2, ..., c_{3t}\}, S)$ of X3C, we construct an instance $\mathcal{E}'$ for CCDV-Copeland^1-UNI restricted to elections with single-peaked width 3 as follows. The candidate set and the single-peaked partition are the same as for CCAV-Copeland^1-UNI.

Votes: There are totally $2|S| - t$ votes with $|S| - t$ votes with peaks at $I(p)$ and all the other $|S|$ votes (corresponding to S) with peaks at $I(c_{3t})$. The central idea is to construct the votes in such way that all deleted votes have peaks at $I(c_{3t})$ whenever $\mathcal{E}'$ is a true-instance. Furthermore, after deleting these votes, each candidate, except p, is beat by at least another candidate which is from the same interval as itself.

We first create the votes corresponding to S. For each $s = \{c_i, c_j, c_k\} \in S$, we create a vote $\succ_s$ with peak at $I(c_{3t})$ and with preference $p \succ p' \succ p''$. For every two candidates $a \in I(c_x)$ and $b \in I(c_y)$ with $x < y$, we have that $b \succ_s a$. With regard to the preference in each $I(c_x)$ with $1 \leq x \leq 3t$, we set $c_x^2 \succ c_x^3 \succ c_x^1$ if $x \in \{i, j, k\}$ and set $c_x^3 \succ c_x^1 \succ c_x^2$ otherwise. Thus, there are $o(c_x)$ votes with preference $c_x^2 \succ c_x^3 \succ c_x^1$ and $\bar{o}(c_x)$ votes with preference $c_x^3 \succ c_x^1 \succ c_x^2$.

We now construct the votes with peaks at $I(p)$. There are totally $|S| - t$ such votes, all of which prefer p to p' to p''. Since all these votes have their peaks at $I(p)$, the preference between every two candidates $a \in I(c_x)$ and $b \in (c_y)$ with $x < y$ is $a \succ b$. Concerning the preference in each interval $I(c_x)$, we set $c_x^1 \succ c_x^2 \succ c_x^3$ in $\frac{1}{2} \cdot o(c_x)$ arbitrary votes. In the remaining votes, we set $c_x^2 \succ c_x^3 \succ c_x^1$ in $\bar{o}(c_x) - t$ many of them and set $c_x^3 \succ c_x^1 \succ c_x^2$ in the rest. Clearly, $|S| - t - \frac{1}{2} \cdot o(c_x) - \bar{o}(c_x) + t = \frac{1}{2} \cdot o(c_x)$.

In summary, for each c_x, there are totally $\frac{1}{2} \cdot o(c_x)$ votes with preference $c_x^1 \succ c_x^2 \succ c_x^3$, $|S| - t$ votes with preference $c_x^2 \succ c_x^3 \succ c_x^1$ and $\frac{1}{2} \cdot o(c_x) + \bar{o}(c_x)$ votes with preference $c_x^3 \succ c_x^1 \succ c_x^2$. Moreover, all votes prefer p to p' to p''.

The proof for CCDV-Copeland^1-NON can be modified from the construction for CCDV-Copeland^1-UNI by deleting the candidate p' from the constructed election. $\square$

Now, we discuss the destructive control problems by adding/deleting votes for Copeland$^\alpha$. In contrast to the $\mathcal{NP}$-completeness of constructive control by adding/deleting votes in Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$ when restricted to elections with single-peaked width 3, we show that the destructive counterparts can be solved in polynomial time, if the single-peaked width is bounded by a constant. More precisely, from the parameterized complexity perspective, we prove that the destructive control problems by adding/deleting votes for Copeland$^\alpha$ with $0 \leq \alpha \leq 1$ are $\mathcal{FPT}$ with respect to the single-peaked width. Recall that all these problems are $\mathcal{NP}$-hard in the general case [10].

Theorem 6. *The destructive control problems by adding or deleting votes for Copeland$^\alpha$ for all $0 \leq \alpha \leq 1$ are $\mathcal{FPT}$ with the single-peaked width k as the parameter, for both unique-winner and nonunique-winner models.*

4. MAXIMIN

In this section, we focus on control problems for Maximin. All the constructive/destructive control problems by adding/deleting votes are $\mathcal{NP}$-complete for Maximin in general [8]. Moreover, all these problems are $\mathcal{W}[1]$-hard with respect to the number of added/deleted votes as the parameter in the general case [17]. Our main results of this section are summarized in Table 2. Even though Maximin and Copeland^1 are two different voting correspondences, our results show that the complexity of the control problems studied in this paper for Maximin behave in the same way as for Copeland^1.

The next theorem follows from the facts that (1) Maximin is weakCondorcet-consistent [3]; (2) there is at least one weak Condorcet winner in every election with single-peaked width 2; and (3) the constructive control problems by adding/deleting votes for (weak) Condorcet are polynomial-time solvable in elections with single-peaked width 2 (implied by Theorem 2).

	Single-peaked width 2	Single-peaked width 3	Single-peaked width k
CCAV	$\mathcal{P}$	$\mathcal{NP}$-c	$\mathcal{NP}$-c
CCDV	$\mathcal{P}$	$\mathcal{NP}$-c	$\mathcal{NP}$-c
DCAV	$\mathcal{P}$	$\mathcal{P}$	$\mathcal{FPT}$
DCDV	$\mathcal{P}$	$\mathcal{P}$	$\mathcal{FPT}$

Table 2: Complexity of the contructive/destructive control problems by adding/deleting votes for Maximin.

Theorem 7. *The constructive control problems by adding or deleting votes for Maximin are polynomial-time solvable when restricted to elections with single-peaked width 2, for both unique-winner and nonunique-winner models.*

Then, we consider single-peaked width 3.

Theorem 8. *The constructive control problems by adding or deleting votes for Maximin are $\mathcal{NP}$-complete when restricted to elections with single-peaked width 3, for both unique-winner and nonunique-winner models.*

Proof. Clearly, both problems are in $\mathcal{NP}$. In the following, we only prove the $\mathcal{NP}$-hardness of CCAV-Maximin-UNI. Given an instance $\mathcal{E} = (U = \{c_1, c_2, ..., c_{3t}\}, S)$ of X3C, we construct an instance $\mathcal{E}'$ as follows.

Candidates: For each $c_x \in U$, we create three candidates c_x^1, c_x^2, c_x^3 which form an interval denoted by $I(c_x)$. In addition, we have three candidates p, p' and p'' which form an interval denoted by $I(p)$. The distinguished candidate is p.

Single-Peaked Partition: $(I(p), I(c_1), I(c_2), ..., I(c_{3t}))$.

Registered Votes: Let η be an integer with $\eta \geq 3t$ and $\eta \equiv 0 \bmod 3$. We create $2\eta + 1$ registered votes. Precisely, we have

(1) $\frac{2}{3} \cdot \eta - t + 1$ votes defined as

$$p \succ p' \succ p'' \succ c_1^1 \succ c_1^2 \succ c_1^3 \succ, ..., \succ c_{3t}^1 \succ c_{3t}^2 \succ c_{3t}^3$$

(2) t votes defined as

$$p' \succ p \succ p'' \succ c_1^1 \succ c_1^2 \succ c_1^3 \succ, ..., \succ c_{3t}^1 \succ c_{3t}^2 \succ c_{3t}^3$$

(3) $\frac{1}{3}\eta$ votes defined as

$$p' \succ p'' \succ p \succ c_1^2 \succ c_1^3 \succ c_1^1 \succ, ..., \succ c_{3t}^2 \succ c_{3t}^3 \succ c_{3t}^1$$

(4) $\frac{1}{3}\eta$ votes defined as

$$c_{3t}^2 \succ c_{3t}^3 \succ c_{3t}^1 \succ, ..., c_1^2 \succ c_1^3 \succ c_1^1 \succ p' \succ p'' \succ p$$

(5) $\frac{2}{3}\eta$ votes defined as

$$c_{3t}^3 \succ c_{3t}^1 \succ c_{3t}^2 \succ, ..., c_1^3 \succ c_1^1 \succ c_1^2 \succ p'' \succ p \succ p'$$

It is easy to verify that p' is the current unique winner.

Unregistered Votes: For each $s = \{c_i, c_j, c_k\} \in S$, we create a vote $\succ_s$ with peak at $I(p)$. Moreover, in the interval $I(p)$, we set $p \succ_s p' \succ_s p''$. For every $I(c_x)$, we set $c_x^2 \succ_s c_x^3 \succ_s c_x^1$ if $x \in \{i, j, k\}$, and $c_x^1 \succ_s c_x^2 \succ_s c_x^3$ otherwise.

In the following, we prove that $\mathcal{E}$ is a true-instance if and only if $\mathcal{E}'$ is a true-instance.

($\Rightarrow$:) Let S' be a solution for $\mathcal{E}$. We claim that the unregistered votes corresponding to S', that is, $\mathcal{V}' = \{\succ_s \mid s \in S'\}$ form a solution for $\mathcal{E}'$. Since S' is an exact 3-set cover, for each $I(c_x)$, there is exactly one vote in $\mathcal{V}'$ with preference $c_x^2 \succ c_x^3 \succ c_x^1$ and exactly $t - 1$ votes with preference

$c_x^1 \succ c_x^2 \succ c_x^3$. Then, it is easy to calculate that, after adding all votes in $\mathcal{V}'$ to the election, c_x^1 has the highest Maximin score $\frac{2}{3}\eta+t$ among all candidates in $I(c_x)$ for all $0 \leq x \leq 3t$. Moreover, since all unregistered votes prefer p to p' to p'', p has the highest Maximin score $\frac{2}{3}\eta + t + 1$ among all candidates in $I(p)$, which is also the highest Maximin score among all candidates in the final election. Hence, p becomes the unique winner.

($\Leftarrow$:) Let $\mathcal{V}'$ be a solution for $\mathcal{E}'$. We claim that the subset S' corresponding to $\mathcal{V}'$ is an exact 3-set cover for $\mathcal{E}$. We first observe that $\mathcal{V}'$ contains exactly t votes, since otherwise, p' would have a Maximin score not less than p. Since all unregistered votes prefer p to p' to p'', p has a final Maximin score $\frac{2}{3}\eta+t+1$. Therefore, for every $c_x \in U$, there is at least one vote $\succ_s$ in $\mathcal{V}'$ with $c_x^2 \succ c_x^3 \succ c_x^1$, since otherwise, c_x^1 would have a Maximin score not less than p. Since U contains exactly $3t$ elements and every unregistered vote contains for three different c_x preference $c_x^2 \succ c_x^3 \succ c_x^1$, S' must be an exact 3-set cover for $\mathcal{E}$.

The reduction for the nonunique-winner model is the same as the above reduction with only the difference that in the registered votes, we have one less vote in the first type of votes. □

In contrast to the $\mathcal{NP}$-completeness of constructive control, the destructive control case turns out to be fixed-parameter tractable.

THEOREM 9. *The destructive control problems by adding or deleting votes for Maximin are $\mathcal{FPT}$ with respect to the single-peaked width, for both unique-winner and nonunique-winner models.*

5. A GENERAL THEOREM

In this section, we consider elections containing an odd number of votes. Elections with an odd number of votes have been studied in different context [3, 11, 18, 21]. In such elections, there is no tie, while comparing two candidates. In addition, several theorems have been achieved for such elections, for example, see page 5 for May's theorem, page 234 for Sen's theorem and page 239 for Black's theorem in [21]. Especially, the Black's theorem implies that, in an odd-votes election, the Condorcet winner always exists in the single-peaked case. Moreover it must be the top candidate of the median vote. The following lemma implies that with the odd-votes elections, the Smith set must be included in the median interval. Observe that if the number of votes is odd, the median group contains exactly one interval.

LEMMA 10. *For every election with the median group containing only one interval, the median interval is a superset of the Smith set.*

Our main contribution of this section is a general theorem which can be used to derive $\mathcal{FPT}$ results for the constructive/destructive control problems by adding/deleting votes in odd-votes elections, that is, adding/deleting votes resulting in elections with odd number of votes.

THEOREM 11. *For an odd-votes election with a voting correspondence passing the Smith-IIA criterion, if a constructive/destructive control problem by adding or deleting votes is $\mathcal{FPT}$ with the number of candidates as parameter, then the same problem is also $\mathcal{FPT}$ with single-peaked width as parameter. This claim holds for both unique-winner and nonunique-winner models.*

PROOF. We only give the proof for the constructive control problems. Since there are odd number of votes in the final election, the median group contains only one interval (the median interval). Due to Lemma 10, all voting correspondences passing the Smith-IIA criterion always select winners from the median interval. Thus, to solve the problems stated in the theorem, we have two objectives. One is to make the interval C_i containing the distinguished candidate p the median interval. The other objective is then to make p a winner. By the Smith-IIA criterion, we can focus on the election restricted to C_i, once the first objective has been reached. These observations motivate us to propose a general reduction rule, which significantly shrinks the size of the candidate set. The main idea of the reduction rule is to replace the "irrelevant candidates" by only two candidates x and y, where $\{x\}$ will be an interval and be placed on the left-side of C_i, and $\{y\}$ will also be an interval and be placed on the right-side of C_i in the single-peaked partition P. The role of the two candidates is to store the information of the peaks of all votes. More precisely, the reduction rule replaces each vote with a new vote containing only the candidates $C_i \cup \{x, y\}$. In particular, if a vote has its peak on the left-side (resp. right-side) of C_i, the new vote will have its peak at $\{x\}$ (resp. $\{y\}$). If the vote has its peak at C_i, the new vote will also have its peak at C_i. In all three cases, the new vote preserves the preference of the original one over the candidates in C_i. A formal description of the reduction rule is as follows.

Reduction Rule. Let $\mathcal{E} = (\mathcal{C}, \mathcal{V})$ be an election and $P = (C_1, C_2, ..., C_i, ..., C_\omega)$ be a single-peaked partition of $\mathcal{E}$. We do the following operations (for the control problem by adding votes, the operations should also be implemented on the unregistered votes) to get a new election $\mathcal{E}'$.

1. Add two new intervals $C_0 = \{x\}$ and $C_{\omega+1} = \{y\}$ such that C_0 is in the leftmost position of P and $C_{\omega+1}$ is in the rightmost position of P;
2. Replace each vote $\succ$ whose peak is on the left-side (resp. right-side) of C_i with a new vote $\succ'$ defined as $x \succ' (\succ|_{C_i}) \succ' y$ (resp. $y \succ' (\succ|_{C_i}) \succ' x$);
3. Replace each vote $\succ$ whose peak is at C_i with a new vote $\succ'$ defined as $(\succ|_{C_i}) \succ' x \succ' y$;
4. Delete all intervals except C_i, C_0 and $C_{\omega+1}$.

It is clear that the single-peaked width of the resulting election $\mathcal{E}'$ is bounded by k. After applying the reduction rule, each instance contains at most $k+2$ candidates. If a control problem can be solved in $O(f(m) \cdot |\mathcal{E}|^{O(1)})$ time for m being the number of candidates, then it admits an $O(f(k) \cdot |\mathcal{E}|^{O(1)})$-time algorithm as well. The correctness of Theorem 11 follows. □

Theorem 11 requires that the voting correspondence must pass the Smith-IIA criterion and the considered problems must be $\mathcal{FPT}$ with respect to the number of candidates as the parameter. At first glance, it seems that the conditions, especially the second one, are very restrictive. However, we show several voting correspondences which satisfy both conditions.

The following voting correspondences pass the Smith-IIA criterion [20]: Ranked Pairs, Schulze's, Copeland$^\alpha$, Condorcet, Kemeny, and Slater's. One can modify a voting correspondence φ which does not pass the Smith-IIA criterion to a new one passing the Smith-IIA criterion by restricting the election to the candidates in the Smith set. We use φ-Smith to denote the new correspondence.

Faliszewski et al. [10] showed that Copeland$^\alpha$ satisfy the second condition. In addition, Hemaspaandra et al. [13] showed both Ranked pairs and Schulze's correspondences satisfy the second condition. We extend these results to more voting correspondences.

Lemma 12. *The constructive/destructive control problems by adding or deleting votes are $\mathcal{FPT}$ with the number of candidates as parameters in the general case for the following voting correspondences: Kemeny, Slater's, all positional scoring correspondences, Bucklin's, Maximin, Nanson's, and Baldwin's.*

Corollary 13. *In an odd-votes election, the constructive/desctructive control problems by adding or deleting votes for both unique-winner and nonunique-winner models are $\mathcal{FPT}$ with single-peaked width as parameter for the following voting correspondences: Ranked Pairs, Schulze's, Copeland$^\alpha$, Kemeny, Slater's, and φ-Smith, where φ can be a positional scoring correspondence, Bucklin's, Maximin, Nanson's or Baldwin's.*

6. OUTLOOKS

The next control scenario to examine could be the control by partitioning votes, where $\mathcal{NP}$-hardness for general and polynomial-time solvability in the single-peaked case have been proven for several voting correspondences. Moreover, a general characterization for elections with even number of votes or other classes of voting correspondences as Theorem 11 could be a challenging but interesting research topic. Finally, it is also demanding to relate other structural parameters like single-peaked crossing width [6] with the complexity study of control problems.

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