# On Parameterized Complexity of Group Activity Selection Problems on Social Networks (Extended Abstract)

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## ABSTRACT

In Group Activity Selection Problem with graph structure (gGASP), players form coalitions to participate in activities and have preferences over pairs of the form (activity, group size); moreover, a group of players can only engage in the same activity if the members of the group form a connected subset of the underlying communication structure. We study the parameterized complexity of finding outcomes of gGASP that are Nash stable, individually stable or core stable. For the parameter 'number of activities', we propose an FPT algorithm for Nash stability for the case where the social network is acyclic and obtain a W[1]-hardness result for cliques (i.e., for classic GASP); similar results hold for individual stability. In contrast, finding a core stable outcome is hard even if the number of activities is bounded by a small constant, both for classic GASP and when the social network is a star. For the parameter 'number of players', all problems we consider are in XP for arbitrary social networks; on the other hand, we prove W[1]-hardness results with respect to the parameter 'number of players' for the case where the social network is a clique (i.e., for classic GASP).

## Keywords

Group activity selection problems; social networks; parameterized complexity

## 1. INTRODUCTION

In mutliagent systems, agents form coalitions to perform tasks. A useful model for analyzing how tasks can be allocated to groups of agents is the *group activity selection problem* (GASP), proposed by Darmann et al. [4]. In GASPs, participants express preferences over pairs of the form (activity, group size). The activities are then assigned to participants so as to achieve the best performance for the whole system as well as to satisfy individual agents. The key idea behind this formulation is that ideal group size depends on the task at hand: in a company, an ideal size of the sales team may differ from that of a web developers' team.

However, there is one important feature missing from the standard GASP model, namely the *feasibility* of resulting groups. In many real-life scenarios, smooth communication among members of a group is crucial in order for different individuals to work together, and hence one needs to take into account communication structures among agents. For instance, a group of employers are unable to realize their full potential if no agent knows each other. Nevertheless, the basic GASP framework imposes no restrictions on how agents can split into different groups.

A succinct way to capture such restrictions is to represent communication structures by undirected graphs. This idea dates back to cooperative games with graph structure, proposed by Myerson [10]. Under Myerson's model, nodes of the graph correspond to players and edges represent communication links between them; feasible coalitions correspond to connected subgraphs of that graph. Recently, Igarashi et al. [9] extended this idea to GASP. They considered group activity selection on social networks (gGASP), where groups need to be connected in their underlying social network in order to achieve certain tasks. The focus of their work was on core stability and Nash stability. In contrast with similar settings in cooperative games [6, 2, 8], many of the problems were shown to be NP-complete for very simple network structures; in particular, deciding the existence of core stable or Nash stable outcomes was shown to be NP-complete even when the social network is either a path, a star, or has connected components of size at most four.

Motivated by the work of Igarashi et al. [9], we investigate the parameterized complexity of finding stable outcomes in group activity selection problems. In particular, we focus on two parameters: the number of activities and the number of players.

## 2. MODEL

We now introduce the model of Igarashi et al. [9].

Definition 1 (gGASP). *An instance of the* Group Activity Selection Problem with graph structure (gGASP) *is given by a finite set of* players $N = [n]$*, a finite set of* activities $A = A^* \cup \{a_\emptyset\}$ *where* $A^* = \{a_1, a_2, \ldots, a_p\}$ *and* $a_\emptyset$ *is the* void activity*, a* profile $(\succeq_i)_{i \in N}$ *of complete and transitive preference relations over the set of* alternatives $X = A^* \times [n] \cup \{(a_\emptyset, 1)\}$*, and a set of communication links between players* $L \subseteq \{\, \{i,j\} \mid i,j \in N \wedge i \neq j \,\}$.

Two non-void activities $a$ and $b$ are said to be *equivalent* if for each player $i \in N$ and every $\ell \in [n]$ it holds that $(a, \ell) \sim_i (b, \ell)$. A non-void activity $a \in A^*$ is called *copyable* if $A^*$ contains at least $n$ activities that are equivalent to $a$ (including $a$ itself).

An outcome of a gGASP is a *feasible assignment* of activities in $A$ to players in $N$, i.e., a mapping $\pi : N \to A$ where for each $a \in A^*$ the set $\pi^a = \{\, i \in N \mid \pi(i) = a \,\}$ of players assigned to $a$ is connected in $(N, L)$. For $i \in N$ with $\pi(i) \neq a_\emptyset$, we let $\pi_i = \{i\} \cup \{\, j \in N \mid \pi(j) = \pi(i) \}$ denote the set of players assigned to the same activity as player $i \in N$; for $i \in N$ with $\pi(i) = a_\emptyset$, we set $\pi_i = \{i\}$. A feasible assignment $\pi : N \to A$ of a gGASP is *individually rational* (IR) if each player weakly prefers her own activity to doing nothing, i.e.$(\pi(i), |\pi_i|) \succeq_i (a_\emptyset, 1)$ for all $i \in N$. A connected coalition $S \subseteq N$ and an activity $a \in A^*$ *strongly block*

**Appears in:** *Proceedings of the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2017), S. Das, E. Durfee, K. Larson, M. Winikoff (eds.), May 8–12, 2017, São Paulo, Brazil.*

Table 1: **Overview of our complexity results. 'NS' stands for Nash stability, 'IS' stands for individual stability, 'CR' stands for core stability. All W[1]-hardness results are accompanied by XP-membership proofs except for finding an individually stable assignment in GASPs on cliques. For all 'XP'-entries, the question whether the problem is fixed-parameter tractable remains open. ♢ indicates that the result is not directly stated, but follows indirectly. New results are listed in boldface font.**

| | | Complexity (general case) | few activities (FPT wrt $p$) | few players (FPT wrt $n$) | copyable activities |
|---|---|---|---|---|---|
| NS | cliques | NP-c. [3] | **W[1]-h.** | **W[1]-h.** | NP-c. [1] |
| | acyclic | NP-c. [9] | **FPT** | **XP** | poly time [9] |
| | paths | NP-c. [9] | FPT [9] | **XP** | poly time [9] |
| | stars | NP-c. [9] | FPT [9] | **XP** | poly time [9] |
| | small components | NP-c.[9] | FPT [9] | **XP** | poly time [9]♢ |
| IS | cliques | NP-c. [3] | **W[1]-h.** | **W[1]-h.** | NP-c. [1] |
| | acyclic | **NP-c.** | **FPT** | **XP** | **poly time** |
| | paths | **NP-c.** | **FPT** | **XP** | **poly time** |
| | stars | **NP-c.** | **FPT** | **XP** | **poly time** |
| | small components | **NP-c.** | FPT [9]♢ | **XP** | poly time [9]♢ |
| CR | cliques | NP-c. [3] | **NP-c. for $p = 4$** | **W[1]-h.** | NP-c. [1] |
| | acyclic | NP-c. [9] | **NP-c. for $p = 2$** | **XP** | poly time [5] |
| | paths | NP-c. [9] | **XP** | **XP** | poly time [5] |
| | stars | NP-c. [9] | **NP-c. for $p = 2$** | **XP** | poly time [5] |
| | small components | NP-c. [9] | FPT [9] | **XP** | poly time [9]♢ |

an assignment $\pi : N \to A$ if $\pi^a \subseteq S$ and $(a, |S|) \succ_i (\pi(i), |\pi_i|)$ for all $i \in S$. A feasible assignment $\pi : N \to A$ of a gGASP is called *core stable* (CR) if it is individually rational, and there is no connected coalition $S \subseteq N$ and activity $a \in A^*$ such that $S$ and $a$ strongly block $\pi$. Given a feasible assignment $\pi : N \to A$ of a gGASP, a player $i \in N$ is said to have

- an *NS-deviation* to activity $a \in A^*$ if $\pi^a \cup \{i\}$ is connected, and $i$ strictly prefers to join the group $\pi^a$, i.e., $(a, |\pi^a|+1) \succ_i (\pi(i), |\pi_i|)$.
- an *IS-deviation* if it is an NS-deviation, and all players in $\pi^a$ accept it, i.e., $(a, |\pi^a| + 1) \succeq_j (a, |\pi^a|)$ for all $j \in \pi^a$.

A feasible assignment $\pi : N \to A$ of a gGASP is called *Nash stable* (NS) (respectively, *individually stable* (IS)) if it is individually rational and no player $i \in N$ has an NS-deviation (respectively, an IS-deviation) to some $a \in A^*$.

# 3. FEW ACTIVITIES

Igarashi et al. [9] demonstrate that for paths and stars the problem of finding a Nash stable outcome is fixed-parameter tractable with respect to the number of activities. We prove that this FPT result extends to arbitrary acyclic networks, thereby solving a problem left open by Igarashi et al. [9]. For general graphs, we obtain a W[1]-hardness result, implying that this problem is unlikely to admit an FPT algorithm; in fact, our hardness result holds even for 'vanilla' GASP, i.e., when the social network imposes no constraints on possible coalitions. On the positive side, for gGASPs on cliques, we prove that finding a Nash stable assignment is polynomial-time solvable when the number of activities is constant. However, it is not clear if this result can be extended to general gGASPs.

Core stability turns out to be more computationally challenging than Nash stability and individual stability when the number of activities is small: we show that core stable assignments are hard to find even if there are only two activities and the underlying graph is a star (and thus one cannot expect an FPT result with respect to the number of activities for this setting). This hardness result can be extended to the case where there are at least four activities and $(N, L)$ is a clique, i.e., to classic GASP, thereby solving a problem left open by the work of Darmann [3]. On the other hand, if there is only one activity, a core stable assignment always exists and can be constructed efficiently, for any network structure.

# 4. FEW PLAYERS

Another parameter we consider is the number of players. Although we expect the number of activities to be small in many realistic settings, there are also situations where players can choose from a huge variety of possible activities. It is then natural to ask if stability-related problems for gGASPs are tractable in the number of players $n$ is small.

We first observe that for all stability concepts considered in this paper the problem of finding a stable feasible assignment is in XP with respect to $n$: we can simply guess the activity of each player (there are at most $(p + 1)^n$ possible guesses) and check whether the resulting assignment is feasible and stable.

We show that gGASP is unlikely to be fixed-parameter tractable with respect to $n$: it is W[1]-hard to determine the existence of stable outcomes in gGASPs on cliques when parameterized by the number of players $n$. This is somewhat surprising, because an FPT algorithm with respect to $n$ could afford to iterate through all possible partitions of the players into coalitions. It is worth noting that in our proofs, we essentially show the hardness of determining whether a fixed coalition structure can be stabilized for some assignment. The computational intractability is thus due to the difficulty in assigning activities to coalitions when players have non-trivial preferences.

Note that although we showed W[1]-hardness for each of the parameters $p$ and $n$, parameterizing by the combined parameter $p+n$ immediately gives fixed-parameter tractability, since the input size is trivially upper-bounded by $n^2 \cdot p$.

# 5. DISCUSSION

We have investigated the parameterized complexity of computing stable outcomes of group activity selection problems on networks, with respect to two natural parameters. Our complexity results, together with those of Igarashi et al. [9], are summarized in Table 1. An extended version of our paper, which includes most of the proofs, is available on arXiv [7].

Many of our hardness results hold for the classic GASP problem, where there are no constraints on possible coalitions. However, some of our positive results only hold for acyclic graphs. Interestingly, one of our tractability results holds for GASPs, but it is not clear if it can be extended to gGASPs; thus, while simple networks may decrease complexity, allowing for arbitrary networks may have the opposite effect.

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