Doxastic Reasoning with Multi-Source Justifications based on Second Order Propositional Modal Logic

(Extended Abstract)

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ABSTRACT

In multi-agent systems, an agent generally forms her belief based on information from multiple sources, such as messages from other agents or perception of the external environment. While modal epistemic logic has been a standard formalism for reasoning about agent's belief, it lacks the expressive power for tracking information sources. Justification logic (JL) provides the missing expressivity by using justification terms to keep track of the belief formation process. However, because JL does not make a clear distinction between potential and actual evidence, the interpretation of justification formulas in JL turns out to be ambiguous. In this paper, we present a justification-based multi-source reasoning formalism built upon second-order propositional modal logic. Our framework not only inherits the source-tracking advantage of JL but also allows the distinction between the actual observation and simply potential admissibility of evidence.

Keywords

second-order propositional modal logic, justification logic, reasoning about belief, evidential reasoning, dynamic epistemic logic, Gettier problem, logical omniscience

1. MOTIVATION

Reasoning about autonomous agents' informational attitudes, such as knowledge and belief, has been a long-standing area in the research of AI and intelligent agents [8, 18]. The typical perception-action cycle for intelligent agents assumes that an agent forms her belief about the environment and acts or makes decisions in accordance with such belief and her preference. Hence, reasoning about belief and knowledge plays a central role in the operational process of agent systems. Since the seminal work by Hintikka [16], modal logic has been a standard formalism for such kind of epistemic or doxastic reasoning.

In the multi-agent environment, an agent generally forms her belief by receiving information from different sources. Therefore, it is crucially important to keep track of the information sources and the derivation process that can be regarded as justifications of the agent's belief. However, the notion of justification was typically ignored in the standard modal formalism. Therefore, in the formalism, the modal formula $\Box\varphi$ is interpreted as "φ is believable" or "φ is knowable". By contrast, justification logics (JL) supply the missing component by adding justification terms to epistemic formulas [5, 2, 4, 13]. The first member of the JL family is the logic of proofs (LP) proposed in [1]. Although the original purpose of LP is to formalize the Brouwer-Heyting-Kolmogorov semantics for intuitionistic logic and establish its completeness with respect to this semantics, in a more general setting, JL has evolved into a kind of explicit epistemic logic and received much attention in computer science and AI [2, 5].

Although JL provides a formalism for reasoning about explicit belief, it is argued in [9, 10] that the logic does not have adequate expressive power to make the necessary distinction between potential and actual evidence. To address the issue, JL is enriched with modalities for expressing the informational contents of justification terms in [10] and the fact that a piece of evidence has been actually observed is definable in the enriched logics. To model the process of belief formation based on multiple information sources, we indeed have to distinguish potential evidence from actual one. Nevertheless, to achieve the definability of actual evidence, the logic in [10] employs a rather complicated language with Boolean modalities. Alternatively, in this paper, we propose a more succinct formalism for reasoning about justified belief based on multiple information sources. Roughly speaking, we use second order propositional modal logic (SOPML) originated from the early work in [7, 12] and its multi-agent extension called epistemic quantified Boolean logic (EQBL) in [6] to denote the information inclusion relation between the agent's belief and different information sources. Then, a piece of evidence is regarded as actually observed iff its informational contents are included in the agent's belief.

2. PRELIMINARIES

2.1 Second Order Propositional Modal logic

Second order propositional modal logic (SOPML) is the extension of modal logic with propositional quantifiers[7, 12]. Let Φ denote the set of propositional variables. Then, the formulas of SOPML are defined as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \to \varphi \mid \Box\varphi \mid \forall p\varphi,$$

where $p \in \Phi$, $\neg$ is the negation, $\to$ is the material implication, $\Box$ is the modality, and $\forall$ is the universal quantifier. Other logical

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connectives such as $\wedge, \vee, \equiv$, the modality $\Diamond$, and the existential quantifier $\exists$ are defined as abbreviations as usual.

2.2 Justification logic

To represent justifications, JL provides formal terms built up from constants and variables using various operation symbols. Constants represent justifications for commonly accepted truths–typically axioms, whereas variables denote unspecified justifications. While different variants of JL allow different operation symbols, most of them contain application and sum. Specifically, the justification terms and formulas of the basic JL are defined as follows:

$$t ::= a \mid x \mid t \cdot t \mid t + t,$$

$$\varphi ::= p \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid t{:}\varphi,$$

where $p \in \Phi$, a is a justification constant, and x is a justification variable. We use Tm to denote the set of all justification terms.

JL furnishes an evidence-based foundation for epistemic logic by using justification formula $t : \varphi$ to denote "t is a justification of φ", or more strictly, "t is accepted as a justification of φ" [2]. Semantically, the formula $t : \varphi$ can be regarded as that t is an admissible evidence for φ and based on the evidence, φ is believed. Based on the semantics, two characteristic properties of basic JL are the Application and Sum axioms:

- Application: $s{:}(\varphi \rightarrow \psi) \rightarrow (t{:}\varphi \rightarrow s \cdot t{:}\psi)$
- Sum: $s{:}\varphi \rightarrow s + t{:}\varphi, t{:}\varphi \rightarrow s + t{:}\varphi$

3. MAIN IDEA

While the formation of an agent's belief from evidence is crucially important in the perception-action cycle for intelligent agents, the aspect has received less attention in epistemic logic, partly due to the lacking of mechanism to represent evidential sources and their relationship with an agent's belief. Although JL can fill the gap by providing an evidence-based foundation for epistemic logic, it has been argued that the intuitive interpretation of a justification formula $t{:}\varphi$ is ambiguous in JL [9, 10].

On one hand, $t : \varphi$ can mean that evidence t is admissible with respect to φ without asserting that t is actually observed. In this interpretation, $t : \varphi$ has the *conditional* reading "If evidence t is observed, then φ is believed.". However, this reading is not compatible with the formal semantics of JL because the semantics explicitly enforces the condition that φ is believed by the agent.

On the other hand, $t{:}\varphi$ can mean that φ is justified belief due to the actual observation of t. In this interpretation, $t{:}\varphi$ has the *conjunctive* reading "Evidence t has been actually observed, and so φ is now believed.". This reading satisfies the principle of *justification yielding belief* (JYB) formalized with the modular semantics of JL introduced in [3]. However, under this interpretation, $t : \varphi$ asserts that t has been actually observed. Hence, $t + s : \varphi$ also asserts that the joint evidence $t + s$ has been actually observed. Then, by the axiom Sum, it means that from $t{:}\varphi$, we can derive that both s and t have been actually observed, even though s is completely irrelevant with t. This seems quite counterintuitive.

The ambiguity arises because, during the evolution from LP to JL, the semantic meaning of justification has been extended from mathematical proof to general evidence; however, the syntax of the language remains unchanged and hence the expressive power is no longer adequate for explicit epistemic reasoning. Therefore, to overcome the problem, a more fine-grained language that can differentiate these two interpretations of justification formulas has been proposed in [10].

The key point to clarify the ambiguity is whether a piece of evidence has been actually observed. The basic idea is that evidence has some informational contents and if a piece of evidence has been observed, then its informational contents should have been assimilated into the current belief. Thus, the language must be extended with modal operators $\Box_t$ to represent the informational contents of t for each justification term t. In addition, the language needs a special constant ϵ that represent the accumulation of evidence so far and a corresponding modal operator $\Box_\epsilon$ to represent an agent's (implicit) belief. However, to represent that the informational contents of a piece of evidence have been assimilated into the current belief, the logic in [10] is based on a complicated formalism of Boolean modal logic [14]. Hence, in this paper, we aim at developing a more succinct logical formalism for reasoning about source-tracking beliefs by using SOPML.

To achieve the goal, we extend JL to a logic for justification-based multi-source reasoning (JLMS). For the language of JLMS, the definition of justification terms remains the same as that of JL and the formation rules of formulas are as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid t{:}\varphi \mid \Box_S\varphi \mid \forall p\varphi,$$

where $p \in \Phi$, $t \in \mathsf{Tm}$ is a justification term, and $S \subseteq \mathsf{Tm} \cup \{\epsilon\}$. In the language, a subset $S \subseteq \mathsf{Tm} \cup \{\epsilon\}$ denotes the accumulation of pieces of evidence (and the agent's belief if $\epsilon \in S$) in S and $\Box_S$ represents its informational contents. When S is a singleton $\{s\}$, we will write $\Box_s$ instead of $\Box_{\{s\}}$. In addition, when $S = \emptyset$, we will omit the subscript and write $\Box_\emptyset\varphi$ simply as $\Box\varphi$. The fact that the evidence t has been actually observed can be defined as abbreviation in the language as follows:

$$\textsc{ao}(t) =_{def} \forall p(\Box_t p \rightarrow \Box_\epsilon p),$$

i.e. the informational contents of t have been assimilated into the current belief. In the language, the justification formula $t : \varphi$ is reserved for representing the admissibility of t with respect to φ. That is, $t : \varphi$ means that t is a good reason for believing φ. Then, we can define a new belief operator as

$$B_t\varphi =_{def} (t{:}\varphi) \wedge \textsc{ao}(t)$$

which intuitively means that φ is a belief justified by the actual observation of t.

4. CONCLUDING REAMRKS

To sum up, we propose a justification-based multi-source belief logic, which is an extension of both JL and SOPML with modalities that can represent informational contents of accumulated evidence. In the enriched languages, we can clarify the ambiguous interpretation of justification formulas. In addition, we have the following results that are not mentioned in preceding sections:

- the integration of dynamic modalities into our logic;
- the investigation of properties about the proposed logic, including its axiomatization and the logical omniscience problem; and
- the application of the formalism to some epistemological problem arising from Gettier cases [15, 17, 11].

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