Multi-Agent Search using Sensors with Heterogeneous Capabilities

(Short Paper)

Guruprasad K. R. and Debasish Ghose *†

ABSTRACT

In this paper we introduce a new concept namely, *generalized Voronoi partition* and use it to formulate two heterogeneous multi-agent search strategies. The core idea is optimal deployment of agents having sensors with heterogeneous capabilities, in a search space so as to maximize search effectiveness. We address a few theoretical issues such as optimality of deployment, convergence and spatial distributedness of the control law and the search strategies.

1. INTRODUCTION

Inspired by nature, scientists and engineers have developed the concept of multi-agent systems with robots, UAVs, etc., as agents. These multi-agent systems can perform a wide variety of tasks such as search and rescue, surveillance, achieve and maintain spatial formations, move as flocks while avoiding obstacles, multiple source identification and many more. In this paper we address the problem of searching for targets in an unknown environment.

Sujit and Ghose [1] partition the search space into hexagonal cells and associate each cell with an uncertainty value representing lack of information about the cell. As the agents move through these cells, they acquire information, reducing the corresponding uncertainty value. Bullo et al. [2] use centroidal Voronoi configuration [3] for deployment of a sensor network and also provide a few fundamental theoretical results in the area of distributed optimization and control.

In this work we provide a new partitioning scheme namely, *generalized Voronoi partition*, as a generalization of the standard Voronoi partition [4] and use it to devise strategies for optimal deployment of heterogeneous agents. The optimal deployment forms the basis for two *heterogeneous multi-agent search strategies* namely, *sequential deploy and search* and *combined deploy and search*. The heterogeneity arises from the fact that the agents have sensors with different capabilities. We provide convergence results for the search strategies and also analyze the strategies for spatial distributedness property.

The paper is organized as follows. In Section 2 we introduce a novel partitioning scheme for a Euclidean space based on Voronoi diagrams, namely *generalized Voronoi partition*. We discuss the heterogeneous multi-agent locational optimization problem in Section 3. The objective function, its critical points, the control law responsible for motion of agents and its convergence and spatial distributedness property are also discussed here. In Section 4 we formulate a multi-agent heterogeneous search problem based on the heterogeneous multi-agent locational optimization problem. We go on to propose and analytically study two multi-agent heterogeneous search strategies, namely *sequential deploy and search* and *combined deploy and search*. The paper concludes with a discussion on possible directions for future work in Section 5.

2. GENERALIZED VORONOI PARTITION

Here we present a new scheme of partitioning a given space. The concept is based on Voronoi decomposition. In case of Voronoi decomposition, a distance measure such as the Euclidean distance forms the basis on which the space is partitioned. A few generalizations such as weighted (multiplicatively and additively weighted) Voronoi partitions have been used in some applications [4]. We propose a partitioning scheme based on a collection of functions called node functions along with nodes or generators.

Consider a space $Q \subset \mathbb{R}^d$, a set of points called *nodes* or *generators* $\mathcal{P} = \{p_1, p_2, \ldots, p_N\} \in Q$, with $p_i \neq p_j$, whenever $i \neq j$, and a strictly decreasing function $f_i : \mathbb{R}^+ \mapsto R$, called *node function* for the i-th node. The collection $\{f_i\}$ satisfies the condition that $f_i - f_j$ is analytic $\forall i, j \in [1, N]$. Define a collection $\{V_i\}$, $i \in [1, N]$, with mutually disjoint interiors, such that $Q = \cup_{i \in [1,N]} V_i$, where V_i is defined as

$$V_i = \{q \in Q | f_i(\|p_i - q\|) \geq f_j(\|p_j - q\|), j \neq i, \quad j \in [1, N]\} \tag{1}$$

We call $\{V_i\}$, $i \in [1, N]$, as a *generalized Voronoi partition* of Q with nodes $\mathcal{P}$ and node functions f_i. Note that

1. Whenever $q = p_i$, either $q \in V_i$ or $V_i = \emptyset$.

---

*Guruprasad K. R. is on leave from the Department of Mechanical Engineering, National Institute of Technology Karnataka, Surathkal, 575025, India and is presently pursuing doctoral studies at Guidance, Control, and Decision Systems Laboratory, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, 560012, India. email: krgprao@gmail.com

†Debasish Ghose is Professor at Guidance, Control, and Decision Systems Laboratory, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, 560012, India. email: dghose@aero.iisc.ernet.in

Cite as: Multi-Agent Search using Sensors with Heterogeneous Capabilities (Short Paper), Guruprasad K. R. and Debasish Ghose, *Proc. of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, Padgham, Parkes, Müller and Parsons (eds.), May,12-16.,2008,Estoril,Portugal,pp. 1397-1400.

2. V_i can be non-connected and may contain other Voronoi cells inside.

3. $q \in V_i$ means that $f_i(\| p_i - q \|)$ is the maximum of all other node functions. In the context of the multi-agent search problem discussed later, $q \in V_i$ means that the i-th agent is most effective in performing the search task at the point q.

4. If f_is are strictly increasing, then "$\geq$" is replaced by "$\leq$" in (1) as in a minimization problem where the f_i are used as a kind of penalty functions.

5. The condition $f_i - f_j$ is analytic ensures that the intersection between any two partitions is a set of measure zero, that is, it should be made up of union of at most $d-1$ dimensional subsets of $\mathbb{R}^d$. Otherwise the requirement that the cells should have mutually disjoint interiors may be violated.

6. The restriction of monotonicity on the node functions can be relaxed. Restriction of monotonicity is imposed so that the partitioning makes some sense in practical applications such as multi-agent search discussed here.

7. Use of a general metric in place of the Euclidean metric leads to further generalization.

8. The generators too can be general objects such as lines, curves, or polytopes instead of points. In such cases the distance measure can be suitably defined.

Remark 1: It can be shown that if the node functions are homogeneous (and strictly increasing/decreasing), then the generalized Voronoi partition is the same as the standard Voronoi partition.
Remark 2: The multiplicatively and/or additively weighted Voronoi partitions, Voronoi partitions with non-Euclidean metric or pseudo-metric, and with general objects such as lines, curves, discs, polytopes etc., in place of points [4] can also be viewed as special cases of the generalized Voronoi partition (1).

2.1 Generalized Delaunay Graph

Delaunay graph is the dual of Voronoi partition. Two nodes are said to be neighbors (connected by an edge), if corresponding Voronoi cells are adjacent. This concept can be extended to generalized Voronoi partitioning scheme. For simplicity we call such a graph a *Delaunay graph*, $\mathcal{G}_D$.

Two nodes are said to be neighbors in a *generalized Delaunay graph*, if the corresponding *generalized Voronoi* cells are adjacent, that is, $(i,j) \in \mathcal{E}_{\mathcal{G}_D}$, the edge set corresponding to the graph $\mathcal{G}_D$, if $V_i \cap V_j \neq \emptyset$.

3. HETEROGENEOUS LOCATIONAL OPTIMIZATION PROBLEM

Here we use the concept of generalized Voronoi partitions to solve heterogeneous locational optimization problem. Let $Q \subset \mathbb{R}^d$ be a convex polytope; $\phi : Q \mapsto [0,1]$, be a density distribution function; $\mathcal{P} = (p_1, p_2, \ldots, p_N) \in Q^N$ be the configuration of N agents, with $p_i \neq p_j$ whenever $i \neq j$; $f_i : \mathbb{R}^+ \mapsto \mathbb{R}$, $i \in [1, N]$, be continuous, strictly decreasing function corresponding to i-th node and $V_i \subset Q$ be the generalized Voronoi cell corresponding to the i-th node.

Consider the objective function to be maximized,

$$\begin{aligned} \mathcal{H}(\mathcal{P}) &= \int_Q \max_i \{ f_i(\|q - p_i\|) \} \phi(q) dQ \\ &= \sum_i \int_{V_i} f_i(\|q - p_i\|) \phi(q) dQ \end{aligned} \tag{2}$$

where $\|.\|$ is the Euclidean distance.

Maximizing the objective function (2) can be interpreted as locating the nodes in such a way, that with respect to the density distribution ϕ, and the node functions f_is, the node configuration $\mathcal{P}^*$ is optimal.

3.1 The critical points

The gradient of the objective function (2) with respect to p_i, the location of the i-th node in Q, is given by,

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial p_i} &= \int_{V_i} \phi(q) \frac{\partial f_i(r_i)}{\partial p_i} dQ \\ &= -\int_{V_i} \tilde{\phi}(q)(p_i - q) dQ = -\tilde{M}_{V_i}(p_i - \tilde{C}_{V_i}) \end{aligned} \tag{3}$$

where $r_i = \|q - p_i\|$ and $\tilde{\phi}(q) = -\phi(q) \partial f_i(r_i) / \partial (r_i)^2$, which is always non-negative as f_i is strictly decreasing function of $r \in \mathbb{R}^+$. Here $\tilde{M}_{V_i}$ and $\tilde{C}_{V_i}$ are the mass and centroid of the cell V_i with $\tilde{\phi}$ as density. Thus the critical points are $p_i = C_{V_i}$, and such a configuration $\mathcal{P}$ of agents is called *centroidal Voronoi configuration.*

THEOREM 1. *The gradient, given by (3), is spatially distributed over the Delaunay graph $\mathcal{G}_D$.*

Here by spatial distributed function, we mean information from only neighboring nodes suffices to compute the value of the function at a node. We skip the proof here as it is fairly straightforward.
Note : If for some $i \in [1, N]$, $V_i = \emptyset$, then $C_{V_i} = p_i$.
Remark 3: The critical points are not unique, as with the standard Voronoi partition. But in the case of a generalized Voronoi partition, some of the cells could become null and such a condition can lead to local minima.

3.2 The control law

Let us consider the system dynamics as

$$\dot{p}_i = u_i \tag{4}$$

Consider the control law

$$u_i = -k_{prop}(p_i - \tilde{C}_{V_i}) \tag{5}$$

Control law (5) makes the robots move towards $\tilde{C}_{V_i}$ for positive k_{prop}.
Remark 4: It is not necessary that $C_{V_i} \in V_i$, but $C_{V_i} \in Q$ is true ensuring that Q is an invariant set for (4) under (5).

THEOREM 2. *The trajectories of the robots governed by the control law (5), starting from any initial condition $\mathcal{P}(0) \in Q^N$, will asymptotically converge to the critical points of $\mathcal{H}$.*

Proof. Here we will use LaSalle's invariance principle [5], which is basically an extension of Lyapunov's theorem requiring $\dot{V}$ to be *negative semi-definite* rather than *negative definite* as in Lyapunov's theorem, and the candidate function V need not be *positive definite* (see Remark on Theorem 3.8 in [5] pp 90-91).

Consider $V(\mathcal{P}) = -\mathcal{H}$, where $\mathcal{P} = (p_1, p_2, ..., p_N)$ represents the configuration of N robots.

$$\dot{V}(\mathcal{P}) = -\frac{d\mathcal{H}}{dt} = -\sum_i \frac{\partial \mathcal{H}}{\partial p_i} \dot{p}_i = -2\alpha k_{prop} \sum_i \tilde{M}_{V_i}(p_i - \tilde{C}_{V_i})^2 \tag{6}$$

We observe that $V : Q \mapsto \mathbb{R}$ is continuously differentiable in Q, $M = Q$ is a compact invariant set, $\dot{V}$ is negative definite in M, $E = \dot{V}^{-1}(0) = \{\tilde{C}_{V_i}\}$and E itself is the largest invariant subset of E by the control law (5).

Thus by LaSalle's invariance principle, the trajectories of the robots governed by control law (5), starting from any initial configuration $\mathcal{P}(0) \in Q^N$, will asymptotically converge to set N, the critical points of $\mathcal{H}$, that is, the centroidal Voronoi partitions with respect to the density as perceived by the sensors. □

4. HETEROGENEOUS MULTI-AGENT SEARCH

We consider a multi-agent search problem with heterogeneous sensors. N agents equipped with sensors are deployed in the search space $Q \subset \mathbb{R}^d$ to perform search operation. Lack of information about the search space is modeled as an uncertainty density distribution $\phi : Q \mapsto [0, 1]$, which is assumed known *a priori* to all the agents at the beginning of the search operation. For simplicity we will refer to ϕ as *density*. If information is known completely at a point $q \in Q$, then $\phi(q) = 0$. Thus, the aim of the search strategy is to ensure $\phi(q) \to 0, \forall q \in Q$, as the search progresses. Let $\mathcal{P} = (p_1, p_2, \ldots, p_N) \in Q^N$ be configuration of agents, with $p_i \neq p_j$ whenever $i \neq j$.

During the search operation the density is updated as,

$$\phi_{n+1}(q) = \phi_n(q) \min_i \{\beta_i(\|p_i - q\|)\} \tag{7}$$

where $\beta_i : \mathbb{R}^+ \mapsto [0, 1]$ is a sensor detection function of i-th agent and n is iteration count which will be discussed later.

Most sensors' effectiveness decreases with Euclidean distance, thus β_i can be assumed to be strictly increasing function of the Euclidean distance. Equation (7) selects the agent i, which is most effective in performing search task at point $q \in Q$.

Now suppose that the agents have to be deployed in Q in such way as to maximize one-step uncertainty reduction, that is, maximize the effectiveness of one-step multi-agent search. Consider the objective function

$$\begin{aligned} \mathcal{H}_n &= \int_Q (\phi_n(q) - \min_i\{\beta_i(\|p_i - q\|)\}\phi_n(q))dQ \\ &= \sum_i \int_{V_i} \phi_n(q) f_i(r_i) dQ \end{aligned} \tag{8}$$

where V_i is the *generalized Voronoi cell* (1) corresponding to the i-th agent, with $f_i(.) = 1 - \beta_i(.)$ as node function and $\mathcal{P}$ as the nodes, and $r_i = \|p_i - q\|$.

The critical points

The gradient of $\mathcal{H}_n$ with respect to i-th agent position p_i can be derived in the same manner as in (3)

$$\frac{\partial \mathcal{H}_n}{\partial p_i} = -\tilde{M}_{V_i}(p_i - \tilde{C}_{V_i}) \tag{9}$$

where $\tilde{\phi}_n(q) = -\phi_n(q)\partial f(r_i)/\partial(r_i)^2$. Here $\tilde{M}_{V_i}$ and $\tilde{C}_{V_i}$ are respectively the mass and centroid of the cell V_i with $\tilde{\phi}$ as density. Thus the critical points of the objective function (8) are $\{C_{V_i}\}$, $i = 1, 2, \ldots N$.

THEOREM 3. *The gradient, given by (9), is spatially distributed over the Delaunay graph* $\mathcal{G}_D$.

Proof: Follows from Theorem 1. □

We use,

$$\beta_i(r) = 1 - k_i e^{-\alpha_i r^2}, \quad k_i \in (0, 1), \quad \alpha_i > 0, \text{ and } \quad i \in [1, N] \tag{10}$$

which models a class of sensors' sensitivity with Euclidean distance fairly well. The exponential function enables us to get a closed form solution for the critical points. We assume first order dynamics for individual agents as given by (4) and same control law as (5).

Remark 5 Theorems 1 and 2 are valid for the heterogeneous multi-agent search problem discussed in this section.

In the following sections we propose and analyze two heterogeneous multi-agent search strategies namely *Sequential Deploy and Search* and *Combined Deploy and Search.*

4.1 Sequential Deploy and Search (SDS)

In this strategy, the agents are first deployed optimally according to the objective function (8) and the search task is performed reducing the density at the end of the deployment step. This iteration of deploy and search in a sequential manner continues till the uncertainty density is reduced below a required level. The iteration count n in (7) refers to the number of deploy and search steps. The control law (5) is used to move the agents towards the critical points, that is, the centroids of the corresponding cells.

THEOREM 4. *The* sequential deploy and search *strategy is spatially distributed over the Delaunay graph* $\mathcal{G}_D$.

Proof : Follows from Theorem 3. □

THEOREM 5. *The sequential deploy and search strategy can reduce the average uncertainty to any arbitrarily small value in a finite number of iterations.*

Proof : The uncertainty density update law (7) for any $q \in Q$ with the exponential sensor detection function takes the form,

$$\phi_n(q) = (1 - k_i e^{-\alpha_i {r_i}^2})\phi_{n-1}(q) := \gamma_{n-1}\phi_{n-1}(q), \, q \in V_i \tag{11}$$

Applying the above update rule recursively, we have,

$$\phi_n(q) = \gamma_{n-1}\gamma_{n-2}\ldots\gamma_1\gamma_0\phi_0(q) \tag{12}$$

Let $D(Q) := \max_{p,q\in Q}(\| \; p - q \; \|)$, $k = \min_i\{k_i\}$ and $\alpha = \max_i\{\alpha_i\}$. It should be noted that, $0 < k < 1$, $0 \leq r_i \leq D(Q)$. $D(Q)$ is bounded as the set Q is bounded and $0 \leq \gamma_j \leq 1 - k e^{-\alpha\{D(Q)^2\}} = l$ (say), $j \in \mathbb{N}$; and $l < 1$.

Now consider the sequence $\{\Gamma\}$ defined by $\Gamma_n := \gamma_n\gamma_{n-1}\ldots \ldots\gamma_1\gamma_0 \leq l^{n+1}$, which vanishes in the limit $n \to \infty$. Thus,

$$\lim_{n\to\infty} \phi_n(q) = \lim_{n\to\infty} \Gamma_{n-1}\phi_0(q) = 0$$

As the uncertainty density ϕ vanishes at each point $q \in Q$ in the limit, the average uncertainty density over Q is bound to vanish in the limit as $n \to \infty$, implying the statement of the Theorem. □

Figure 1 shows a simulation result for SDS strategy with 5 agents.

4.2 Combined Deploy and Search (CDS)

In the *sequential deploy and search* strategy, the search task is carried out only at the end of each deployment step.

In *combined deploy and search* (CDS) strategy, the robots are governed by the same control law, but as they move towards the respective centroid, the search task is performed simultaneously. From a more practical point of view the search task could be performed in discrete intervals.

The formulation is based on the *sequential deploy and search* strategy.

$$\begin{aligned}\Delta_n \phi(q) &= \phi_{n+1}(q) - \phi_n(q) \\ &= \phi_n(q) \min_i (1 - \beta(\| p_i - q \|))\end{aligned} \tag{13}$$

which has the continuous version given by,

$$\dot{\phi}(q,t) = \min_i (1 - \beta(r_i)) \phi(q,t) \tag{14}$$

where $r_i = (\| p_i - q \|)$.

The objective function (8), used for *sequential deploy and search* strategy, is fixed for each iteration as $\phi_n(q)$ is fixed for the n-th iteration. In *combined deploy and search*, the search task takes place as the robots move continuously in time (or in every time step in case of discrete time implementation). Thus, it is appropriate to modify the objective function to be maximized as,

$$\mathcal{H}(t) = \sum_i \int_{V_i} \phi(q,t)(1 - \beta(\|p_i - q\|))dQ \tag{15}$$

The objective function depends on ϕ, which is now a function of time. In this case we do not solve the optimization problem in a formal way. It is easy to see that at any time t, the critical points of the instantaneous objective function (15) are the same as those of (8). We borrow the control law (5) from the *sequential deploy and search* strategy. It can be shown that the control law (5) will make the robots move towards the respective instantaneous centroids. The centroids depend on time explicitly in addition to implicit time dependence through the robot positions.

THEOREM 6. *The continuous time* combined deploy and search *strategy is spatially distributed over the Delaunay graph* $\mathcal{G}_D$.

Proof. The proof follows from Theorem 4. □

THEOREM 7. *The continuous time* c*ombined deploy and search strategy can reduce the average uncertainty to any arbitrarily small value in finite time.*

Proof : At any point $q \in Q$, the uncertainty density $\phi(q,t)$ is bounded below by 0. We shall look at the upper bound for the same.

We can extract a sequence from the function $\phi(q,t)$, at a given $q \in Q$. Let $\{t_0, t_1, \dots\}$ be a monotonically increasing sequence of real numbers such that $t_{i+1} = t_i + 1$.

The equation (14) can be re-written as

$$\dot{\phi}(q,t) = \gamma(t)\phi(q,t) \tag{16}$$

where $\gamma(t) = \min_i(1 - \beta(r_i))$.

Now let us define a sequence $\{\rho\}$ as, $\rho_0 = \phi(q, t_0)$ and

$$\rho_{n+1} = \{\min_{\{t_n \geq t > t_{n+1}\}} \gamma(t)\}\rho_n = \gamma_n \rho_n \tag{17}$$

where $\gamma_n = \min_{\{t_n \geq t > t_{n+1}\}} \gamma(t) < 1$. Note that $\rho_n \geq \phi(q, t_n)$,$\forall n \in \mathbb{N}$.

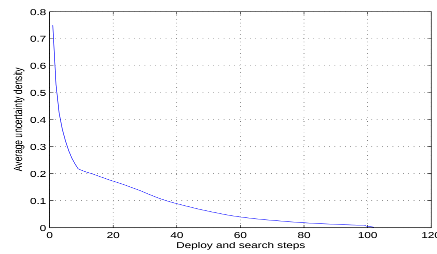

Figure 1: The average uncertainty density vs number of deploy and search steps for SDS strategy with $k = 0.8$ **and** $\alpha_1 = 0.4$, $\alpha_2 = 0.7$, $\alpha_3 = 0.1$, $\alpha_4 = 0.07$, $\alpha_5 = 0.1$ **for 5 agents**

The above equation (17) can be applied recursively to get the relationship

$$\rho_{n+1} = \gamma_n \gamma_{n-1} \dots \gamma_1 \gamma_0 \rho_0 \tag{18}$$

The sequence $\{\Gamma\}$ defined by $\Gamma_n = \prod_0{}^n \gamma_i$ converges to 0 as proved in Theorem 5, so also the sequence $\{\rho\}$. Thus, the function $\phi(q,t)$, for a given $q \in Q$, is bounded above by a sequence which converges to zero and is bounded below by the constant function 0. Thus, $\lim_{t \to \infty} \phi(q,t) = 0$, for any $q \in Q$. Hence, the average density too is bound to vanish as $t \to \infty$, implying the statement of the Theorem. □

5. CONCLUSIONS

We have introduced the concept of a *generalized Voronoi partition*. The standard Voronoi partitions and its variations were shown as special cases of the general partitioning scheme. We used *generalized Voronoi partition* to devise a locational optimization problem using heterogeneous sensors. Then we went on to formulate a heterogeneous multi-agent search problem based on these concepts. The objective function, its critical points, a control law moving the agents, its spatial distributedness and convergence properties were discussed. Two heterogeneous multi-agent search strategies namely *sequential deploy and search* and *combined deploy and search* have been proposed and their spatial distributedeness and convergence properties have been studied.

6. REFERENCES

[1] P.B. Sujit and D. Ghose, Search using multiple UAVs with flight time constraints , *IEEE Transactions on Aerospace and Electronic Systems*, Volume 40, Issue 2, April 2004 pp. 491 - 509

[2] J. Cortes, S. Martinez, and F. Bullo, Spatially-distributed coverage optimization and control with limited-range interactions, *ESAIM: Control, Optimization and Calculus of Variations* 11 (4), 2005, pp. 691-719.

[3] Q. Du, V. Faber, M. Gunzburger, Centroidal Voronoi tessellations: Applications and algorithms, *SIAM Review*, Vol 41, no. 4, 1999, pp. 637-676.

[4] F. Aurenhammer, Voronoi Diagrams - A survey of a fundamental geometric data structure. *ACM Computing Surveys*, 23(3), 1991, pp. 345-405.

[5] H. J. Marquez, *Nonlinear Control Systems - Analysis and Design*, John Wiley & Sons, Inc., 2003.