A constrained argumentation system for practical reasoning

Leila Amgoud
IRIT–CNRS
118, route de Narbonne
31062 Toulouse - France
amgoud@irit.fr

Caroline Devred
LERIA–UFR Sciences
2 Boulevard Lavoisier
49045 Angers - France
devred@info.univ-angers.fr

Marie-Christine Lagasquie-Schiex
IRIT–UPS
118, route de Narbonne
31062 Toulouse - France
lagasq@irit.fr

ABSTRACT

Practical reasoning (PR), which is concerned with the generic question of what to do, is generally seen as a two steps process: (1) *deliberation*, in which an agent decides what state of affairs it wants to reach –that is, its *desires*; and (2) *means-ends reasoning*, in which the agent looks for plans for achieving these desires. A desire is *justified* if it holds in the current state of the world, and *feasible* if there is a plan for achieving it. The agent's *intentions* are thus a consistent subset of desires that are both justified and feasible. This paper proposes the first argumentation system for PR that computes in one step the intentions of an agent, allowing thus to avoid the drawbacks of the existing systems. The proposed system is grounded on a recent work on constrained argumentation systems, and satisfies the rationality postulates identified in argumentation literature, namely the *consistency* and the *completeness* of the results.

Categories and Subject Descriptors

I.2.3 [**Deduction and Theorem Proving**]: Nonmonotonic reasoning and belief revision; I.2.11 [**Distributed Artificial Intelligence**]: Intelligent agents

General Terms

Human Factors, Theory

Keywords

Argumentation, Practical reasoning

1. INTRODUCTION

Practical reasoning (PR) [15], is concerned with the generic question "what is the right thing to do for an agent in a given situation". In [21], it has been argued that PR is a two steps process. The first step, often called *deliberation*, consists of identifying the desires of an agent. In the second step, called *means-end reasoning*, one looks for ways for achieving those desires, *i.e.* for actions or plans. A desire is *justified* if it holds in the current state of the world, and is *feasible* if it has a plan for achieving it. The agent's intentions, what the agent decides to do, is a consistent subset of desires that are both justified and feasible.

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What is worth noticing in most works on practical reasoning is the use of arguments for providing reasons for choosing or discarding a desire as an intention. Indeed, several argumentation-based systems for PR have been proposed in the literature [3, 13, 14]. However, in most of these works, the problem of PR is modeled in terms of at least two separate systems, each of them capturing a given step of the process. Such an approach may suffer from a serious drawback. In fact, some desires that are not feasible may be accepted at the deliberation step to the detriment of other justified and feasible desires. Moreover, the properties of those systems are not investigated.

This paper proposes the first argumentation system that computes the intentions of an agent in one step. The system is grounded on a recent work on *constrained* argumentation systems [9]. These last extend the well-known general system of Dung [10] by adding constraints on arguments that need to be satisfied by the extensions returned by the system. Our system takes then as input i) three categories of arguments: *epistemic* arguments that support beliefs, *explanatory* arguments that show that a desire holds in the current state of the world, and *instrumental* arguments that show that a desire is feasible, ii) different conflicts among those arguments, and iii) a particular constraint on arguments that captures the idea that for a desire to be pursued it should be both feasible and justified. This is translated by the fact that in a given extension each instrumental argument for a desire should be accompanied by at least an explanatory argument in favor of that desire. The output of our system is different sets of arguments as well as different sets of intentions. The use of a constrained system makes it possible to compute directly the intentions from the extensions. The properties of this system are deeply investigated. In particular, we show that its results are safe, and satisfy the rationality postulates identified in [5], namely consistency and completeness.

The paper is organized as follows: Section 2 recalls the basics of a constrained argumentation system. Section 3 presents the logical language. Section 4 studies the different types of arguments involved in a practical reasoning problem, and Section 5 investigates the conflicts that may exist between them. Section 6 presents the constrained argumentation system for PR, and its properties are given in Section 7. The system is then illustrated in Section 8.

2. BASICS OF CONSTRAINED ARGUMENTATION

Argumentation is an established approach for reasoning with inconsistent knowledge, based on the construction and the comparison of arguments. Many argumentation formalisms are built around an underlying logical language and an associated notion of logical consequence, defining the notion of argument. The argument construction is a monotonic process: new knowledge cannot rule out an argument but gives rise to new arguments which may interact with the first argument. Since knowledge bases may give rise to inconsistent conclusions, the arguments may be conflicting too. Consequently, it is important to determine among all the available arguments, the ones that are ultimately "acceptable". In [10], an abstract argumentation system has been proposed, and different acceptability semantics have been defined.

DEF. 1. **([10] – Basic argumentation system)** *An* argumentation system *is a pair* $\mathsf{AF} = \langle \mathcal{A}, \mathcal{R} \rangle$ *with* $\mathcal{A}$ *is a set of arguments, and* $\mathcal{R}$ *is an attack relation (*$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$*).*

Before recalling the acceptability semantics of Dung [10], let us first introduce some useful concepts.

DEF. 2. **([10] – Conflict-free, Defence)** *Let* $\mathcal{E} \subseteq \mathcal{A}$. $\mathcal{E}$ *is* conflict-free *iff* $\nexists\, \alpha, \beta \in \mathcal{E}$ *such that* $\alpha\, \mathcal{R}\, \beta$. $\mathcal{E}$ defends *an argument* α *iff* $\forall\, \beta \in \mathcal{A}$*, if* $\beta\, \mathcal{R}\, \alpha$*, then* $\exists\, \delta \in \mathcal{E}$ *such that* $\delta\, \mathcal{R}\, \beta$*.*

Dung's semantics are all based on a notion of admissibility.

DEF. 3. **([10] – Acceptability semantics)** *Let* $\mathcal{E}$ *be a set of arguments.* $\mathcal{E}$ *is an* admissible *set iff it is conflict-free and defends every element in* $\mathcal{E}$*.* $\mathcal{E}$ *is a* preferred extension *iff it is a maximal (w.r.t. set-inclusion) admissible set.* $\mathcal{E}$ *is a* stable extension *iff it is a preferred extension that attacks all arguments in* $\mathcal{A}\backslash\mathcal{E}$*.*

Note that every stable extension is also a preferred one, but the converse is not always true.

The above argumentation system has been generalized in [9]. The basic idea is to explicit *constraints* on arguments that should be satisfied by the above Dung's extensions. For instance, one may want that the two arguments α and β belong to the same stable extension. These constraints are generally expressed in terms of a propositional formula built from a language using $\mathcal{A}$ as an alphabet.

DEF. 4. **([9] – Constraints on arguments, Completion of a set of arguments)** *Let* $\mathcal{A}$ *be a set of arguments and* $\mathcal{L}_{\mathcal{A}}$ *be a propositional language defined using* $\mathcal{A}$ *as the set of propositional variables.* C *is a* constraint *on* $\mathcal{A}$ *iff* C *is a formula of* $\mathcal{L}_{\mathcal{A}}$*. The* completion *of a set* $\mathcal{E} \subseteq \mathcal{A}$ *is:* $\widehat{\mathcal{E}} = \{\alpha \mid \alpha \in \mathcal{E}\} \cup \{\neg\alpha \mid \alpha \in \mathcal{A} \setminus \mathcal{E}\}$*. A set* $\mathcal{E} \subseteq \mathcal{A}$ satisfies C *iff* $\widehat{\mathcal{E}}$ *is a model of* C *(*$\widehat{\mathcal{E}} \vdash C$*).*

A constrained system is defined as follows:

DEF. 5. **([9] – Constrained argumentation system)** *A* constrained argumentation system *is a triple* $\mathsf{CAF} = \langle \mathcal{A}, \mathcal{R}, C \rangle$ *with* C *is a constraint on arguments of* $\mathcal{A}$*.*

Let us recall how Dung's extensions are extended in constrained systems. As said before, the basic idea is to compute Dung's extensions, and then to keep among those extensions the ones that satisfy the constraint C.

DEF. 6. **([9] – C-admissible set)** *Let* $\mathcal{E} \subseteq \mathcal{A}$. $\mathcal{E}$ *is* C-admissible *iff i)* $\mathcal{E}$ *is admissible, ii)* $\mathcal{E}$ *satisfies the constraint* C*.*

Note that the empty set is admissible, however, it is not always C-admissible since $\widehat{\varnothing}$ does not always imply C.

DEF. 7. **([9] – C-extensions)** *Let* $\mathcal{E} \subseteq \mathcal{A}$. $\mathcal{E}$ *is a* C-preferred extension *iff* $\mathcal{E}$ *is maximal for set-inclusion among the* C*-admissible sets.* $\mathcal{E}$ *is a* C-stable extension *iff* $\mathcal{E}$ *is a* C*-preferred extension that attacks all arguments in* $\mathcal{A}\backslash\mathcal{E}$*.*

Now that the acceptability semantics are defined, we are ready to define the status of any argument.

DEF. 8. **(Argument status)** *Let* CAF *be a constrained argumentation system, and* $\mathcal{E}_1, \ldots, \mathcal{E}_x$ *its extensions under a given semantics. Let* $\alpha \in \mathcal{A}$. α *is* accepted *iff* $\alpha \in \mathcal{E}_i$, $\forall \mathcal{E}_i$ *with* $i = 1, \ldots, x$. α *is* rejected *iff* $\nexists \mathcal{E}_i$ *such that* $\alpha \in \mathcal{E}_i$. α *is* undecided *iff* α *is neither accepted nor rejected.*

One can easily check that if an argument is rejected in the basic system AF, then it will also be rejected in CAF.

PROP. 1. *Let* $\alpha \in \mathcal{A}$*. If* α *is rejected in* AF*, then* α *is also rejected in* CAF*.*

PROOF. Let $\alpha \in \mathcal{A}$. Assume that α is rejected in AF, and that α is not rejected in CAF.
Case of stable semantics: Since α is not rejected in CAF, then there exists $\mathcal{E}_i$ that is a C-stable extension of CAF, and $\alpha \in \mathcal{E}_i$. In [9], it has been shown (Prop. 6) that every C-stable extension is also a stable extension. Consequently, $\mathcal{E}_i$ is also a stable extension. Since α is rejected in AF, then $\alpha \notin \mathcal{E}_i$, contradiction.
Case of preferred semantics: Since α is not rejected in CAF, then there exists $\mathcal{E}_i$ that is a C-preferred extension of CAF, and $\alpha \in \mathcal{E}_i$. In [9], it has been shown (Prop. 4) that each C-preferred extension is a subset of a preferred extension. This means that $\exists \mathcal{E}$ such $\mathcal{E}$ is a preferred extension of AF and $\mathcal{E}_i \subseteq \mathcal{E}$. However, since α is rejected in AF, then $\alpha \notin \mathcal{E}$, contradiction with the fact that $\alpha \in \mathcal{E}_i$. $\square$

3. LOGICAL LANGUAGE

This section presents the logical language that will be used throughout the paper. Let $\mathcal{L}$ be a *propositional language*, and $\equiv$ be the classical equivalence relation. >From $\mathcal{L}$, a subset $\mathcal{D}$ is distinguished and is used for encoding *desires*. By desire we mean a state of affairs that an agent wants to reach. Elements of $\mathcal{D}$ are *literals*. We will write $d_1, \ldots, d_n$ to denote desires and the lowercase letters will denote formulas of $\mathcal{L}$.

>From the above sets, *desire-generation* rules can be defined. A desire-generation rule expresses under which conditions an agent may adopt a given desire. A desire may come from beliefs. For instance, "if the weather is sunny, then I desire to go to the park". In this case, the desire of going to the park depends on my belief about the weather. A desire may also come from other desires. For example, if there is a conference in India, *and* I have the desire to attend, then I desire also to attend the tutorials. Finally, a desire may be unconditional, this means that it depends on neither beliefs nor desires. These three sources of desires are captured by the following desire-generation rules.

DEF. 9. **(Desire-Generation Rules)** *A* desire-generation rule *(or a* desire rule*) is an expression of the form*

$$b \wedge d_1 \wedge \cdots \wedge d_{m-1} \hookrightarrow d_m, \textit{ where}$$

b *is a propositional formula of* $\mathcal{L}$ *and* $\forall d_i$, $d_i \in \mathcal{D}$*. Moreover,* $\nexists d_i, d_j$ *with* $i, j \leq m$ *such that* $d_i \equiv d_j$. $b \wedge d_1 \wedge \cdots \wedge d_{m-1}$ *is called the* body *of the rule (this body may be empty; this is the case of an unconditional desire), and* d_m *is its* consequent*.*

The meaning of the rule is "if the agent *believes* b and *desires* $d_1, \ldots, d_{m-1}$, then the agent will *desire* d_m as well". Note that the same desire d_i may appear in the consequent of several rules. This means that the same desire may depend on different beliefs or desires. In what follows, a desire rule is consistent if it depends on consistent beliefs and on non contradictory desires.

DEF. 10. **(Consistent Desire Rule)** *A desire rule* $b \wedge d_1 \wedge \cdots \wedge d_{m-1} \hookrightarrow d_m$ *is* consistent *iff* $b \nvdash \bot$, $\forall i = 1 \ldots m$, $b \nvdash \neg d_i$ *and* $\nexists d_i, d_j$ *with* $i, j \leq m$ *such that* $d_i \equiv \neg d_j$*. Otherwise, the rule is said* inconsistent*.*

An agent is assumed to be equipped with *plans* provided by a given planning system. The generation of such plans is beyond the scope of this paper. A plan is a way of achieving a desire. It is defined as a triple: i) a set of pre-conditions that should be satisfied before executing the plan, ii) a set of post-conditions that hold after executing the plan, and iii) the desire that is reached by the plan.

DEF. 11. **(Plan)** *A* plan *is a triple* $\langle S, T, x\rangle$ *such that*

- S *and* T *are* consistent *sets of formulas of* $\mathcal{L}$,
- $x \in \mathcal{D}$,
- $T \vdash x$ *and* $S \not\vdash x$.

Of course, there exists a link between S and T. But this link is not explicitly defined here because we are not interested by this aspect of the process. We just consider that the plan is given by a correct and sound planning system (for instance [11, 16]).

In the remaining of the paper, we suppose that an agent is equipped with three *finite bases*: i) a base $\mathcal{K} \neq \varnothing$ and $\mathcal{K} \neq \{\bot\}$ containing its *basic beliefs* about the environment (elements of $\mathcal{K}$ are propositional formulas of the language $\mathcal{L}$), ii) a base $\mathcal{B}_d$ containing its "consistent" desire rules, iii) a base $\mathcal{P}$ containing its plans. Using $\mathcal{B}_d$, we can characterize the *potential desires* of an agent as follows:

DEF. 12. **(Potential Desires)** *The set of* potential desires *of an agent is* $\mathcal{PD} = \{d_m | \exists b \wedge d_1 \wedge \cdots \wedge d_{m-1} \hookrightarrow d_m \in \mathcal{B}_d\}$.

These are "potential" desires because it is not yet clear whether these desires are justified and feasible or not.

4. TYPOLOGY OF ARGUMENTS

The aim of this section is to present the different kinds of arguments involved in practical reasoning. There are mainly three categories of arguments: one category for supporting/attacking beliefs, and two categories for justifying the adoption of desires. Note that the arguments will be denoted with lowercase greek letters.

4.1 Justifying beliefs

The first category of arguments is that studied in argumentation literature, especially for handling inconsistency in knowledge bases. Indeed, arguments are built from a knowledge base in order to support or to attack potential conclusions or inferences. These arguments are called *epistemic* in [12]. In our application, such arguments are built from the base $\mathcal{K}$. In what follows, we will use the definition proposed in [17].

DEF. 13. **(Epistemic Argument)** *Let* $\mathcal{K}$ *be a knowledge base. An* epistemic argument α *is a pair* $\alpha = \langle H, h\rangle$ *s.t: 1)* $H \subseteq \mathcal{K}$*, 2)* H *is consistent, 3)* $H \vdash h$ *and 4)* H *is minimal (for set* $\subseteq$*) among the sets satisfying conditions 1, 2, 3.*
The support *of the argument is given by the function* $\mathtt{SUPP}(\alpha) = H$, *whereas its* conclusion *is returned by* $\mathtt{CONC}(\alpha) = h$. $\mathcal{A}_b$ *stands for the set of all epistemic arguments that can be built from the base* $\mathcal{K}$.

4.2 Justifying desires

A desire may be pursued by an agent only if it is *justified* and *feasible*. Thus, there are two kinds of reasons for adopting a desire: i) the conditions underlying the desire hold in the current state of world; ii) there is a plan for reaching the desire. The definition of the first kind of arguments involves two bases: the belief base $\mathcal{K}$ and the base of desire rules $\mathcal{B}_d$. In what follows, we will use a tree-style definition of arguments [19]. Before presenting that definition, let us first introduce the functions $\mathtt{BELIEFS}(\delta)$, $\mathtt{DESIRES}(\delta)$, $\mathtt{CONC}(\delta)$ and $\mathtt{SUB}(\delta)$ that return respectively, for a given argument δ, the beliefs used in δ, the desires supported by δ, the conclusion and the set of sub-arguments of the argument δ.

DEF. 14. **(Explanatory Argument)** *Let* $\langle \mathcal{K}, \mathcal{B}_d\rangle$ *be two bases.*

- *If* $\exists \hookrightarrow d \in \mathcal{B}_d$ *then* $\longrightarrow d$ *is an* explanatory argument *(δ) with* $\mathtt{BELIEFS}(\delta) = \varnothing$, $\mathtt{DESIRES}(\delta) = \{d\}$, $\mathtt{CONC}(\delta) = d$, $\mathtt{SUB}(\delta) = \{\delta\}$.
- *If* α *is an epistemic argument, and* $\delta_1, \ldots, \delta_m$ *are explanatory arguments, and* $\exists\, \mathtt{CONC}(\alpha) \wedge \mathtt{CONC}(\delta_1) \wedge \ldots \wedge \mathtt{CONC}(\delta_m) \hookrightarrow d \in \mathcal{B}_d$ *then* $\alpha, \delta_1, \ldots, \delta_m \longrightarrow d$ *is an* explanatory argument *(δ) with* $\mathtt{BELIEFS}(\delta) = \mathtt{SUPP}(\alpha) \cup \mathtt{BELIEFS}(\delta_1) \cup \ldots \cup \mathtt{BELIEFS}(\delta_m)$, $\mathtt{DESIRES}(\delta) = \mathtt{DESIRES}(\delta_1) \cup \ldots \cup \mathtt{DESIRES}(\delta_m) \cup \{d\}$, $\mathtt{CONC}(\delta) = d$, $\mathtt{SUB}(\delta) = \{\alpha\} \cup \mathtt{SUB}(\delta_1) \cup \ldots \cup \mathtt{SUB}(\delta_m) \cup \{\delta\}$.

$\mathcal{A}_d$ *stands for the set of all explanatory arguments that can be built from* $\langle \mathcal{K}, \mathcal{B}_d\rangle$ *with a consistent* $\mathtt{DESIRES}$ *set.*

One can easily show that the set $\mathtt{BELIEFS}$ of an explanatory argument is a subset of the knowledge base $\mathcal{K}$, and that the set $\mathtt{DESIRES}$ is a subset of $\mathcal{PD}$.

PROP. 2. *Let* $\delta \in \mathcal{A}_d$. $\mathtt{BELIEFS}(\delta) \subseteq \mathcal{K}$, $\mathtt{DESIRES}(\delta) \subseteq \mathcal{PD}$.

PROOF. Let $\delta \in \mathcal{A}_d$. Let us show that $\mathtt{BELIEFS}(\delta) \subseteq \mathcal{K}$. $\mathtt{BELIEFS}(\delta) = \bigcup \mathtt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{A}_b \cap \mathtt{SUB}(\delta)$. According to the definition of an epistemic argument α_i, $\mathtt{SUPP}(\alpha_i) \subseteq \mathcal{K}$, thus $\mathtt{BELIEFS}(\delta) \subseteq \mathcal{K}$.
Let us show that $\mathtt{DESIRES}(\delta) \subseteq \mathcal{PD}$. This is a direct consequence from the definition of an explanatory argument and the definition of the set $\mathcal{PD}$. □

Note that a desire may be supported by several explanatory arguments since it may be the consequent of different desire rules.

The last category of arguments claims that "a desire may be pursued since it has a plan for achieving it". The definition of this kind of arguments involves the belief base $\mathcal{K}$ and the base of plans $\mathcal{P}$.

DEF. 15. **(Instrumental Argument)** *Let* $\langle \mathcal{K}, \mathcal{P}\rangle$ *be two bases, and* $d \in \mathcal{PD}$. *An* instrumental *argument is a pair* $\pi = \langle\langle S, T, x\rangle, d\rangle$ *where 1)* $\langle S, T, x\rangle \in \mathcal{P}$*, 2)* $S \subseteq \mathcal{K}$*, 3)* $x \equiv d$.
$\mathcal{A}_p$ *stands for the set of all instrumental arguments that can be built from* $\langle \mathcal{K}, \mathcal{P}, \mathcal{PD}\rangle$. *The function* $\mathtt{CONC}$ *returns for an argument* π *the desire* d. *The function* $\mathtt{Prec}$ *returns the pre-conditions* S *of the plan, whereas* $\mathtt{Postc}$ *returns its post-conditions* T.

The second condition of the above definition says that the pre-conditions of the plan hold in the current state of the world. In other words, the plan can be executed. Indeed, it may be the case that the base $\mathcal{P}$ contains plans whose pre-conditions are not true. Such plans cannot be executed and their corresponding instrumental arguments do not exist.
In what follows, $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$. Note that $\mathcal{A}$ is *finite* since the three initial bases ($\mathcal{K}$, $\mathcal{B}_d$ and $\mathcal{P}$) are finite.

5. INTERACTIONS AMONG ARGUMENTS

An argument constitutes a reason for believing, or adopting a desire. However, it is not a proof that the belief is true, or in our case that the desire should be adopted. The reason is that an argument can be attacked by other arguments. In this section, we will investigate the different kinds of conflicts among the arguments identified in the previous section.

5.1 Conflicts among epistemic arguments

An argument can be attacked by another argument for three main reasons: i) they have contradictory conclusions (this is known as *rebuttal*), ii) the conclusion of an argument contradicts a premise of another argument (*assumption attack*), iii) the conclusion of an argument contradicts an inference rule used in order to build the other argument (*undercutting*). Since the base $\mathcal{K}$ is built around a propositional language, it has been shown in [2] that the notion of assumption attack is sufficient to capture conflicts between epistemic arguments.

DEF. 16. *Let* $\alpha_1, \alpha_2 \in \mathcal{A}_b$. $\alpha_1 \mathcal{R}_b \alpha_2$ *iff* $\exists h' \in \texttt{SUPP}(\alpha_2)$ *such that* $\texttt{CONC}(\alpha_1) \equiv \neg h'$.

Note that the relation $\mathcal{R}_b$ *is not symmetric*. Moreover, one can show that there are no self-defeating arguments.

In [6], the argumentation system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has been applied for handling inconsistency in a knowledge base, say $\mathcal{K}$. In this particular case, a full correspondence has been established between the stable extensions of the system and the maximal consistent subsets of the base $\mathcal{K}$. Before presenting formally the result, let us introduce some useful notations. Let $\mathcal{E} \subseteq \mathcal{A}_b$, $\texttt{Base}(\mathcal{E}) = \bigcup H_i$ such that $\langle H_i, h_i \rangle \in \mathcal{E}$. Let $T \subseteq \mathcal{K}$, $\texttt{Arg}(T) = \{\langle H_i, h_i \rangle | H_i \subseteq T\}$.

PROP. 3 ([6]). *Let* $\mathcal{E}$ *be a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. $\texttt{Base}(\mathcal{E})$ *is a maximal (for set* $\subseteq$*) consistent subset of* $\mathcal{K}$ *and* $\texttt{Arg}(\texttt{Base}(\mathcal{E})) = \mathcal{E}$.

PROP. 4 ([6]). *Let* T *be a maximal (for set* $\subseteq$*) consistent subset of* $\mathcal{K}$.
$\texttt{Arg}(T)$ *is a stable extension of* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ *and* $\texttt{Base}(\texttt{Arg}(T)) = T$.

A direct consequence of the above result is that if the base $\mathcal{K}$ is not reduced to $\perp$, then the system $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ has at least one non-empty stable extension.

PROP. 5. *The argumentation system* $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$ *has non-empty stable extensions.*

PROOF. Since $\mathcal{K} \neq \{\perp\}$ and $\mathcal{K} \neq \varnothing$ then the base $\mathcal{K}$ has at least one maximal (for set inclusion) consistent subset, say T. According to Prop. 4, $\texttt{Arg}(T)$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. □

5.2 Conflicts among explanatory arguments

Explanatory arguments may also be conflicting. Indeed, two explanatory arguments may be based on two contradictory desires.

DEF. 17. *Let* $\delta_1, \delta_2 \in \mathcal{A}_d$. $\delta_1 \mathcal{R}_d \delta_2$ *iff* $\exists d_1 \in \texttt{DESIRES}(\delta_1)$, $d_2 \in \texttt{DESIRES}(\delta_2)$ *such that* $d_1 \equiv \neg d_2$.

PROP. 6. *The relation* $\mathcal{R}_d$ *is symmetric and irreflexive.*

PROOF. The proof follows directly from the definition of $\mathcal{R}_d$. □

Note that from the definition of an explanatory argument, its set DESIRES cannot be inconsistent. However, it is worth noticing that the set BELIEFS may be inconsistent, or even the union of the beliefs of two explanatory arguments is inconsistent. However, later in the paper, we will show that it is useless to explicit this kind of conflicts, since they are captured by conflicts between the explanatory arguments and epistemic ones (see Prop. 9 and Prop. 10).

5.3 Conflicts among instrumental arguments

Two plans may be conflicting for four main reasons:

- their pre-conditions are incompatible (*i.e.* the two plans cannot be executed at the same time),
- their post-conditions are incompatible (the execution of the two plans will lead to contradictory states of the world),
- the post-conditions of a plan and the preconditions of the other are incompatible (*i.e.* the execution of a plan will prevent the execution of the second plan in the future),
- their supporting desires are incompatible (indeed, plans for achieving contradictory desires are conflicting; their execution will in fact lead to a contradictory state of the world).

The above reasons are captured in the following definition of attack among instrumental arguments. Note that a plan cannot be incompatible with itself.

DEF. 18. *Let* $\pi_1, \pi_2 \in \mathcal{A}_p$ *with* $\pi_1 \neq \pi_2$. $\pi_1 \mathcal{R}_p \pi_2$ *iff:*

- $\texttt{Prec}(\pi_1) \wedge \texttt{Prec}(\pi_2) \models \perp$, *or*
- $\texttt{Postc}(\pi_1) \wedge \texttt{Postc}(\pi_2) \models \perp$, *or*
- $\texttt{Postc}(\pi_1) \wedge \texttt{Prec}(\pi_2) \models \perp$ *or* $\texttt{Prec}(\pi_1) \wedge \texttt{Postc}(\pi_2) \models \perp$.

PROP. 7. *The relation* $\mathcal{R}_p$ *is symmetric and irreflexive.*

PROOF. The proof follows directly from the definition of $\mathcal{R}_p$. □

One can show that if two plans realize conflicting desires, then their corresponding instrumental arguments are conflicting too.

PROP. 8. *Let* $d_1, d_2 \in \mathcal{PD}$. *If* $d_1 \equiv \neg d_2$, *then* $\forall \pi_1, \pi_2 \in \mathcal{A}_p$ *s.t.* $\texttt{CONC}(\pi_1) = d_1$ *and* $\texttt{CONC}(\pi_2) = d_2$, *then* $\pi_1 \mathcal{R}_p \pi_2$.

PROOF. Let $d_1, d_2 \in \mathcal{PD}$. Suppose that $d_1 \equiv \neg d_2$. Let us also suppose that $\exists\, \pi_1, \pi_2 \in \mathcal{A}_p$ with $\texttt{CONC}(\pi_1) = d_1$, and $\texttt{CONC}(\pi_2) = d_2$. According to Definition 15, it holds that $\texttt{Postc}(\pi_1) \vdash d_1$ and $\texttt{Postc}(\pi_2) \vdash d_2$. Since $d_1 \equiv \neg d_2$, then $\texttt{Postc}(\pi_2) \vdash \neg d_1$. However, the two sets $\texttt{Postc}(\pi_1)$ and $\texttt{Postc}(\pi_2)$ are both consistent (according to Definition 11), thus $\texttt{Postc}(\pi_1) \cup \texttt{Postc}(\pi_2) \vdash \perp$. Thus, $\pi_1 \mathcal{R}_p \pi_2$. □

In this section, we have considered only binary conflicts between plans, and consequently between their corresponding instrumental arguments. However, in every-day life, one may have for instance three plans such that any pair of them is not conflicting, but the three together are incompatible. For simplicity reasons, in this paper we suppose that we do not have such conflicts.

5.4 Conflicts among mixed arguments

In the previous sections we have shown how arguments of the same category can interact with each other. In this section, we will show that arguments of different categories can also interact. Indeed, epistemic arguments play a key role in ensuring the acceptability of explanatory or instrumental arguments. Namely, an epistemic argument can attack both types of arguments. The idea is to invalidate any belief used in an explanatory or instrumental argument. An explanatory argument may also conflict with an instrumental argument when this last achieves a desire whose negation is among the desires of the explanatory argument.

DEF. 19. *Let* $\alpha \in \mathcal{A}_b$, $\delta \in \mathcal{A}_d$, $\pi \in \mathcal{A}_p$.

- $\alpha \mathcal{R}_{bd} \delta$ *iff* $\exists h \in \texttt{BELIEFS}(\delta)$ *s.t.* $h \equiv \neg\texttt{CONC}(\alpha)$.
- $\alpha \mathcal{R}_{bp} \pi$ *iff* $\exists h \in \texttt{Prec}(\pi)$, *s.t.* $h \equiv \neg\texttt{CONC}(\alpha)$.
- $\delta \mathcal{R}_{pdp} \pi$ *and* $\pi \mathcal{R}_{pdp} \delta$ *iff* $\texttt{CONC}(\pi) \equiv \neg d$ *and* $d \in \texttt{DESIRES}(\delta)$[1].

As already said, the set of beliefs of an explanatory argument may be inconsistent. In such a case, the explanatory argument is attacked (in the sense of $\mathcal{R}_{bd}$) for sure by an epistemic argument.

PROP. 9. *Let* $\delta \in \mathcal{A}_d$. *If* $\texttt{BELIEFS}(\delta) \vdash \perp$, *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha \mathcal{R}_{bd} \delta$.

PROOF. Let $\delta \in \mathcal{A}_d$. Suppose that $\texttt{BELIEFS}(\delta) \vdash \perp$. This means that $\exists T$ that is minimal for set inclusion among subsets of $\texttt{BELIEFS}(\delta)$ with $T \vdash \perp$. Thus[2], $\exists h \in T$ such that $T \backslash \{h\} \vdash \neg h$ with $T \backslash \{h\}$ is consistent. Since $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$ (according to Prop. 2), then $T \backslash \{h\} \subseteq \mathcal{K}$. Consequently, $\exists\, \langle T \backslash \{h\}, \neg h \rangle \in \mathcal{A}_b$ with $h \in \texttt{BELIEFS}(\delta)$. Thus, $\langle T \backslash \{h\}, \neg h \rangle \mathcal{R}_{bd} \delta$. □

[1]Note that if $\delta_1 \mathcal{R}_{pdp} \pi_2$ and there exists δ_2 such that $\texttt{CONC}(\delta_2) = \texttt{CONC}(\pi_2)$ then $\delta_1 \mathcal{R}_d \delta_2$.
[2]Since T is $\subseteq$-minimal among inconsistent subsets of $\texttt{BELIEFS}(\delta)$, then each subset of T is consistent.

Similarly, when the beliefs of two explanatory arguments are inconsistent, it can be checked that there exists an epistemic argument that attacks at least one of the two explanatory arguments.

PROP. 10. *Let* δ_1, $\delta_2 \in \mathcal{A}_d$ *respecting* $\texttt{BELIEFS}(\delta_1) \not\vdash \bot$ *and* $\texttt{BELIEFS}(\delta_2) \not\vdash \bot$. *If* $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \bot$, *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha\ \mathcal{R}_{bd}\ \delta_1$, *or* $\alpha\ \mathcal{R}_{bd}\ \delta_2$.

PROOF. Let $\delta_1, \delta_2 \in \mathcal{A}_d$ with $\texttt{BELIEFS}(\delta_1) \not\vdash \bot$ and $\texttt{BELIEFS}(\delta_2) \not\vdash \bot$. Suppose that $\texttt{BELIEFS}(\delta_1) \cup \texttt{BELIEFS}(\delta_2) \vdash \bot$. So, $\exists T_1 \subseteq \texttt{BELIEFS}(\delta_1)$ and $\exists T_2 \subseteq \texttt{BELIEFS}(\delta_2)$ with $T_1 \cup T_2 \vdash \bot$ and $T_1 \cup T_2$ is minimal for set inclusion, *i.e.* $T_1 \cup T_2$ is a minimal conflict. Since $\texttt{BELIEFS}(\delta_1) \not\vdash \bot$ and $\texttt{BELIEFS}(\delta_2) \not\vdash \bot$, then $T_1 \neq \varnothing$ and $T_2 \neq \varnothing$. Thus, $\exists h \in T_1 \cup T_2$ such that $(T_1 \cup T_2) \setminus \{h\} \vdash \neg h$. Since $T_1 \cup T_2$ is a minimal conflict, then each subset of $T_1 \cup T_2$ is consistent, thus the set $(T_1 \cup T_2) \setminus \{h\}$ is consistent. Moreover, according to Prop. 2, $\texttt{BELIEFS}(\delta_1) \subseteq \mathcal{K}$ and $\texttt{BELIEFS}(\delta_2) \subseteq \mathcal{K}$. Thus, $T_1 \subseteq \mathcal{K}$ and $T_2 \subseteq \mathcal{K}$. It is then clear that $(T_1 \cup T_2) \setminus \{h\} \subseteq \mathcal{K}$. Consequently $\langle (T_1 \cup T_2) \setminus \{h\}, \neg h\rangle$ is an argument of $\mathcal{A}_b$.

If $h \in T_1$, then $\langle (T_1 \cup T_2) \setminus \{h\}, \neg h\rangle\ \mathcal{R}_{bd}\ \delta_1$, and if $h \in T_2$, then $\langle (T_1 \cup T_2) \setminus \{h\}, \neg h\rangle\ \mathcal{R}_{bd}\ \delta_2$. □

Conflicts may also exist between an instrumental argument and an explanatory one since the beliefs of the explanatory argument may be conflicting with the preconditions of the instrumental one. Here again, we'll show that there exists an epistemic argument that attacks at least one of the two arguments.

PROP. 11. *Let* $\delta \in \mathcal{A}_d$ *and* $\pi \in \mathcal{A}_p$ *with* $\texttt{BELIEFS}(\delta) \not\vdash \bot$. *If* $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi) \vdash \bot$ *then* $\exists \alpha \in \mathcal{A}_b$ *such that* $\alpha\ \mathcal{R}_{bd}\ \delta$, *or* $\alpha\ \mathcal{R}_{bp}\ \pi$.

PROOF. Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$. Suppose that $\texttt{BELIEFS}(\delta) \not\vdash \bot$. Since $\texttt{BELIEFS}(\delta) \not\vdash \bot$ and $\texttt{Prec}(\pi) \not\vdash \bot$, then $\exists T \subseteq \texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi)$ with $\texttt{BELIEFS}(\delta) \cap T \neq \varnothing$, $\texttt{Prec}(\pi) \cap T \neq \varnothing$ and T is the smallest inconsistent subset of $\texttt{BELIEFS}(\delta) \cup \texttt{Prec}(\pi)$.

Since $T \vdash \bot$, then $\exists h \in T$ such that $T \setminus \{h\} \vdash \neg h$ with $T \setminus \{h\}$ is consistent. Since $\texttt{BELIEFS}(\delta) \subseteq \mathcal{K}$ and since $\texttt{Prec}(\pi) \subseteq \mathcal{K}$, then $T \subseteq \mathcal{K}$. Consequently, $T \setminus \{h\} \subseteq \mathcal{K}$. Thus, $\langle T \setminus \{h\}, \neg h\rangle \in \mathcal{A}_b$.

If $h \in \texttt{BELIEFS}(\delta)$, then $\langle T \setminus \{h\}, \neg h\rangle\ \mathcal{R}_{bd}\ \delta$. If $h \in \texttt{Prec}(\pi)$, then $\langle T \setminus \{h\}, \neg h\rangle\ \mathcal{R}_{bp}\ \pi$. □

Later in the paper, it will be shown that the three above propositions are sufficient for ignoring these conflicts (between two explanatory arguments, and between an explanatory argument and an instrumental one). Note also that explanatory arguments and instrumental arguments are not allowed to attack epistemic arguments. In fact, a desire cannot invalidate a belief. Let us illustrate this issue by an example borrowed from [18]. An agent thinks that it will be raining, and that when it is raining, she gets wet. It is clear that this agent does not desire to be wet when it is raining. Intuitively, we should get one extension $\{rain, wet\}$. The idea is that if the agent believes that it is raining, and she will get wet if it rains, then she should believe that she will get wet, regardless of her likings. To do otherwise would be to indulge in *wishful thinking*.

6. ARGUMENTATION SYSTEM FOR PR

The notion of constraint which forms the backbone of constrained argumentation systems allows, in the context of PR, the representation of the link between the justification of a desire and the plan for achieving it (so between the explanatory argument in favor of a given desire and the instrumental arguments in favor of that desire). A constrained argumentation system for PR is defined as follows:

DEF. 20. **(Constrained argumentation system for PR)** *The constrained argumentation system for practical reasoning is the triple* $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C\rangle$ *with:*

- $\mathcal{A} = \mathcal{A}_b \cup \mathcal{A}_d \cup \mathcal{A}_p$,
- $\mathcal{R} = \mathcal{R}_b \cup \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{bd} \cup \mathcal{R}_{bp} \cup \mathcal{R}_{pdp}$
- *and* C *a constraint on arguments defined on* $\mathcal{A}$ *respecting* $C = \wedge_i(\pi_i \Rightarrow (\vee_j \delta_j))$ *for each* $\pi_i \in \mathcal{A}_p$ *and* $\delta_j \in \mathcal{A}_d$ *such that* $\texttt{CONC}(\pi_i) \equiv \texttt{CONC}(\delta_j)$.

Note that the satisfaction of the constraint C implies that each plan of a desire must be taken into account only if this desire is justified. Note also that we consider that there may be several plans for one desire but only one desire for each plan. Nevertheless, for each desire there may exist several explanatory arguments.

An important remark concerns the notion of defence. This notion has two different semantics in a PR context. When we consider only epistemic or explanatory arguments, the defence corresponds exactly to the notion defined in Dung's argumentation systems and in its constrained extension: an argument α attacks the attacker of another argument β; so α "reinstates" β; without the defence, β cannot be kept in an admissible set. Things are different with instrumental arguments: when an instrumental argument attacks another argument, this attack is always symmetric (so, each argument defends itself against an instrumental argument). In this case, it would be sufficient to take into account the notion of conflict-free in order to identify the plans which belong to an admissible set. However, in order to keep an homogeneous definition of admissibility, the notion of defence is also used for instrumental arguments knowing that it is without impact when conflicts from an instrumental argument are concerned.

Note that $\varnothing$ is always a C-admissible set of $\mathsf{CAF}_{\mathsf{PR}}$ (since $\varnothing$ is admissible and all π_i variables are false in $\widehat{\varnothing}$, so $\widehat{\varnothing} \vdash C$)[3]. Thus, $\mathsf{CAF}_{\mathsf{PR}}$ has at least one C-preferred extension. Moreover, the extensions do not contain the "good" plans of non-justified desires. The use of a constraint makes it possible to filter the content of the extensions and to keep only useful information.

At some places of the paper, we will refer by $\mathsf{AF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}\rangle$ to a basic argumentation system for PR, *i.e.* an argumentation system without the constraint, and $\mathcal{A}$ and $\mathcal{R}$ are defined as in Def. 20.

Remember that the purpose of a practical reasoning problem is to compute the intentions to be pursued by an agent, *i.e.* the desires that are both justified and feasible.

DEF. 21 (SET OF INTENTIONS). *Let* $\mathcal{I} \subseteq \mathcal{PD}$. $\mathcal{I}$ *is a* set of intentions *iff there exists a* C*-extension* $\mathcal{E}$ *(under a given semantics) of* $\mathsf{CAF}_{\mathsf{PR}}$ *such that for each* $d \in \mathcal{I}$, *there exists* $\pi \in \mathcal{A}_p \cap \mathcal{E}$ *such that* $d = \texttt{CONC}(\pi)$.

Our system provides an interesting solution to the PR problem. It computes directly sets of intentions, and identifies the state of the world as well as the plans necessary for achieving these intentions.

7. PROPERTIES OF THE SYSTEM

The aim of this section is to study the properties of the proposed argumentation system for PR. The system inherits most of the results got in [9]. However, the following result, whose proof is obvious, holds in the context of PR but not in the general case.

PROP. 12. *Let* $\mathsf{CAF}_{\mathsf{PR}} = \langle \mathcal{A}, \mathcal{R}, C\rangle$. *The set* Ω *of* C*-admissible sets defines a complete partial order for* $\subseteq$.

An important property shows that the set of epistemic arguments in a given stable extension of $\mathsf{AF}_{\mathsf{PR}}$ is itself a stable extension of the system $\langle \mathcal{A}_b, \mathcal{R}_b\rangle$. This shows clearly that stable extensions are "complete" w.r.t. epistemic arguments.

[3]This is due to the particular form of the constraint for PR. This is not true for any constraints (see Section2 and [9]).

PROP. 13. *If $\mathcal{E}$ is a stable extension of $\mathsf{AF}_{\mathsf{PR}}$, then the set $\mathcal{E}\cap\mathcal{A}_b$ is a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$.*

PROOF. Let $\mathcal{E}$ be a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Let us suppose that $\mathcal{E}'$ = $\mathcal{E} \cap \mathcal{A}_b$ is not a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$. Two cases exist:
Case 1: $\mathcal{E}'$ is not conflict-free. This means that there exist $\alpha, \alpha' \in \mathcal{E}'$ such that $\alpha\mathcal{R}_b\alpha'$. Since $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$, then $\alpha, \alpha' \in \mathcal{E}$. This means that $\mathcal{E}$ is not conflict-free. This contradicts the fact that $\mathcal{E}$ is a stable extension.
Case 2: $\mathcal{E}'$ does not attack every argument that is not in $\mathcal{E}'$. This means that $\exists\alpha \in \mathcal{A}_b$ and $\notin \mathcal{E}'$ and $\mathcal{E}'$ does not attack (w.r.t. $\mathcal{R}_b$) α. This means that $\mathcal{E}' \cup \{\alpha\}$ is conflict-free, thus $\mathcal{E} \cup \{\alpha\}$ is also conflict-free, and does not attack an argument that is not in it (because only an epistemic argument can attack another epistemic argument and all epistemic arguments of $\mathcal{E}$ belong to $\mathcal{E}'$). This contradicts the fact that $\mathcal{E}$ is a stable extension. □

Another important property of $\mathsf{AF}_{\mathsf{PR}}$ is that it has stable extensions.

PROP. 14. *The system $\mathsf{AF}_{\mathsf{PR}}$ has at least one non-empty stable extension.*

PROOF. (Sketch) $\mathsf{AF}_{\mathsf{PR}}$ can be viewed as the union of 2 argumentation systems: $\mathsf{AF}_b = \langle\mathcal{A}_b, \mathcal{R}_b\rangle$ and $\mathsf{AF}_{dp} = \langle\mathcal{A}_d \cup \mathcal{A}_p, \mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp}\rangle$ plus the $\mathcal{R}_{bd} \cup \mathcal{R}_{bp}$ relation. The system AF_b has stable extensions (according to Prop. 5). Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be those extensions. The system AF_{dp} is symmetric in the sense of [8] since the relation $\mathcal{R}_d \cup \mathcal{R}_p \cup \mathcal{R}_{pdp}$ is symmetric. In [8], it has been shown that such a system has stable extensions which correspond to maximal (for $\subseteq$) sets of arguments that are conflict-free. Let $\mathcal{E}'_1, \ldots, \mathcal{E}'_m$ be those extensions.
These two systems are linked with the $\mathcal{R}_{bd} \cup \mathcal{R}_{bp}$ relation. Two cases can be distinguished:

- **case1:** $\mathcal{R}_{bd}\cup\mathcal{R}_{bp} = \varnothing$**.** $\forall\mathcal{E}_i, \mathcal{E}'_j$, the set $\mathcal{E}_i\cup\mathcal{E}'_j$ is a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Indeed, $\mathcal{E}_i \cup \mathcal{E}'_j$ is conflict-free since $\mathcal{E}_i$, $\mathcal{E}'_j$ are both conflict-free, and the relation $\mathcal{R}_{bd} \cup \mathcal{R}_{bp} = \varnothing$. Moreover, $\mathcal{E}_i \cup \mathcal{E}'_j$ defeats every argument that is not in $\mathcal{E}_i \cup \mathcal{E}'_j$, since if $\alpha \notin \mathcal{E}_i \cup \mathcal{E}'_j$, then: i) if $\alpha \in \mathcal{A}_b$, then $\mathcal{E}_i$ defeats w.r.t. $\mathcal{R}_b$ α since $\mathcal{E}_i$ is a stable extension. Now, assume that $\alpha \in \mathcal{A}_d \cup \mathcal{A}_p$. Then, $\mathcal{E}'_j \cup \{\alpha\}$ is conflicting since $\mathcal{E}'_j$ is a maximal (for $\subseteq$) set that is conflict-free. Thus, $\mathcal{E}'_j$ defeats α.
- **case2:** $\mathcal{R}_{bd} \cup \mathcal{R}_{bp} \neq \varnothing$**.** Let $\mathcal{E}$ be a maximal (for set inclusion) set of arguments that is built with the following algorithm:
  1. $\mathcal{E} = \mathcal{E}_i$
  2. while ($\exists\beta \in \mathcal{A}_p \cup \mathcal{A}_d$ such that $\mathcal{E} \cup \{\beta\}$ is conflict-free) do $\mathcal{E} = \mathcal{E} \cup \{\beta\}$

  This algorithm stops after a finite number of steps (because $\mathcal{A}_p\cup\mathcal{A}_d$ is a finite set) and gives a set of arguments which is $\subseteq$-maximal among the conflict-free sets which include $\mathcal{E}_i$. It is easy to see that $\mathcal{E}$ is stable because, by construction, $\forall\gamma \in (\mathcal{A}_p \cup \mathcal{A}_d) \setminus \mathcal{E}$, $\exists\gamma' \in \mathcal{E}$ such that $\gamma'\mathcal{R}\gamma$, and, because $\mathcal{E}_i \subseteq \mathcal{E}$, we also have $\forall\alpha \in \mathcal{A}_b \setminus \mathcal{E}$, $\exists\alpha' \in \mathcal{E}$ such that $\alpha'\mathcal{R}\alpha$.

So there is always a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. □

It is easy to check that explanatory argument with contradictory beliefs are rejected in the system $\mathsf{CAF}_{\mathsf{PR}}$.

PROP. 15. *Let $\delta \in \mathcal{A}_d$ with $\mathtt{BELIEFS}(\delta) \vdash \bot$. The argument δ is rejected in $\mathsf{CAF}_{\mathsf{PR}}$.*

PROOF. (Sketch) Let $\delta \in \mathcal{A}_d$ with $\mathtt{BELIEFS}(\delta) \vdash \bot$. According to Prop. 14, the system $\mathsf{AF}_{\mathsf{PR}}$ has at least one stable extension. Let $\mathcal{E}$ be one of these stable extensions. Suppose that $\delta \in \mathcal{E}$. According to Prop. 13, the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$. Moreover, we can show that $\exists\alpha \in \mathcal{E} \cap \mathcal{A}_b$ such that $\alpha\mathcal{R}_{bd}\delta$. This contradicts the fact that a stable extension is conflict-free. Thus, δ is rejected in $\mathsf{AF}_{\mathsf{PR}}$. According to Prop. 1, δ is also rejected in $\mathsf{CAF}_{\mathsf{PR}}$. □

Similarly, it can be checked that if two explanatory arguments have conflicting beliefs, then they will never belong to the same stable extension at the same time.

PROP. 16. *Let δ_1, $\delta_2 \in \mathcal{A}_d$ respecting $\mathtt{BELIEFS}(\delta_1) \not\vdash \bot$ and $\mathtt{BELIEFS}(\delta_2) \not\vdash \bot$. If $\mathtt{BELIEFS}(\delta_1) \cup \mathtt{BELIEFS}(\delta_2) \vdash \bot$, then $\nexists\mathcal{E}$ C-stable extension of $\mathsf{CAF}_{\mathsf{PR}}$ such that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$.*

PROOF. (Sketch) Let $\delta_1, \delta_2 \in \mathcal{A}_d$ respecting $\mathtt{BELIEFS}(\delta_1) \not\vdash \bot$, $\mathtt{BELIEFS}(\delta_2) \not\vdash \bot$, and $\mathtt{BELIEFS}(\delta_1) \cup \mathtt{BELIEFS}(\delta_2) \vdash \bot$. Let $\mathcal{E}$ be a C-stable extension of $\mathsf{CAF}_{\mathsf{PR}}$. Thus, $\mathcal{E}$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Suppose that $\delta_1 \in \mathcal{E}$ and $\delta_2 \in \mathcal{E}$. According to Property 13, the set $\mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$. Moreover, we can easily show that $\exists\alpha \in \mathcal{E} \cap \mathcal{A}_b$ such that $\alpha\mathcal{R}_{bd}\delta_1$, or $\alpha\mathcal{R}_{bd}\delta_2$. This contradicts the fact that a stable extension is conflict-free. □

Similarly, if the beliefs of an explanatory argument and an instrumental one are conflicting, the two arguments will not appear in the same stable extension.

PROP. 17. *Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\mathtt{BELIEFS}(\delta) \not\vdash \bot$. If $\mathtt{BELIEFS}(\delta) \cup \mathtt{Prec}(\pi) \vdash \bot$ then $\nexists\mathcal{E}$ with $\mathcal{E}$ is a C-stable extension of $\mathsf{CAF}_{\mathsf{PR}}$ such that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$.*

PROOF. (Sketch) Let $\delta \in \mathcal{A}_d$ and $\pi \in \mathcal{A}_p$ with $\mathtt{BELIEFS}(\delta) \not\vdash \bot$ and $\mathtt{BELIEFS}(\delta) \cup \mathtt{Prec}(\pi) \vdash \bot$. Let $\mathcal{E}$ be a C-stable extension of $\mathsf{CAF}_{\mathsf{PR}}$. Thus, $\mathcal{E}$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$. Let us assume that $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$. Since $\mathcal{E}$ is a stable extension of $\mathsf{AF}_{\mathsf{PR}}$, then $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}_b$ is a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$ (according to Prop. 13). Moreover, it can easily be checked that when $\mathtt{BELIEFS}(\delta) \cup \mathtt{Prec}(\pi) \vdash \bot$ then $\exists\alpha \in \mathcal{E}'$ such that $\alpha\mathcal{R}_{bd}\delta$ or $\alpha\mathcal{R}_{bp}\pi$. This means that $\mathcal{E}$ attacks δ or $\mathcal{E}$ attacks π. However, $\delta \in \mathcal{E}$ and $\pi \in \mathcal{E}$. This contradicts the fact that $\mathcal{E}$ is conflict free. □

The next results are of great importance. They show that the proposed argumentation system for PR satisfies the "consistency" rationality postulate identified in [5]. Indeed, we show that each stable extension of our system supports a consistent set of desires and a consistent set of beliefs. Let $\mathcal{E} \subseteq \mathcal{A}$, the following notations are defined: $\mathtt{Bel}(\mathcal{E}) = (\bigcup_{\alpha_i\in\mathcal{E}\cap\mathcal{A}_b}\mathtt{SUPP}(\alpha_i)) \cup (\bigcup_{\delta_j\in\mathcal{E}\cap\mathcal{A}_d} \mathtt{BELIEFS}(\delta_j)) \cup (\bigcup_{\pi_k\in\mathcal{E}\cap\mathcal{A}_p} \mathtt{Prec}(\pi_k))$ and $\mathtt{Des}(\mathcal{E}) = (\bigcup_{\delta_j\in\mathcal{E}\cap\mathcal{A}_d} \mathtt{DESIRES}(\delta_j)) \cup (\bigcup_{\pi_k\in\mathcal{E}\cap\mathcal{A}_p} \mathtt{CONC}(\pi_k))$.

THEOREM 1. *(Consistency) Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the C-stable extensions of $\mathsf{CAF}_{\mathsf{PR}}$. $\forall\mathcal{E}_i$, $i = 1, \ldots, n$, it holds that:*

- $\mathtt{Bel}(\mathcal{E}_i) = \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$*,*
- $\mathtt{Bel}(\mathcal{E}_i)$ *is a $\subseteq$-maximal consistent subset of $\mathcal{K}$ and*
- $\mathtt{Des}(\mathcal{E}_i)$ *is consistent.*

PROOF. Let $\mathcal{E}$ be a C-stable extension of $\mathsf{CAF}_{\mathsf{PR}}$. Thus, $\mathcal{E}$ is also a stable extension of $\mathsf{AF}_{\mathsf{PR}}$.
1. Let us show that the set $\mathtt{Bel}(\mathcal{E}_i) = \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. In order to prove this, one should handle two cases:
1.1. $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \mathtt{Bel}(\mathcal{E}_i)$. This is implied by $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = \bigcup \mathtt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$ (cf. definition of $\mathtt{Bel}(\mathcal{E})$).
1.2. $\mathtt{Bel}(\mathcal{E}_i) \subseteq \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. Let us suppose that $\exists h \in \mathtt{Bel}(\mathcal{E}_i)$ and $h \notin \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. According to Property 13, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle\mathcal{A}_b, \mathcal{R}_b\rangle$. Moreover, according to [6], $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$[4]. However, $\mathtt{Bel}(\mathcal{E}_i) \subseteq \mathcal{K}$, then $h \in \mathcal{K}$. Since $h \notin \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$, then $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \cup \{h\} \vdash \bot$ (this is due to the fact that $\mathtt{Bel}(\mathcal{E}_i\cap\mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$). Thus, $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \vdash \neg h$. This means that $\exists H \subseteq \mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ such that H is the minimal consistent subset of $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$, thus $H \vdash \neg h$. Since $H \subseteq \mathcal{K}$ (since $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) \subseteq \mathcal{K}$), then $\langle H, \neg h\rangle \in \mathcal{A}_b$. However, according to [6], $\mathtt{Arg}(\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)) = \mathcal{E}_i \cap \mathcal{A}_b$. Besides, $h \in \mathtt{Bel}(\mathcal{E}_i)$, there are three possibilities:

- $h \in \mathtt{BELIEFS}(\delta)$ with $\delta \in \mathcal{E}_i$. In this case, $\langle H, \neg h\rangle$ $\mathcal{R}_{bd}$ δ. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.
- $h \in \mathtt{Prec}(\pi)$ with $\pi \in \mathcal{E}_i$. In this case, $\langle H, \neg h\rangle$ $\mathcal{R}_{bp}$ π. This contradicts the fact that $\mathcal{E}_i$ is a stable extension that is conflict-free.

[4]Because $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = \bigcup \mathtt{SUPP}(\alpha_i)$ with $\alpha_i \in \mathcal{E}_i \cap \mathcal{A}_b$; so, $\mathtt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b) = \mathtt{Base}(\mathcal{E}_i \cap \mathcal{A}_b)$.

- $h \in \texttt{SUPP}(\alpha)$ with $\alpha \in \mathcal{E}_i$. This is impossible since the set $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension, thus it is conflict free.

2. Let us show that the set $\texttt{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$. According to the first item of Theorem 1, $\texttt{Bel}(\mathcal{E}_i)$ = $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. However, according to Property 13, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$, and according to [6], $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$ is a maximal (for set-$\subseteq$) consistent subset of $\mathcal{K}$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is a maximal (for set inclusion) consistent subset of $\mathcal{K}$.
3. Let us show that the set $\texttt{Des}(\mathcal{E}_i)$ is consistent. Let us suppose that $\texttt{Des}(\mathcal{E}_i)$ is inconsistent, this means that $\bigcup \texttt{DESIRES}(\delta_k) \cup \bigcup \texttt{CONC}(\pi_j) \vdash \perp$ with $\delta_k \in \mathcal{E}_i$ and $\pi_j \in \mathcal{E}_i$. Since $\texttt{Des}(\mathcal{E}_i) \subseteq \mathcal{PD}$ (according to Property 2), then $\exists d_1, d_2 \in \texttt{Des}(\mathcal{E}_i)$ such that $d_1 \equiv \neg d_2$. Three possible situations may occur:
a. $\exists \pi_1, \pi_2 \in \mathcal{E}_i \cap \mathcal{A}_p$ such that $\texttt{CONC}(\pi_1) = d_1$, and $\texttt{CONC}(\pi_2) = d_2$. This means that $\pi_1 \mathcal{R}_p \pi_2$, thus $\pi_1 \mathcal{R} \pi_2$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.
b. $\exists \delta_1, \delta_2 \in \mathcal{E}_i \cap \mathcal{A}_d$ such that $d_1 \in \texttt{DESIRES}(\delta_1)$ and $d_2 \in \texttt{DESIRES}(\delta_2)$. This means that $\delta_1 \mathcal{R}_d \delta_2$, thus $\delta_1 \mathcal{R} \delta_2$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.
c. $\exists \delta \in \mathcal{E}_i \cap \mathcal{A}_d$, $\exists \pi \in \mathcal{E}_i \cap \mathcal{A}_p$ such that $d_1 \in \texttt{DESIRES}(\delta)$ and d_2 = $\texttt{CONC}(\pi)$. Since $d_1 \in \texttt{DESIRES}(\delta)$, thus $\exists \delta' \in \texttt{SUB}(\delta)$ such that $\texttt{CONC}(\delta')$ = d_1. This means that $\delta' \mathcal{R}_{pdp} \pi$, thus $\delta' \mathcal{R} \pi$. However, since $\delta \in \mathcal{E}_i$, thus $\delta' \in \mathcal{E}_i$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free. □

As direct consequence of the above result, an intention set is consistent. Formally:

THEOREM 2. *Under the stable semantics, each set of intentions of* $\mathsf{CAF_{PR}}$ *is consistent.*

PROOF. Let $\mathcal{I}$ be a set of intentions of $\mathsf{CAF_{PR}}$. Let us suppose that $\mathcal{I}$ is inconsistent. From the definition of an intention set, it is clear that $\mathcal{I} \subseteq \texttt{Des}(\mathcal{E}_i)$ with $\mathcal{E}_i$ is a C-stable extension of $\mathsf{CAF_{PR}}$. However, according to Theorem 1 the set $\texttt{Des}(\mathcal{E}_i)$ is consistent. □

Our system satisfies also the rationality postulate concerning the closedness of the extensions [5]. Namely, the set of arguments that can be built from the beliefs, desires, and plans involved in a given stable extension, is that extension itself. Let $\mathcal{E}_i$ be a C-stable extension. $\mathcal{A}$s is the set of arguments built from $\texttt{Bel}(\mathcal{E}_i)$, $\texttt{Des}(\mathcal{E}_i)$, the plans involved in building arguments of $\mathcal{E}_i$, and the base $\mathcal{B}_d$.

THEOREM 3. *(Closedness) Let $\mathcal{E}_1, \ldots, \mathcal{E}_n$ be the C-stable extensions of* $\mathsf{CAF_{PR}}$. *$\forall \mathcal{E}_i, i = 1, \ldots, n$, it holds that: $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$ and $\mathcal{A}s$ = $\mathcal{E}_i$.*

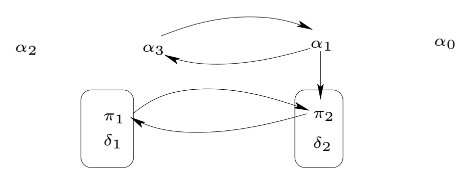
PROOF. Let $\mathcal{E}_i$ be a C-stable extension of the system $\mathsf{CAF_{PR}}$. $\mathcal{E}_i$ is also a stable extension of $\mathsf{AF_{PR}}$ (according to [9]).
1. Let us show that $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$. According to Theorem 1, it is clear that $\texttt{Bel}(\mathcal{E}_i)$ = $\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b)$. Moreover, according to Property 13, $\mathcal{E}_i \cap \mathcal{A}_b$ is a stable extension of $\langle \mathcal{A}_b, \mathcal{R}_b \rangle$. Besides, according to [6] $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i \cap \mathcal{A}_b))$ = $\mathcal{E}_i \cap \mathcal{A}_b$, thus $\texttt{Arg}(\texttt{Bel}(\mathcal{E}_i))$ = $\mathcal{E}_i \cap \mathcal{A}_b$.
2. Let us show that $\mathcal{A}s$ = $\mathcal{E}_i$. The case $\mathcal{E}_i \subseteq \mathcal{A}s$ is trivial. Let us show that $\mathcal{A}s \subseteq \mathcal{E}_i$. Let us suppose that $\exists y \in \mathcal{A}s$ and $y \notin \mathcal{E}_i$. There are three possible situations:
2.1. $y \in \mathcal{A}s \cap \mathcal{A}_b$: Since $y \notin \mathcal{E}_i$, this means that $\exists \alpha \in \mathcal{E}_i \cap \mathcal{A}_b$ such that $\alpha \mathcal{R}_b y$. Thus, $\texttt{SUPP}(\alpha) \cup \texttt{SUPP}(y) \vdash \perp$. However, $\texttt{SUPP}(\alpha) \subseteq \texttt{Bel}(\mathcal{E}_i)$ and $\texttt{SUPP}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$, thus $\texttt{SUPP}(\alpha) \cup \texttt{SUPP}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. This means that $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. According to Theorem 1 this is impossible.
2.2. $y \in \mathcal{A}s \cap \mathcal{A}_d$: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:
2.2.1. $x \in \mathcal{A}_b$ This means that $\texttt{BELIEFS}(y) \cup \texttt{SUPP}(x) \vdash \perp$. However, $\texttt{BELIEFS}(y) \cup \texttt{SUPP}(x) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
2.2.2 $x \in \mathcal{A}_d$This means that $\texttt{DESIRES}(y) \cup \texttt{DESIRES}(x) \vdash \perp$. However, $\texttt{DESIRES}(y) \cup \texttt{DESIRES}(x) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
2.2.3. $x \in \mathcal{A}_p$ This means that $\texttt{DESIRES}(y) \cup \texttt{CONC}(x) \vdash \perp$. However, $\texttt{DESIRES}(y) \cup \texttt{CONC}(x) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
2.3. $y \in \mathcal{A}s \cap \mathcal{A}_p$: Since $y \notin \mathcal{E}_i$, this means that $\exists x \in \mathcal{E}_i$ such that $x \mathcal{R} y$. There are three situations:
2.3.1. $x \in \mathcal{A}_b$ This means that $x \mathcal{R}_{bp} y$, thus $\texttt{SUPP}(x) \cup \texttt{Prec}(y) \vdash \perp$. However, $\texttt{SUPP}(x) \cup \texttt{Prec}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
2.3.2. $x \in \mathcal{A}_d$ This means that $x \mathcal{R}_{pdp} y$, thus $\texttt{DESIRES}(x) \cup \texttt{CONC}(y) \vdash \perp$. However, $\texttt{DESIRES}(x) \cup \texttt{CONC}(y) \subseteq \texttt{Des}(\mathcal{E}_i)$. Thus, $\texttt{Des}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
2.3.3. $x \in \mathcal{A}_p$ This means that $x \mathcal{R}_p y$. There are three different cases:

- $\texttt{Prec}(x) \cup \texttt{Prec}(y) \vdash \perp$. However, $\texttt{Prec}(x) \cup \texttt{Prec}(y) \subseteq \texttt{Bel}(\mathcal{E}_i)$. Thus, $\texttt{Bel}(\mathcal{E}_i)$ is inconsistent. This contradicts Theorem 1.
- $\texttt{Postc}(x) \cup \texttt{Prec}(y) \vdash \perp$. We know that y is built using one of the plans of $\mathcal{E}_i$, say $p = \langle S, T, d \rangle$. Thus, $\exists \pi \in \mathcal{E}_i$ such that π = $\langle p, d' \rangle$. Thus, $\texttt{Postc}(x) \cup \texttt{Prec}(\pi) \vdash \perp$, consequently, $x \mathcal{R} \pi$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.
- $\texttt{Postc}(x) \cup \texttt{Postc}(y) \vdash \perp$. Since $y \in \mathcal{A}s$, thus y is built using one of the plans of $\mathcal{E}_i$, say $p = \langle S, T, d \rangle$. Thus, $\exists \pi \in \mathcal{E}_i$ such that π = $\langle p, d' \rangle$. Thus, $\texttt{Postc}(x) \cup \texttt{Postc}(\pi) \vdash \perp$, consequently, $x \mathcal{R} \pi$. This is impossible since $\mathcal{E}_i$ is a stable extension, thus it is supposed to be conflict-free.

□

8. ILLUSTRATIVE EXAMPLE

In this section, we illustrate the above system on a simple example.



The meaning of these arguments is the following:

- α_0: My AAMAS paper is accepted and AAMAS conference is in Portugal so I go to AAMAS in Portugal
- α_1: My AAMAS paper is accepted and it is scheduled Day D so I am not available Day D
- α_2: My sister's wedding is scheduled Day D
- α_3: My sister's wedding is scheduled Day D so I must be available Day D
- δ_1: I go to AAMAS in Portugal so I desire to visit Portugal
- δ_2: My sister's wedding is scheduled Day D so I desire to go to my sister's wedding Day D
- π_1: My AAMAS paper is accepted, my institute pays my AAMAS mission, AAMAS is in Portugal so I can realize my desire to visit Portugal
- π_2: I am available Day D, my sister's wedding is scheduled Day D, I know where and how to go to my sister's wedding Day D so I can realize my desire to go to my sister's wedding Day D

So, we have:

- the constraint: $C = (\pi_1 \Rightarrow \delta_1) \wedge (\pi_2 \Rightarrow \delta_2)$;
- the C-preferred and C-stable extensions are $\mathcal{E}_1$ = {α_2, α_0, α_3, π_2, δ_2, δ_1}, $\mathcal{E}_2$ = {α_2, α_0, α_3, π_1, δ_1, δ_2}, $\mathcal{E}_3$ = {α_2, α_0, α_1, π_1, δ_1, δ_2},
- the sets of intentions are { visit Portugal }, { go to my sister's wedding }.

9. RELATED WORKS

A number of attempts have been made to use formal models of argumentation as a basis for PR. In fact the use of arguments for justifying an action has already been advocated by philosophers like Walton [20] who proposed the famous *practical syllogism*:

- G is a goal for agent X
- Doing action A is sufficient for agent X to carry out G
- Then, agent X ought to do action A

The above syllogism, which would apply to the means-end reasoning step, is in essence already an argument in favor of doing action A. However, this does not mean that the action is warranted, since other arguments (called counter-arguments) may be built or provided against the action.

In [1], an argumentation system is presented for generating consistent plans from a given set of desires and planning rules. This was later extended with argumentation systems that generate the desires themselves [3]. This system suffers from three main drawbacks: i) exhibiting a form of wishful thinking, ii) desires may depend only on beliefs, and iii) some undesirable results may be returned due to the separation of the two steps of PR. Due to lack of space, we will unfortunately not give an example where anomalies occur using that approach. In [14], the problem of wishful thinking has been solved. However, the separation of the two steps was kept. Other researchers in AI like Atkinson and Bench Capon [4] are more interested in studying the different argument schemes that one may encounter in practical reasoning. Their starting point was the above practical syllogism of Walton. The authors have defined different variants of this syllogism as well as different ways of attacking it. However, it is not clear how all these arguments can be put together in order to answer the critical question of PR "what is the right thing to do in a given situation?". Our work can be viewed as a way for putting those arguments all together.

10. CONCLUSION

The paper has tackled the problem of practical reasoning, which is concerned with the question "what is the best thing to do at a given situation?" The approach followed here for answering this question is based on argumentation theory, in which choices are explained and justified by arguments. The contribution of this paper is two-fold. To the best of our knowledge, this paper proposes the first argumentation system that computes the intentions in one step, *i.e.* by combining desire generation and planning. This avoids undesirable results encountered by previous proposals in the literature. This has been possible due to the use of constrained argumentation systems developed in [9]. The second contribution of the paper consists of studying deeply the properties of argumentation-based practical reasoning.

This work can be extended in different ways. First, we are currently working on relaxing the assumption that the attack relation among instrumental arguments is binary. Indeed, it may be the case that more than two plans may be conflicting while each pair of them is compatible. Another important extension would be to introduce preferences to the system. The idea is that beliefs may be pervaded with uncertainty, desires may not have equal priorities, and plans may have different costs. Thus, taking into account these preferences will help to reduce the intention sets into more relevant ones. In [7], it has been shown that an argument may not only be attacked by other arguments, but may also be supported by arguments. It would be interesting to study the impact of such a relation between arguments in the context of PR. Another area of future work is investigating the proof theories of this system. The idea is to answer the question "is a given potential desire a possible intention of the agent ?" without computing the whole preferred extensions. Finally, an interesting area of future work is investigating the relationship between our framework and axiomatic approaches to BDI agents.

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