Using Enthymemes in an Inquiry Dialogue System

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ABSTRACT

A common assumption for logic-based argumentation is that an argument is a pair $\langle \Phi, \alpha \rangle$ where Φ is a minimal subset of the knowledgebase such that Φ is consistent and Φ entails the claim α. However, real arguments (i.e. arguments presented by humans) usually do not have enough explicitly presented premises for the entailment of the claim (i.e. they are enthymemes). This is because there is some common knowledge that can be assumed by a proponent of an argument and the recipient of it. This allows the proponent of an argument to encode an argument into a real argument by ignoring the common knowledge, and it allows a recipient of a real argument to decode it into the intended argument by drawing on the common knowledge. If both the proponent and recipient use the same common knowledge, then this process is straightforward. Unfortunately, this is not always the case, and this raises interesting issues for dialogue systems in which the recipient has to cope with the disparities between the different views on what constitutes common knowledge. Here we investigate the use of enthymemes in inquiry dialogues. For this, we propose a generative inquiry dialogue system and show how, in this dialogue system, enthymemes can be managed by the agents involved, and how common knowledge can evolve through dialogue.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence:—*Multi-agent systems; coherence & coordination.*

General Terms

Theory, design.

Keywords

Argumentation, dialogue, inquiry, enthymemes, cooperation.

1. INTRODUCTION

Arguments used in dialogues do not normally fit the mould of being logical arguments. Real arguments (i.e. those presented by people in general) are normally enthymemes [17]. An enthymeme only explicitly represents some of the premises for entailing its claim. So if Γ is the set of premises explicitly given for an enthymeme, and α is the claim, then Γ does not entail α, but there are some implicitly assumable premises Γ' such that $\Gamma \cup \Gamma'$ is a minimal consistent set of formulae that entails α.

For example, for a claim *you need an umbrella today*, a man may give his wife the premise *forecast predicts rain*. The premise does not entail the claim, but the wife may easily identify the common knowledge used by her husband in order to reconstruct the intended argument correctly.

Whilst humans are constantly handling examples like this, the logical formalization that characterizes the process remains underdeveloped. Therefore, we need to investigate enthymemes because of their ubiquity in the real world, and because of the difficulties they raise for formalizing and automating argumentation. If we want to build agents that can understand real arguments coming from humans, they need to identify the missing premises with some reliability. And if we want to build agents that can generate real arguments for humans, they need to identify the premises that can be omitted without causing undue confusion. To address this need, in this paper we consider how enthymemes can be characterized in dialogues.

Dialogue games are now a common approach to characterizing argumentation-based dialogues (e.g. [12, 13]). Dialogue games are normally made up of a set of communicative acts called moves, and sets of rules stating: which moves it is legal to make at any point in a dialogue (the *protocol*); the effect of making a move; and when a dialogue terminates. Most of the work so far has looked only at *modelling* different types of dialogue from the Walton and Krabbe typology [18], rather than systems that can *generate* dialogues.

Here we focus on inquiry dialogues that use enthymemes, which may stand alone or be embedded within other dialogues. The goal of the participants is to share knowledge in order to jointly construct arguments; without embedded inquiry dialogues, the arguments that can be exchanged in dialogues such as persuasion or deliberation potentially miss out on useful arguments that involve unexpressed beliefs of the other agent. Inquiry dialogues aim for the expression of these beliefs in order to find these arguments.

In [4], the first generative inquiry dialogue system was proposed, which incorporates a strategy for each agent to uniquely select a legal move to make for each step of the dialogue, but enthymemes were not considered. In [8], the first framework for using enthymemes in logic-based argumentation was proposed, but the framework was oriented to monological as opposed to dialogical argumentation. Here,

Cite as: Using Enthymemes in an Inquiry Dialogue System, Elizabeth Black and Anthony Hunter, *Proc. of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, Padgham, Parkes, Müller and Parsons (eds.), May, 12-16., 2008, Estoril, Portugal,pp. 437-444.
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we adapt and integrate the proposals in [4, 8] in order to define a new framework for generating inquiry dialogues that use enthymemes. The agents involved can send and receive enthymemes, they can query the other agent if they do not understand an enthymeme they have received, and they can update their perception of what can be used as common knowledge based on the information exchanged during the dialogue.

2. LOGICAL ARGUMENTS

The usual paradigm for **logic-based argumentation** is that there is a large repository of information, represented by Δ, from which logical arguments can be constructed for and against arbitrary claims (e.g. [1, 3, 5, 7]). There is no *a priori* restriction on the contents, and the pieces of information in the repository can be arbitrarily complex. Therefore, Δ is not expected to be consistent. It need not even be the case that every single formula in Δ is consistent.

The framework adopts a very common intuitive notion of a logical argument. Essentially, an argument is a set of relevant formulae that can be used to prove some claim, together with that claim. Each claim is represented by a formula. Provability is represented by a consequence relation that may be for a logic such as classical logic or perhaps a defeasible logic. Here we focus on a classical propositional language $\mathcal{L}$ with classical deduction denoted by the symbol $\vdash$. We use $\alpha, \beta, \gamma, \ldots$ to denote formulae, $\Delta, \Phi, \Psi, \ldots$ to denote sets of formulae, and $\mathsf{Atoms}(\alpha)$ to denote the set of atoms from which a formula α is composed.

Definition 2.1. *A* **logical argument** *is a pair $\langle\Phi, \alpha\rangle$ s.t.: (1) $\Phi \subseteq \Delta$; (2) $\Phi \not\vdash \perp$; (3) $\Phi \vdash \alpha$; and (4) there is no $\Phi' \subset \Phi$ s.t. $\Phi' \vdash \alpha$. We say that $\langle\Phi, \alpha\rangle$ is a logical argument for α. We call α the* **claim** *of the logical argument and Φ the* **support** *of the logical argument (we also say that Φ is a support for α).*

Example 2.1. *Let $\Delta = \{\alpha, \alpha \rightarrow \beta, \gamma \rightarrow \neg\beta, \gamma, \delta, \delta \rightarrow \beta, \neg\alpha, \neg\gamma\}$. Some logical arguments are: $\langle\{\alpha, \alpha \rightarrow \beta\}, \beta\rangle$, $\langle\{\neg\alpha\}, \neg\alpha\rangle$, $\langle\{\alpha \rightarrow \beta\}, \neg\alpha \vee \beta\rangle$, and $\langle\{\neg\gamma\}, \delta \rightarrow \neg\gamma\rangle$.*

An **approximate argument** is a pair $\langle\Phi, \alpha\rangle$ where $\Phi \subseteq \mathcal{L}$ and $\alpha \in \mathcal{L}$. This is a very general definition. It does not assume that Φ is consistent, or that it even entails α. In this paper, we restrict consideration to particular kinds of approximate arguments that relax the definition of a logical argument: If $\Phi \vdash \alpha$, then $\langle\Phi, \alpha\rangle$ is **valid**; If $\Phi \not\vdash \perp$, then $\langle\Phi, \alpha\rangle$ is **consistent**; If $\Phi \vdash \alpha$, and there is no $\Phi' \subset \Phi$ such that $\Phi' \vdash \alpha$, then $\langle\Phi, \alpha\rangle$ is **minimal**; If $\Phi \vdash \alpha$, and $\Phi \not\vdash \perp$, then $\langle\Phi, \alpha\rangle$ is **expansive** (i.e. it is valid and consistent, but it may have unnecessary premises).

In addition, we require a further kind of approximate argument that has the potential to be transformed into a logical argument: If $\Phi \not\vdash \alpha$, and $\Phi \not\vdash \neg\alpha$, then $\langle\Phi, \alpha\rangle$ is a **precursor** (i.e. it is a precursor for an argument). Therefore, if $\langle\Phi, \alpha\rangle$ is a precursor, then there exists some $\Psi \subset \mathcal{L}$ such that $\Phi \cup \Psi \vdash \alpha$ and $\Phi \cup \Psi \not\vdash \perp$, and hence $\langle\Phi \cup \Psi, \alpha\rangle$ is expansive. Finally, we say $\langle\Phi, \alpha\rangle$ is a **simple argument** iff $\langle\Phi, \alpha\rangle$ is a logical argument or a precursor.

Example 2.2. *Let $\Delta = \{\alpha, \neg\alpha \vee \beta, \gamma, \neg\beta, \beta, \neg\gamma, \neg\beta \vee \gamma\}$. Some approximate arguments from Δ that are valid include $\{A_1, A_2, A_3, A_4, A_5\}$ of which $\{A_1, A_3, A_5\}$ are expansive, $\{A_2, A_5\}$ are minimal, and A_5 is a logical argument. Also, $\{A_6, A_7\}$ are approximate arguments that are not valid of which A_6 is a precursor. So A_5 and A_6 are simple arguments.*

$$
\begin{aligned}
A_1 &= \langle\{\alpha, \neg\alpha \vee \beta, \gamma, \beta\}, \beta\rangle\\
A_2 &= \langle\{\gamma, \neg\gamma\}, \beta\rangle\\
A_3 &= \langle\{\alpha, \neg\alpha \vee \beta, \gamma\}, \beta\rangle\\
A_4 &= \langle\{\alpha, \neg\alpha \vee \beta, \gamma, \neg\gamma\}, \beta\rangle\\
A_5 &= \langle\{\alpha, \neg\alpha \vee \beta\}, \beta\rangle\\
A_6 &= \langle\{\neg\alpha \vee \beta\}, \beta\rangle\\
A_7 &= \langle\{\neg\alpha \vee \beta, \neg\beta \vee \gamma, \neg\gamma\}, \beta\rangle
\end{aligned}
$$

Some observations that we can make concerning approximate arguments include: (1) If $\langle\Gamma, \alpha\rangle$ is expansive, then there is a $\Phi \subseteq \Gamma$ such that $\langle\Phi, \alpha\rangle$ is a logical argument; (2) If $\langle\Phi, \alpha\rangle$ is minimal, and $\langle\Phi, \alpha\rangle$ is expansive, then $\langle\Phi, \alpha\rangle$ is a logical argument; (3) If $\langle\Phi, \alpha\rangle$ is a logical argument, and $\Psi \subset \Phi$, then $\langle\Psi, \alpha\rangle$ is a precursor; and (4) If $\langle\Gamma, \alpha\rangle$ is a precursor, then $\langle\Gamma, \alpha\rangle$ is consistent.

3. ENTHYMEMES

We now adapt the framework for enthymemes in monological argumentation [8] in which an enthymeme is simply a precursor that can be generated from a logical argument.

Definition 3.1. *Let $\langle\Psi, \alpha\rangle$ be a logical argument. $\langle\Phi, \alpha\rangle$ is an* **enthymeme** *for $\langle\Psi, \alpha\rangle$ iff $\Phi \subset \Psi$.*

So if a proponent has a logical argument that it wishes a recipient to be aware of (we refer to this argument as the **intended argument**), then the proponent may send an enthymeme instead of the intended argument to the recipient. We refer to whatever the proponent sends to the recipient (whether the intended argument or an enthymeme for that intended argument) as the **real argument**. So a real argument is a simple argument.

Example 3.1. *Let α be "you need an umbrella today", and β be "forecast predicts rain". So for an intended argument $\langle\{\beta, \beta \rightarrow \alpha\}, \alpha\rangle$, the real argument sent by the proponent to the recipient may be $\langle\{\beta\}, \alpha\rangle$.*

In general, since there can be more than one real argument that can be generated from an intended argument, a proponent i needs to choose which to send to a recipient j.

To facilitate this selection, the proponent consults what it believes can be used as common knowledge between i and j. We assume that each agent i has a finite knowledgebase Δ_i, called a **perbase**, that is its personal knowledgebase, and so if i is a proponent, the support of the intended argument comes from Δ_i. In addition, agent i has a function $\pi_i^j : \mathcal{L} \mapsto [0, 1]$, called a **cobase function**, that represents the degree to which an agent i believes each formula in the language can be used as common knowledge between i and j. For $\alpha \in \mathcal{L}$, the higher the value of $\pi_i^j(\alpha)$, the more that i regards it is possible to use α as common knowledge between i and j. So if $\pi_i^j(\alpha) = 0$, then i believes that it cannot use α as common knowledge between i and j, whereas if $\pi_i^j(\alpha) = 1$, then i believes that there is no knowledge that it is more able to use as common knowledge between i and j than α. In this paper, we assume that for each cobase function, only a finite part of the language has a non-zero value, and so using π_i^j, agent i can construct a finite set of knowledge Π_i^j, called a **cobase**, that represents what an agent i believes it can use as common knowledge between i and j where $\Pi_i^j = \{\alpha \in \mathcal{L} \mid \text{ and } \pi_i^j(\alpha) \geq 0.5\}$.

Example 3.2. *In Ex. 3.1, with $\beta, \beta \rightarrow \alpha \in \Delta_i$, proponent i could have the cobase Π_i^j where $\beta \rightarrow \alpha \in \Pi_i^j$ representing that the premise $\beta \rightarrow \alpha$ is superfluous in any real argument consigned by proponent i to recipient j.*

Note, Π_i^j reflects the perception i has of the formulae that can be used as common knowledge between i and j, and Π_j^i reflects the perception j has of the formulae that can be used as common knowledge between i and j, and so it is not necessarily the case that $\Pi_i^j = \Pi_j^i$. Furthermore, it is not necessarily the case that i regards the set of formulae that can be used as common knowledge between i and j as being consistent, and so it is possible, for some α, that $\alpha \in \Pi_i^j$ and $\neg\alpha \in \Pi_i^j$.

Now consider an agent i who has an intended argument $\langle \Phi, \alpha \rangle$ that it wants agent j to be aware of. So Φ is a subset of Δ_i, i is the proponent of the argument and j is the recipient of the argument. By reference to its representation of the formulae that can be used as common knowledge Π_i^j, agent i will remove premises ϕ from Φ which are in Π_i^j.

Definition 3.2. *For a logical argument $\langle \Phi, \alpha \rangle$, the* **encodation** *of $\langle \Phi, \alpha \rangle$ from a proponent i for a recipient j, denoted $\mathsf{Encode}(\langle \Phi, \alpha \rangle, \Pi_i^j)$, is the approximate argument $\langle \Psi, \alpha \rangle$, where $\Psi = \Phi \setminus \Pi_i^j$.*

Example 3.3. *In Ex. 3.2, when $\beta \rightarrow \alpha \in \Pi_i^j$, and $\beta \notin \Pi_i^j$, $\mathsf{Encode}(\langle \{\beta, \beta \rightarrow \alpha\}, \alpha \rangle, \Pi_i^j)$ is $\langle \{\beta\}, \alpha \rangle$.*

So given a cobase Π_i^j, it is simple for a proponent i to obtain an encodation for a recipient j. Note, for an intended argument $\langle \Phi, \alpha \rangle$, it is possible that $\mathsf{Encode}(\langle \Phi, \alpha \rangle, \Pi_i^j) = \langle \emptyset, \alpha \rangle$. This raises the question of whether a proponent would want to send a real argument with empty support to another agent, since it is in effect "stating the obvious". Nevertheless, there may be a rhetorical or pragmatic motivation for such a real argument. For example when a husband issues a reminder like *don't forget your umbrella* to his wife when the common knowledge includes the facts that the month is April, the city is London, and London has many showers in April. Hence, *don't forget your umbrella* is the claim, and the support for this real argument is empty.

So it is easy for a proponent to generate a real argument from an intended argument and send it to the recipient. However, it is more difficult for the recipient to recover the intended argument from the real argument. When $\langle \Psi, \alpha \rangle$ is an encodation of $\langle \Phi, \alpha \rangle$, it is either the intended argument or an enthymeme for the intended argument. If it is an enthymeme, then the recipient has to decode its cobase Π_j^i (i.e. the knowledge that j believes can be used as common knowledge between i and j) by adding formulae Ψ' to the support of the enthymeme, creating $\langle \Psi \cup \Psi', \alpha \rangle$, which will be expansive but not necessarily minimal. It would be desirable for $\langle \Psi \cup \Psi', \alpha \rangle$ to be the intended argument, but this cannot be guaranteed. It may be that the wrong formulae from Π_j^i are used, it could be that common knowledge as viewed by agent i is not the same as that viewed by agent j (i.e. $\Pi_i^j \neq \Pi_j^i$), or it could be that there are insufficient formulae in Π_j^i. Nevertheless, we can aim for a reasonable decoding of an enthymeme. Note, in [8], the ranking in π_i^j is also used to improve the decoding.

Definition 3.3. *For an encodation $\langle \Psi, \alpha \rangle$ from a proponent i for a recipient j, a* **decodation** *is of the form $\langle \Psi \cup \Psi', \alpha \rangle$, where $\Psi' \subseteq \Pi_j^i$, and $\langle \Psi \cup \Psi', \alpha \rangle$ is expansive, and there is no Ψ'' s.t. $\Psi'' \subset \Psi'$ and $\langle \Psi \cup \Psi'', \alpha \rangle$ is expansive. Let $\mathsf{Decode}(\langle \Psi, \alpha \rangle, \Pi_j^i)$ denote the set of decodations of $\langle \Psi, \alpha \rangle$.*

Example 3.4. *If $\langle \{\alpha, \alpha \rightarrow \beta\}, \beta \rangle$ is an intended argument from proponent i to recipient j, where $\Pi_i^j = \{\alpha \rightarrow \beta\}$, then the encodation is $\langle \{\alpha\}, \beta \rangle$. Now suppose, $\Pi_j^i = \{\alpha \rightarrow \beta, \alpha \rightarrow \epsilon, \epsilon \rightarrow \beta\}$. So for $\langle \{\alpha\}, \beta \rangle$, the decodations are $\langle \{\alpha, \alpha \rightarrow \beta\}, \beta \rangle$ and $\langle \{\alpha, \alpha \rightarrow \epsilon, \epsilon \rightarrow \beta\}, \beta \rangle$.*

Example 3.5. *If $\langle \{\beta, \gamma, \beta \wedge \gamma \rightarrow \alpha\}, \alpha \rangle$ is an intended argument from proponent i to recipient j, where $\Pi_i^j = \{\beta \wedge \gamma \rightarrow \alpha\}$, then the encodation is $\langle \{\beta, \gamma\}, \alpha \rangle$. If $\Pi_j^i = \{\beta \rightarrow \alpha\}$,the decodation is $\langle \{\beta, \gamma, \beta \rightarrow \alpha\}, \alpha \rangle$.*

So when a recipient decodes an enthymeme, it does not know for certain what the intended argument is, and it is not guaranteed to find it even if $\mathsf{Decode}(\langle \Psi, \alpha \rangle, \Pi_j^i)$ contains only one logical argument. However, if the proponent and recipient have identical beliefs in what can be used as common knowledge between them, then the intended argument is one of the decodations. If there is a unique decodation that is a logical argument, and a high confidence that $\Pi_i^j = \Pi_j^i$, then the recipient may have high confidence that the decodation is the same as the intended argument. Furthermore, if the real argument is a logical argument, then the decodation is unique and correct.

4. REPRESENTING DIALOGUES

The communicative acts in a dialogue are called *moves*. We assume that there are always exactly two agents (*participants*) taking part in a dialogue, each with its own identifier taken from the set $\mathcal{I} = \{1, 2\}$. Each participant takes it in turn to make a move to the other participant. For a dialogue involving participants $1, 2 \in \mathcal{I}$, we also refer to participants using the variables x and $\overline{x}$ such that if x is 1 then $\overline{x}$ is 2 and if x is 2 then $\overline{x}$ is 1.

A move in our system is of the form $\langle Agent, Act, Content \rangle$. *Agent* is the identifier of the agent generating the move, *Act* is the type of move, and the *Content* gives the details of the move. The format for moves used in enthymeme inquiry dialogues is shown in Table 1, and the set of all enthymeme inquiry moves meeting the format defined in Table 1 is denoted $\mathcal{M}$. Note that the system allows for other types of dialogues to be generated and these might require the addition of extra moves. Also, $\mathsf{Sender} : \mathcal{M} \mapsto \mathcal{I}$ is a function such that $\mathsf{Sender}(\langle Agent, Act, Content \rangle) = Agent$.

We now informally explain the different types of move: An open move $\langle x, \mathsf{open}, \alpha \rangle$ opens a search for a real argument for α; A posit move $\langle x, \mathsf{posit}, \langle \Phi, \alpha \rangle \rangle$ asserts a real argument that may be for the topic, or for a premise that may be used in a real argument for the topic; A quiz move $\langle x, \mathsf{quiz}, \langle \Phi, \alpha \rangle \rangle$ that a recipient of an enthymeme uses when its cobase is inadequate to decode the enthymeme; An agree move $\langle x, \mathsf{agree}, \langle \Phi, \alpha \rangle \rangle$ accepts a posit by the other agent when it can decode it; A close move $\langle x, \mathsf{close}, \alpha \rangle$ closes the search for a real argument for α when an agent has no other moves it can make.

A dialogue is simply a sequence of moves, each of which is made from one participant to the other. As a dialogue progresses over time, we denote each timepoint by a natural number. Each move is indexed by the timepoint when the

Move	Format
open	$\langle x, \mathsf{open}, \gamma\rangle$
posit	$\langle x, \mathsf{posit}, \langle\Phi, \phi\rangle\rangle$
quiz	$\langle x, \mathsf{quiz}, \langle\Phi, \phi\rangle\rangle$
agree	$\langle x, \mathsf{agree}, \langle\Phi, \phi\rangle\rangle$
close	$\langle x, \mathsf{close}, \gamma\rangle$

Table 1: The format for moves used in enthymeme inquiry dialogues, where γ is a formula, $\langle\Phi, \phi\rangle$ is a simple argument and x is an agent ($x \in \{1, 2\}$).

move was made. Exactly one move is made at each timepoint.

DEFINITION 4.1. *A* **dialogue**, *denoted D^t, is a sequence of moves of the form $[m_1, \ldots, m_t]$ involving two participants in $\mathcal{I} = \{1, 2\}$, where $t \in \mathbb{N}$ and the following conditions hold:*

1. *m_1 is a move of the form $\langle x, \mathsf{open}, \gamma\rangle$ where $x \in \mathcal{I}$*
2. *$\mathsf{Sender}(m_s) \in \mathcal{I}$ for $1 \leq s \leq t$*
3. *$\mathsf{Sender}(m_s) \neq \mathsf{Sender}(m_{s+1})$ for $1 \leq s < t$*

The **topic** *of the dialogue D^t is returned by $\mathsf{Topic}(D^t)$ s.t. $\mathsf{Topic}(D^t) = \gamma$. The set of all dialogues is denoted $\mathcal{D}$.*

The first move of a dialogue D^t must always be an open move (condition 1 of the previous definition), every move of the dialogue must be made to a participant of the dialogue (condition 2), and the agents take it in turns to receive moves (condition 3). In order to terminate a dialogue, two close moves must appear one immediately after the other in the sequence (called a *matched-close*), or a posit move, with the claim being the topic of the dialogue, must be followed immediately by an agree move (called an *agreed-close*).

DEFINITION 4.2. *Let D^t be a dialogue s.t. $\mathsf{Topic}(D^t) = \gamma$. We say that m_s $(1 < s \leq t)$, is*

- *a* **matched-close for** *D^t iff $m_{s-1} = \langle x, \mathsf{close}, \gamma\rangle$ and $m_s = \langle \overline{x}, \mathsf{close}, \gamma\rangle$.*
- *an* **agreed-close for** *D^t iff $m_{s-1} = \langle x, \mathsf{posit}, \langle\Phi, \gamma\rangle\rangle$ and $m_s = \langle \overline{x}, \mathsf{agree}, \langle\Phi, \gamma\rangle\rangle$.*

We say D^t has a **failed outcome** *iff m_t is a matched close, whereas we say D^t has a* **successful outcome** *of $\langle\Phi, \gamma\rangle$ iff $m_t = \langle \overline{x}, \mathsf{agree}, \langle\Phi, \gamma\rangle\rangle$.*

So a matched-close or an agreed-close will terminate a dialogue D^t but only if D^t has not already terminated.

DEFINITION 4.3. *Let D^t be a dialogue. D^t* **terminates at** *t iff m_t is a matched-close or an agreed-close for D^t and $\neg\exists s$ s.t. $s < t$ and D^s terminates at s.*

In the remainder of this section, we provide two examples of dialogues, and sketch the process behind their generation. In the next section, we will formalize the generation of these inquiry dialogues. In the first example of a dialogue below, we see that agent 2 presents an enthymeme, but then agent 1 follows this with a quiz move since it is unable to decode the enthymeme. In response, agent 1 provides another enthymeme that contains a superset of the premises in the origin posit. Agent 1 is then able to decode it and so it is able to agree to it.

EXAMPLE 4.1. *Let $\Delta_1 = \{\delta\}$, $\Delta_2 = \{\beta, \delta, \beta \wedge \delta \rightarrow \alpha\}$, $\Pi_1^2 = \{\delta\}$, and $\Pi_2^1 = \{\beta, \delta\}$. For this, the following is a dialogue.*

$$\begin{array}{l}
\langle 1, \mathsf{open}, \alpha\rangle \\
\langle 2, \mathsf{posit}, \langle\{\beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 1, \mathsf{quiz}, \langle\{\beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 2, \mathsf{posit}, \langle\{\beta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 1, \mathsf{agree}, \langle\{\beta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle
\end{array}$$

The next example is similar to the previous example, but because agent 1 has a weaker cobase, it repeatedly quizzes until agent 2 provides the intended argument.

EXAMPLE 4.2. *Let $\Delta_1 = \{\beta \wedge \delta \rightarrow \neg\psi, \neg\phi\}$, $\Delta_2 = \{\beta \wedge \delta \rightarrow \alpha, \psi \vee \phi \vee \gamma, \beta, \delta\}$, $\Pi_1^2 = \{\}$, and $\Pi_2^1 = \{\beta, \delta\}$.*

$$\begin{array}{l}
\langle 1, \mathsf{open}, \alpha\rangle \\
\langle 2, \mathsf{posit}, \langle\{\beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 1, \mathsf{quiz}, \langle\{\beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 2, \mathsf{posit}, \langle\{\beta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 1, \mathsf{quiz}, \langle\{\beta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 2, \mathsf{posit}, \langle\{\beta, \delta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle \\
\langle 1, \mathsf{agree}, \langle\{\beta, \delta, \beta \wedge \delta \rightarrow \alpha\}, \alpha\rangle\rangle
\end{array}$$

After a dialogue has closed, the common knowledge of each agent is updated. We explain how this is done in the next section.

5. GENERATING DIALOGUES

We adapt the standard approach of associating a commitment store with each agent participating in a dialogue. We assume an agent's commitment store grows monotonically over the course of the dialogue. If an agent makes a posit move, all the formulae in the support are added to the agent's commitment store. A commitment store is therefore the union of the supports of all the arguments that have been publicly posited by the agent so far in the course of the dialogue. For this reason, when constructing an argument, an agent may make use of not only its own beliefs but also those from the other agent's commitment store.

DEFINITION 5.1. *A* **commitment store** *Σ_x^t is obtained for each agent x and each timepoint t as follows.*

$$\Sigma_x^t = \begin{cases} \emptyset & \text{iff } t = 0, \\ \Sigma_x^{t-1} \cup \Phi & \text{iff } m_t = \langle x, \mathsf{posit}, \langle\Phi, \phi\rangle\rangle, \\ \Sigma_x^{t-1} & \text{otherwise.} \end{cases}$$

The goal of an enthymeme inquiry dialogue is for a pair of agents to jointly construct a real argument for a particular claim, γ, that is the topic of the dialogue. So the first move of an enthymeme inquiry dialogue is an open move with the formula γ as its content. Then at any point in the dialogue, if either agent can construct a real argument for γ using the perbase and the other agent's commitment store, it does so. Otherwise, the agents will put forward premises that could potentially be used in a real argument for the topic of the dialogue.

A protocol is a function that returns the set of moves that are legal for an agent to make at a particular point in a particular type of dialogue. Here we give the specific protocol for enthymeme inquiry dialogues. It takes the dialogue that the agents are participating in and the identifier of the agent whose turn it is to move, and returns the set of legal moves that the agent may make.

DEFINITION 5.2. *The* **enthymeme inquiry protocol** *for agent x is a function* $\mathsf{Protocol}_x : \mathcal{D} \mapsto \wp(\mathcal{M})$. *Let D^t be a dialogue s.t. $1 \leq t$, and $\mathsf{Sender}(m_t) = \overline{x}$, and $\mathsf{Topic}(D^t) = \gamma$, then $\mathsf{Protocol}_x(D^t)$ is*

$$P_x^{\mathsf{quiz}}(D^t) \cup P_x^{\mathsf{agree}}(D^t) \cup P_x^{\mathsf{expd}}(D^t) \\ \cup P_x^{\mathsf{arg}}(D^t) \cup P_x^{\mathsf{prem}}(D^t) \cup \{\langle x, \mathsf{close}, \gamma\rangle\}$$

where the following are sets of moves including 3 types of posit move.

$$P_x^{\mathsf{quiz}}(D^t) \quad = \{\langle x, \mathsf{quiz}, \langle\Phi,\gamma\rangle\rangle \mid m_t = \langle\overline{x}, \mathsf{posit}, \langle\Phi,\gamma\rangle\rangle\}$$

$$P_x^{\mathsf{agree}}(D^t) \quad = \{\langle x, \mathsf{agree}, \langle\Phi,\phi\rangle\rangle \mid m_t = \langle\overline{x}, \mathsf{posit}, \langle\Phi,\phi\rangle\rangle\}$$

$$P_x^{\mathsf{expd}}(D^t) \quad = \{\langle x, \mathsf{posit}, \langle\Phi,\gamma\rangle\rangle \mid m_t = \langle\overline{x}, \mathsf{quiz}, \langle\Psi,\gamma\rangle\rangle \\ \textit{and } \exists\beta \textit{ s.t. } \beta \notin \Psi \textit{ and } \Phi = \Psi \cup \{\beta\}\}$$

$$P_x^{\mathsf{arg}}(D^t) \quad = \{\langle x, \mathsf{posit}, \langle\Phi,\gamma\rangle\rangle \mid \neg\exists s \textit{ s.t. } s \leq t \\ \textit{and } m_s = \langle x, \mathsf{posit}, \langle\Phi,\gamma\rangle\rangle\}$$

$$P_x^{\mathsf{prem}}(D^t) \quad = \{\langle x, \mathsf{posit}, \langle\{\phi\},\phi\rangle\rangle \mid \neg\exists s \textit{ s.t. } s \leq t \\ \textit{and } m_s = \langle x, \mathsf{posit}, \langle\{\phi\},\phi\rangle\rangle \textit{ and } \phi \not\equiv \gamma\}$$

Note that it is straightforward to check conformance with the protocol as the protocol only refers to public elements of the dialogue (i.e. it does not refer to perbases or cobases). For instance, the dialogues in Examples 4.1 and 4.2 conform to the protocol.

We will shortly give a specific strategy function that allows an agent to select exactly one legal move to make at each timepoint in an enthymeme inquiry dialogue. A strategy is personal to an agent and the move that it returns depends on the agent's private beliefs (i.e. its perbase Δ_x and its cobase $\Pi_x^{\overline{x}}$). The enthymeme inquiry strategy states that if there are any legal moves allowed for the agent then a single one of these moves is selected. The conditions for the strategy function are such that a quiz or agree move is made in preference to a posit move, and if there are no such moves then a close move is made.

In order to take the agent's private beliefs into account in the strategy function, we require the following definition. Each real argument used in a dialogue is obtained by forming a logical argument with support from the agent's perbase and the other agent's commitment store, then removing zero or more formulae that the agent regards as usable as common knowledge.

DEFINITION 5.3. *Let Δ_x be a perbase, $\Pi_x^{\overline{x}}$ be a cobase, and $\Sigma_{\overline{x}}^t$ be a commitment store. The set of real arguments for α, denoted* $\mathsf{RealArguments}(\Delta_x, \Pi_x^{\overline{x}}, \Sigma_{\overline{x}}^t, \alpha)$, *is*

$$\{\langle\Psi,\alpha\rangle \mid \langle\Phi,\alpha\rangle \textit{ is a logical argument} \\ \textit{and } \Phi \subseteq \Delta_x \cup \Sigma_{\overline{x}}^t \\ \textit{and } \Psi = \Phi \setminus \Gamma \textit{ s.t. } \Gamma \subseteq \Pi_x^{\overline{x}}\}$$

In order to select a single one of the legal posit moves sanctioned by the protocol, we assign a unique number to the move content and carry out a comparison of these numbers using the usual $<$ ordering. Let us assume that there is a registration function μ over the z formulae in $\Delta_x \cup \Sigma_{\overline{x}}^t \cup \Pi_x^{\overline{x}}$. So, for each formula ϕ in this set, $\mu(\phi)$ returns a unique single digit number base z (this number is only like an id number and can be arbitrarily assigned). We can similarly assign a z digit number to each real argument obtained from this set using a registration function λ together with μ. The function λ returns a unique base z number composed of z digits for each simple argument where the first $(z-(n+1))$ digits are zeros, and the last $n+1$ digits are non-zero. For a simple argument $\langle\{\phi_1,\ldots,\phi_n\},\phi_{n+1}\rangle$,

$$\lambda(\langle\{\phi_1,\ldots,\phi_n\},\phi_{n+1}\rangle) = 0\ldots0\ d_1\ldots d_n\ d_{n+1}$$

where $d_1 < \ldots < d_n < d_{n+1}$ and $\langle d_1,\ldots,d_n,d_{n+1}\rangle$ is a permutation of $\langle\mu(\phi_1),\ldots,\mu(\phi_n),\mu(\phi_{n+1})\rangle$ s.t. μ is the registration function for $\Delta_x \cup \Sigma_x^t \cup \Pi_x^{\overline{x}}$.

DEFINITION 5.4. *Consider the set of posit moves $\Psi = \{\langle x, \mathsf{posit}, \langle\Phi_1,\phi_1\rangle\rangle, \ldots, \langle x, \mathsf{posit}, \langle\Phi_k,\phi_k\rangle\rangle\}$. The* Pick *function returns a posit move s.t.* $\mathsf{Pick}(\Psi) = \langle x, \mathsf{posit}, \langle\Phi_i,\phi_i\rangle\rangle$ *where $(1 \leq i \leq k)$ and for all j $(1 \leq j \leq k)$ if $i \neq j$, then $\lambda(\langle\Phi_i,\phi_i\rangle) < \lambda(\langle\Phi_j,\phi_j\rangle)$.*

We now use these functions to define the enthymeme inquiry strategy. It takes the dialogue D^t and returns exactly one of the legal moves.

DEFINITION 5.5. *The* **enthymeme inquiry strategy** *for agent x is a function* $\mathsf{Strategy}_x : \mathcal{D} \mapsto \mathcal{M}$ *given in Figure 1.*

We are now able to define a well-formed enthymeme inquiry dialogue. This is a dialogue that does not continue after it has terminated and that is generated by the enthymeme inquiry strategy.

DEFINITION 5.6. *A dialogue D^t is a* **well-formed enthymeme inquiry dialogue** *iff, for all s $(s < t)$, (1) D^s does not terminate at s and (2) if $\mathsf{Sender}(m_s) = \overline{x}$, then* $\mathsf{Strategy}_x(D^s) = m_{s+1}$.

The following example illustrates the generation of arguments. Note how each premise that is posited has an atom in common either with the topic of the dialogue or a formula in the commitment store.

EXAMPLE 5.1. *Let $\Delta_1 = \{\epsilon, \psi \vee \phi \vee \gamma, \beta \wedge \delta \rightarrow \neg\psi\}$, $\Delta_2 = \{\beta, \delta, \neg\phi\}$, $\Pi_1^2 = \{\delta\}$, and $\Pi_2^1 = \{\delta, \neg\phi\}$.*

$\langle 2, \mathsf{open}, \gamma\rangle$
$\langle 1, \mathsf{posit}, \langle\{\psi \vee \phi \vee \gamma\}, \psi \vee \phi \vee \gamma\rangle\rangle$
$\langle 2, \mathsf{agree}, \langle\{\psi \vee \phi \vee \gamma\}, \psi \vee \phi \vee \gamma\rangle\rangle$
$\langle 1, \mathsf{posit}, \langle\{\beta \wedge \delta \rightarrow \neg\psi\}, \beta \wedge \delta \rightarrow \neg\psi\rangle\rangle$
$\langle 2, \mathsf{agree}, \langle\{\beta \wedge \delta \rightarrow \neg\psi\}, \beta \wedge \delta \rightarrow \neg\psi\rangle\rangle$
$\langle 1, \mathsf{close}, \gamma\rangle\rangle$
$\langle 2, \mathsf{posit}, \langle\{\beta, \beta \wedge \delta \rightarrow \neg\psi, \psi \vee \phi \vee \gamma\}, \gamma\rangle\rangle$
$\langle 1, \mathsf{quiz}, \langle\{\beta, \beta \wedge \delta \rightarrow \neg\psi, \psi \vee \phi \vee \gamma\}, \gamma\rangle\rangle$
$\langle 2, \mathsf{posit}, \langle\{\beta, \beta \wedge \delta \rightarrow \neg\psi, \psi \vee \phi \vee \gamma, \neg\phi\}, \gamma\rangle\rangle$
$\langle 1, \mathsf{agree}, \langle\{\beta, \beta \wedge \delta \rightarrow \neg\psi, \psi \vee \phi \vee \gamma, \neg\phi\}, \gamma\rangle\rangle$

When a dialogue has terminated, the cobase function of each agent is updated in the following four ways (as formalized in Definition 5.7): (1) If agent $\overline{x}$ opens a dialogue with topic γ, then agent x has decreased belief that γ is common knowledge; (2) If agent $\overline{x}$ has ϕ in its commitment store, then agent x has increased belief that ϕ is common knowledge; (3) If agent $\overline{x}$ quizzes a posit, and agent x adds ϕ in the subsequent posit, then agent x has decreased belief that ϕ is common knowledge; and (4) If agent $\overline{x}$ agrees a posit, and ϕ is in the intended argument by agent x but not in

$$\mathsf{Strategy}_x(D^t) = \begin{cases} S_x^{\mathsf{quiz}}(D^t) & \text{iff } S_x^{\mathsf{quiz}}(D^t) \neq \emptyset \\ S_x^{\mathsf{agree}}(D^t) & \text{iff } S_x^{\mathsf{quiz}}(D^t) = \emptyset \text{ and } S_x^{\mathsf{agree}}(D^t) \neq \emptyset \\ \mathsf{Pick}(S_x^{\mathsf{expd}}(D^t)) & \text{iff } S_x^{\mathsf{quiz}}(D^t) = S_x^{\mathsf{agree}}(D^t) = \emptyset \text{ and } S_x^{\mathsf{expd}}(D^t) \neq \emptyset \\ \mathsf{Pick}(S_x^{\mathsf{arg}}(D^t)) & \text{iff } S_x^{\mathsf{quiz}}(D^t) = S_x^{\mathsf{agree}}(D^t) = S_x^{\mathsf{expd}}(D^t) = \emptyset \text{ and } S_x^{\mathsf{arg}}(D^t) \neq \emptyset \\ \mathsf{Pick}(S_x^{\mathsf{prem}}(D^t)) & \text{iff } S_x^{\mathsf{quiz}}(D^t) = S_x^{\mathsf{agree}}(D^t) = S_x^{\mathsf{expd}}(D^t) = S_x^{\mathsf{arg}}(D^t) = \emptyset \text{ and } S_x^{\mathsf{prem}}(D^t) \neq \emptyset \\ \langle x, \mathsf{close}, \mathsf{Topic}(D^t)\rangle & \text{iff } S_x^{\mathsf{quiz}}(D^t) = S_x^{\mathsf{agree}}(D^t) = S_x^{\mathsf{expd}}(D^t) = S_x^{\mathsf{arg}}(D^t) = S_x^{\mathsf{prem}}(D^t) = \emptyset \end{cases}$$

where the choices for the moves are given by the following subsidiary functions.

$$\begin{array}{lll} S_x^{\mathsf{quiz}}(D^t) & = \{\langle x, \mathsf{quiz}, \langle \Phi, \phi\rangle\rangle \in P_x^{\mathsf{quiz}}(D^t) & \mid \mathsf{Decode}(\langle \Phi, \alpha\rangle, \Pi_x^{\overline{x}}) = \emptyset\} \\ S_x^{\mathsf{agree}}(D^t) & = \{\langle x, \mathsf{agree}, \langle \Phi, \phi\rangle\rangle \in P_x^{\mathsf{agree}}(D^t) & \mid \mathsf{Decode}(\langle \Phi, \alpha\rangle, \Pi_x^{\overline{x}}) \neq \emptyset\} \\ S_x^{\mathsf{expd}}(D^t) & = \{\langle x, \mathsf{posit}, \langle \Phi, \phi\rangle\rangle \in P_x^{\mathsf{expd}}(D^t) & \mid \langle \Phi, \phi\rangle \in \mathsf{RealArguments}(\Delta_x, \Pi_x^{\overline{x}}, \Sigma_{\overline{x}}^t, \phi)\} \\ S_x^{\mathsf{arg}}(D^t) & = \{\langle x, \mathsf{posit}, \langle \Phi, \phi\rangle\rangle \in P_x^{\mathsf{arg}}(D^t) & \mid \langle \Phi, \phi\rangle \in \mathsf{RealArguments}(\Delta_x, \Pi_x^{\overline{x}}, \Sigma_{\overline{x}}^t, \phi)\} \\ S_x^{\mathsf{prem}}(D^t) & = \{\langle x, \mathsf{posit}, \langle \{\phi\}, \phi\rangle\rangle \in P_x^{\mathsf{prem}}(D^t) & \mid \phi \in \Delta_x \text{ and } \mathsf{Relevant}(\phi, D^t)\} \end{array}$$

Figure 1: The enthymeme inquiry strategy function uniquely selects a move according to the following preference ordering (starting with the most preferred): a quiz move (quiz), an agree move (agree), an expand posit move (expd), an argue posit move (arg), a premise posit move (prem), and a close move (close). The conditions on the r.h.s. of each iff statement above imposes this ordering. For premise posits, $\mathsf{Relevant}(\phi, D^t)$ holds iff $\mathsf{Atoms}(\phi) \cap \mathsf{Atoms}(\mathsf{Topic}(D^t)) \neq \emptyset$ or $\exists \psi \in \Sigma_x^t \cup \Sigma_{\overline{x}}^t$ s.t. $\mathsf{Atoms}(\phi) \cap \mathsf{Atoms}(\psi) \neq \emptyset$ (i.e. to be relevant ϕ must have an atom in common with a formula already in the commitment stores or with the topic of the dialogue in order to ensure that it can potentially be used with other formulae in an argument for $\mathsf{Topic}(D^t)$).

the agreed posit, then agent x has increased belief that ϕ is common knowledge. For this, we assume a function Add that increments the value for $\pi_x^{\overline{x}}(\gamma)$ and a function Sub that decrements the value for $\pi_x^{\overline{x}}(\gamma)$. We leave the magnitude of the increment or decrement as a parameter for the framework. But as an example, if $\pi_x^{\overline{x}}(\gamma) \leq 0.9$, $\mathsf{Add}(\pi_x^{\overline{x}}(\gamma)) := \pi_x^{\overline{x}}(\gamma) + 0.1$, and if $\pi_x^{\overline{x}}(\gamma) > 0.9$, $\mathsf{Add}(\pi_x^{\overline{x}}(\gamma)) := 1$

Definition 5.7. *Let D^t be a dialogue that has terminated at t. Agent x updates its cobase function $\pi_x^{\overline{x}}$ as follows.*

If $m_1 = \langle \overline{x}, \mathsf{open}, \gamma\rangle$, then $\pi_x^{\overline{x}}(\gamma) := \mathsf{Sub}(\pi_x^{\overline{x}}(\gamma))$.

If $\phi \in \Sigma_{\overline{x}}^t$, then $\pi_x^{\overline{x}}(\phi) := \mathsf{Add}(\pi_x^{\overline{x}}(\phi))$.

If $\exists i$ s.t. $m_i = \langle \overline{x}, \mathsf{quiz}, \langle \Phi, \gamma\rangle\rangle$,
and $m_{i+1} = \langle x, \mathsf{posit}, \langle \Phi \cup \{\psi\}, \gamma\rangle\rangle$,
then $\pi_x^{\overline{x}}(\psi) := \mathsf{Sub}(\pi_x^{\overline{x}}(\psi))$.

If $\exists i$ s.t. $m_i = \langle \overline{x}, \mathsf{agree}, \langle \Phi, \gamma\rangle\rangle$,
and the intended argument by x is $\langle \Psi, \gamma\rangle$ and $\psi \in \Psi \setminus \Phi$,
then $\pi_x^{\overline{x}}(\psi) := \mathsf{Add}(\pi_x^{\overline{x}}(\psi))$.

Example 5.2. *For the cobase function π_2^1 prior to the dialogue in Example 5.1, the updates after the dialogue are $\pi_2^1(\psi \vee \phi \vee \gamma) := \mathsf{Add}(\pi_2^1(\psi \vee \phi \vee \gamma))$, $\pi_2^1(\beta \wedge \delta \rightarrow \neg\psi) := \mathsf{Add}(\pi_2^1(\beta \wedge \delta \rightarrow \neg\psi))$, $\pi_2^1(\delta) := \mathsf{Add}(\pi_2^1(\delta))$ and $\pi_2^1(\neg\phi) := \mathsf{Sub}(\pi_2^1(\neg\phi))$.*

The above example illustrates how the common knowledge of a pair of agents can be refined at the closure of each dialogue, with the aim of using enthymemes more effectively in future dialogues between the agents.

Whilst we do not impose that the cobase is a subset of the perbase, we can specify the Add and Sub functions to bias formulae that are in the perbase (e.g. increments are greater for formulae in the perbase) with the general aim that the majority of the cobase should be in the perbase. In addition, the membership of the cobase could be a factor in deciding membership of the perbase. For example, if for agent i, and a formula α, there are a number of other agents j such that $\pi_i^j(\alpha) = 1$, then this would be strong meta-level evidence for i that α should be in Δ_i.

6. PROPERTIES OF DIALOGUE SYSTEM

The use of enthymemes make the exchanges made by agents more sensible, since they avoid the use of common knowledge. But, in case of disparities in the cobases of the two agents, the enthymeme inquiry strategy forces the agents to exchange quiz and posit moves when the recipient of an enthymeme cannot decode it.

Proposition 6.1. *Let D^t be a well-formed enthymeme inquiry dialogue.*

If $\mathsf{Strategy}_x(D^t) = \langle x, \mathsf{posit}, \langle \Phi, \phi\rangle\rangle$,
and $\mathsf{Decode}(\langle \Phi, \phi\rangle, \Pi_{\overline{x}}^{x}) = \emptyset$,
then $\mathsf{Strategy}_{\overline{x}}(D^{t+1}) = \langle \overline{x}, \mathsf{quiz}, \langle \Phi, \phi\rangle\rangle$,
and $\mathsf{Strategy}_x(D^{t+2}) = \langle x, \mathsf{posit}, \langle \Phi \cup \{\beta\}, \phi\rangle\rangle$
s.t. $\beta \in \Delta_x \cup \Sigma_{\overline{x}}^t$ and $\langle \Phi \cup \{\beta\}, \phi\rangle$ is a simple argument

The quiz-posit cycle is repeated until the recipient can decode the posited simple argument. This is guaranteed to occur because, in the worst case, the sender gives the logical argument. So every enthymeme is understandable eventually after a finite number of quiz-posit cycles.

More generally the constraints on the strategy function are such that we can show that all enthymeme inquiry dialogues terminate (as agents' belief bases are finite, hence there are only a finite number of different moves that can be generated and agents cannot repeat these moves). To show termination, we require the following subsidiary definition.

Definition 6.1. *A dialogue D^u* **extends** *a dialogue D^t iff the first t moves of D^u are given by the sequence D^t.*

Proposition 6.2. *For any well-formed enthymeme inquiry dialogue D^t, there exists a D^u s.t. $t \leq u$ and D^u terminates at u and D^u extends D^t.*

Based in the definition of the Pick function, the strategy function will posit arguments with smaller supports in preference to arguments with larger supports. This means the use of common knowledge is maximized in enthymemes, and simple arguments based on fewer premises from the perbases and commitment stores are preferred.

PROPOSITION 6.3. *For the moves* $\Psi = \{\langle x, \mathsf{posit}, \langle \Phi_1, \phi_1\rangle\rangle, \ldots, \langle x, \mathsf{posit}, \langle \Phi_k, \phi_k\rangle\rangle\}$. *If* $\mathsf{Pick}(\Psi) = \langle x, \mathsf{posit}, \langle \Phi_i, \phi_i\rangle\rangle$ *then for all* j *s.t.* $(1 \leq j \leq k)$ $|\Phi_i| \leq |\Phi_j|$.

The goal of an enthymeme inquiry dialogue is for the agents to share beliefs in order to construct an argument for a specific claim. The benchmark that we compare the outcome of the dialogue with is the set of arguments that can be constructed from the union of the agents' beliefs. So this benchmark is, in a sense, the 'ideal situation' where there are clearly no constraints on the sharing of beliefs.

From this notion of an ideal situation, and from our simple examples, it may seem that it would be more straightforward to pool both agents' beliefs and apply a reasoning procedure to this set. However, given a real-world scenario this would not necessarily be possible. For example, when dealing with the medical domain we have to consider privacy issues that would restrict agents from simply pooling all beliefs. It may also be the case that the agents have vast belief bases and the communication cost involved in sharing all beliefs may be prohibitive. Moreover, pooling beliefs in this way would not enable us to develop artificial agents that can participate with human agents in dialogues using enthymemes.

An enthymeme inquiry dialogue is sound if and only if, if an argument is generated by the dialogue, then it can also be constructed from the union of the agents' beliefs.

DEFINITION 6.2. *Let* D^t *be a well-formed enthymeme inquiry dialogue. We say that* D^t *is* **sound** *iff, for each* s, *if* $s \leq t$ *and* $m_s = \langle x, \mathsf{posit}, \langle \Phi, \phi\rangle\rangle$, *then* $\langle \Phi, \phi\rangle$ *is a simple argument s.t.* $\Phi \subseteq (\Delta_x \cup \Delta_{\overline{x}})$.

The first lemma states that if an agent posits an argument, then it must be able to construct the argument from its beliefs and the other agent's commitment store. This is clear from the definition of the enthymeme inquiry strategy.

LEMMA 6.1. *Let* D^t *be a well-formed enthymeme inquiry dialogue. If* $\mathsf{Strategy}_x(D^t) = \langle x, \mathsf{posit}, \langle \Phi, \phi\rangle\rangle$, *then* $\langle \Phi, \phi\rangle$ *is a simple argument s.t.* $\Phi \subseteq (\Delta_x \cup \Sigma^t_{\overline{x}})$.

From the above lemma and the fact that the commitment stores are only updated when a posit move is made, we get the following lemma that a commitment store is always a subset of the union of the two agents' beliefs.

LEMMA 6.2. *If* D^t *is a well-formed enthymeme inquiry dialogue, then* $\Sigma^t_x \cup \Sigma^t_{\overline{x}} \subseteq \Delta_x \cup \Delta_{\overline{x}}$.

Using these lemmas, we can now show that enthymeme inquiry dialogues are sound.

PROPOSITION 6.4. *If* D^t *is a well-formed enthymeme inquiry dialogue, then* D^t *is sound.*

Similarly, an enthymeme inquiry dialogue is complete if and only if, if the dialogue terminates at t and it is possible to construct a logical argument for the topic of the dialogue from the union of the two participating agents' beliefs, then there will be an agreement to a simple argument for the topic of the dialogue posited by one of the agents at t.

DEFINITION 6.3. *Let* D^t *be a well-formed enthymeme inquiry dialogue and* $\mathsf{Topic}(D^t) = \gamma$. *We say that* D^t *is* **complete** *iff, if* D^t *terminates at* t, *and there is a logical argument* $\langle \Phi, \gamma\rangle$ *s.t.* $\Phi \subseteq (\Delta_x \cup \Delta_{\overline{x}})$, *then* $m_t = \langle x, \mathsf{agree}, \langle \Psi, \gamma\rangle\rangle$

In order to show that all enthymeme inquiry dialogues are complete, we need some further lemmas. The first states: If neither agent can produce, given their perbase and the other agent's commitment store, an argument for the topic of the dialogue, then the strategy forces them to posit formulae from their perbase, thus adding to their commitment store.

LEMMA 6.3. *Let* D^t *be a well-formed enthymeme inquiry dialogue with* $\mathsf{Topic}(D^t) = \gamma$. *If there is no* $\Phi \subseteq (\Delta_x \cup \Sigma^t_{\overline{x}})$ *and no* $\Phi \subseteq (\Delta_{\overline{x}} \cup \Sigma^t_x)$ *s.t.* $\langle \Phi, \gamma\rangle$ *is a logical argument, and there is a* $\beta \in \Delta_x$ *s.t.* $\mathsf{Relevant}(\beta, D^t)$ *and there is no* s *s.t.* $s < t$ *where* $m_s = \langle x, \mathsf{posit}, \langle \{\beta\}, \beta\rangle\rangle$, *then there is a* $\delta \in \Delta_x$ *s.t.* $\mathsf{Strategy}_x(D^t) = \langle x, \mathsf{posit}, \langle \{\delta\}, \delta\rangle\rangle$.

Following from the above lemma, we obtain the following lemma that says if there is a logical argument for the topic of the dialogue that can be obtained by pooling the agents perbases, then, once the dialogue has terminated, there is a logical argument for the topic of the dialogue that can be obtained from one of the agent's perbase together with the commitment store of the other agent.

LEMMA 6.4. *Let* D^t *be a well-formed enthymeme inquiry dialogue that terminates at* t *with* $\mathsf{Topic}(D^t) = \gamma$. *If there is a* $\Phi \subseteq (\Delta_x \cup \Delta_{\overline{x}})$ *s.t.* $\langle \Phi, \gamma\rangle$ *is a logical argument, then there is a* $\Psi \subseteq (\Delta_x \cup \Sigma^t_{\overline{x}})$ *s.t.* $\langle \Psi, \gamma\rangle$ *is a logical argument.*

The next lemma says that if there is a logical argument for the topic of the dialogue that can be obtained from one of the agent's perbase together with the commitment store of the other agent, then the strategy will force the posit of a simple argument for the topic of the dialogue at some point in the dialogue.

LEMMA 6.5. *Let* D^t *be a well-formed enthymeme inquiry dialogue that terminates at* t *with* $\mathsf{Topic}(D^t) = \gamma$. *If there is a* $\Psi \subseteq (\Delta_x \cup \Sigma^t_{\overline{x}})$ *s.t.* $\langle \Psi, \gamma\rangle$ *is a logical argument, then there is an* s *and a* Φ *s.t.* $s < t$ *and* $m_s = \langle x, \mathsf{posit}, \langle \Phi, \gamma\rangle\rangle$ *or* $m_s = \langle \overline{x}, \mathsf{posit}, \langle \Phi, \gamma\rangle\rangle$.

Using the above lemmas, it is straightforward to now show that enthymeme inquiry dialogues are complete.

PROPOSITION 6.5. *If* D^t *is a well-formed enthymeme inquiry dialogue, then* D^t *is complete.*

From Propositions 6.2 and 6.5, we get the following.

PROPOSITION 6.6. *Let* D^t *be a well-formed enthymeme inquiry dialogue. If* $\mathsf{Topic}(D^t) = \gamma$ *and there exists a logical argument* $\langle \Phi, \gamma\rangle$ *s.t.* $\Phi \subseteq (\Delta_x \cup \Delta_{\overline{x}})$, *then there exists* u *s.t.* D^u *extends* D^t *and* $m_u = \langle x, \mathsf{agree}, \langle \Psi, \phi\rangle\rangle$.

The above gives the desired result that if a logical argument can be constructed from the union of the two participating agents' beliefs whose claim is the topic of the dialogue, then there will come a timepoint in the dialogue at which an agent agrees that it can decode a simple argument for the topic that has been posited by the other agent.

7. CONCLUSIONS

We have presented a dialogue system and given details of a specific protocol and strategy for generating inquiry dialogues between two agents. Our proposal follows the approach in [4] but the types of moves are different, and the protocol and strategy functions are substantially altered, in order to support the use of enthymemes. Since enthymemes are ubiquitous in real-world argumentation, we believe it is important to develop argument-based dialogue systems that can support them. Furthermore, we believe this is the first proposal for a generative inquiry dialogue system that uses enthymemes.

Inquiry dialogues are particularly useful in cooperative domains such as healthcare and science, and they can be usefully embedded within other dialogue types. Other than [4], two groups have proposed protocols for inquiry dialogues: the Liverpool-Toulouse group proposed a protocol for general inquiry dialogues (e.g. [12]), however this protocol can lead to unsuccessful dialogues in which no argument for the topic is found even when such an argument does exist in the union of the two agents beliefs; McBurney and Parsons [10] present a specialised inquiry protocol for use in scientic domains, such as in assessments of carcinogenic risk of new chemicals, however this protocol is too complicated for general use, containing over thirty specialised moves. Neither group have proposed a strategy for use with their inquiry protocol, i.e. their systems model inquiry dialogues but are not sufficient to generate them.

In fact, most existing dialogue systems are only capable of modelling dialogues, and not of generating them. Some work has considered dialogue generation, for example: [14] propose a methodology for designing strategies for negotiation dialogues; both [2] and [9] propose a formalism for representing the private strategy of an agent to which argumentation is then applied to determine the move to be made at a point in a dialogue. None, however, provide a specific strategy for inquiry dialogues.

Other than [4], we believe the only similar works that consider soundness and completeness properties are [11, 15]. In [11], the focus is not on inquiry dialogues searching for arguments for a specific topic, but rather on dialogues during which argumentation graphs (representing the interactions between a set of arguments) are constructed. [11] defines different classes of protocol based on types of move relevance, and look at completeness properties for these protocols. [15] defines different agent programs for negotiation. If such an agent program is both exhaustive and deterministic then exactly one move is suggested by the program at a timepoint, making such a program generative and allowing consideration of soundness and completeness properties.

We believe our proposal could be adapted for other notions of argument (e.g. [1, 7]), and there are diverse ways that the notion of common knowledge could be refined. In particular, we would like to refine the decoding so that it takes a notion of relevance into account (e.g. [16]), and refine the notion of quiz moves so they are more focused on the uncertainty. Finally, decodation is a form of abduction, and so techniques and algorithms developed for abduction could be harnessed for improving the quality of decodation (e.g. [5, 6]).

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