Beyond quasi-linear utility: strategy/false-name-proof multi-unit auction protocols

(Short Paper)

Yuko Sakurai   Yasumasa Saito   Atsushi Iwasaki   Makoto Yokoo
Graduate School of Information Science and Electrical Engineering
Kyushu University, Fukuoka, 819-0395 Japan
{sakurai, saito}@agent.is.kyushu-u.ac.jp
{iwasaki, yokoo}@is.kyushu-u.ac.jp

ABSTRACT

We develop strategy/false-name-proof multi-unit auction protocols that can handle non-quasi-linear utilities.

One almost universal assumption in auction theory literature is that each bidder has quasi-linear utility, except for some works on budget-constrained bidders. In particular, the celebrated VCG protocol is strongly believed to critically depend on the quasi-linear assumption and will break down if this assumption does not hold. We show that with a simple modification, the VCG can handle non-quasi-linear utilities by sacrificing efficiency to a certain extent. The basic idea of this modification is that tentative allocation and payments are determined assuming quasi-linear utilities, but each bidder can choose the actual number of units to obtain based on his non-quasi-linear utility. The modified VCG only uses the gross utility of each bidder. Requiring gross utilities only is an advantage since collecting the entire utility function can be costly. However, determining tentative allocation and payments without considering actual non-quasi-linear utilities can cause significant efficiency loss. Furthermore, the VCG is not robust against false-name-proof.

Thus, we propose a new false-name-proof open ascending auction protocol in which each bidder declares his demand for a series of prices. This protocol can improve efficiency without collecting entire utility functions.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence; K.4.4 [**Computers and Society**]: Electronic Commerce

General Terms

Economics, Theory

Keywords

Electronic Commerce, Auction mechanism design, Non-quasi-linear utility, Strategy-proofness, False-name-proofness

1. INTRODUCTION

Many theoretical/practical studies on Internet auctions have been conducted [3, 7]. One almost universal assumption of these works is that each bidder has quasi-linear utility, i.e., a bidder's utility is defined as the difference between his gross utility of allocated goods and his payment [5]. Here, gross utility means a bidder's utility for units when his payment is zero. One notable exception is works on budget-constrained bidders in single-item/multi-unit auctions [2, 6]. In these works, a bidder is assumed to have budget constraints, i.e., he cannot pay more than a predefined budget limit. His utility becomes minus infinity when his payment exceeds his limit. We can generalize such utilities to more general cases, such as piece-wise quasi-linear utility. For example, a bidder can pay up to certain amount from his own budget. If his payment exceeds that amount, he needs a loan and must pay interest. Such a utility can be represented as a separable utility function, where a bidder's utility is defined as the difference between his gross utility and an increasing function of his payment. In more general cases, we consider inseparable utility in which allocated goods and payment interact with each other and determine the utility. We can apply the well-known Vickrey-Clarke-Groves (VCG) protocol for multi-unit auctions. However, the VCG is strongly believed to critically depend on the quasi-linear assumption and will break down if this assumption does not hold. Borgs *et al.* showed the VCG is no longer strategy-proof if bidders have budget limits [2].

Furthermore, for Internet auctions, Yokoo *et al.* pointed out the possibility of a new type of fraud called false-name bids that exploit the anonymity available on the Internet [8, 11]. False-name bids are submitted under fictitious names, e.g., multiple e-mail addresses. Such deception is very difficult to detect, since identifying each participant on the Internet is virtually impossible. Although several false-name-proof multi-unit auction protocols have been developed, they also assume quasi-linear utilities.

We show a simple modification in which the VCG can handle non-quasi-linear utilities. The basic idea of this modification is that tentative allocation and payments are determined assuming quasi-linear utilities, but each bidder can choose the actual number of units to obtain based on his non-quasi-linear utility. More specifically, we utilize a general framework for describing strategy-proof protocols called the Price-Oriented Rationing-Free (PORF) protocol [9].

The modified VCG only uses the gross utility of each bid-

der. Requiring only gross utility can be an advantage since collecting entire utility functions can be costly. However, determining tentative allocation and payments without considering actual non-quasi-linear utilities can cause significant efficiency loss. Furthemore, the VCG is not false-name-proof. Therefore, we develop a new false-name-proof, open ascending auction protocol based on an existing protocol called Ascending-price OPtion allocation protocol (AOP)[4]. The AOP was developed by modifying the Ausubel auction [1]. In this protocol, a bidder declares a demand for a series of prices announced by the auctioneer. For each announced price and for each bidder, the auctioneer calculates the aggregated demand of other bidders and allocates an option to a bidder to obtain the remaining units at the current price. When the total demand becomes less than the available units, a bidder can choose the optimal option among the allocated options. This can improve efficiency without collecting entire utility functions.

The rest of this paper is organized as follows. We describe the model of this problem setting (Section 2). Next, we explain a limitation of the VCG for budget-constrained bidders (Section 3). Then, with non-quasi-linear utilities, we describe how to modify the VCG to make it strategy-proof (Section 4). Furthermore, we describe our newly developed false-name-proof open ascending auction (Section 5).

2. MODEL

Assume K units of homogeneous goods and a set of bidders $N = \{1, 2, \ldots, n\}$ where $n \geq K$. Bidder i determines his utility by privately observing a parameter or signal, θ_i. We refer to θ_i as the *type* of bidder i and assume θ_i is drawn from set Θ. Let $u(\theta_i, k, p)$ denote the utility of the bidder with type θ_i when he obtains k units and pays price p. We assume $u(\theta_i, 0, 0)$ is normalized to 0.

A utility function $u(\theta_i, k, p)$ is a *quasi-linear utility* if it is defined as the difference between the gross utility of the allocated goods and the payment: $u(\theta_i, k, p) = v(\theta_i, k) - p$. Here, $v(\theta_i, k)$ indicates bidder i's valuation of k units and is identical to $u(\theta_i, k, 0)$.

Next, *separable utility* $u(\theta_i, k, p)$ is represented as the difference between gross utility and an increasing function of payment:

$$u(\theta_i, k, p) = v(\theta_i, k) - f(\theta_i, p). \quad (1)$$

Here, we assume that f is an increasing function of p and is normalized at $f(\theta_i, 0) = 0$. Without loss of generality, we assume $\forall p, f(\theta_i, p) \geq p$ holds[1]. Utility with budget constraints and a piece-wise quasi-linear utility can be represented as special case of a separable utility function. Separable utility is a *utility with budget constraints*, if for a budget limit b_i,

$$f(\theta_i, p) = \begin{cases} p, & p \leq b_i \\ \infty, & \text{otherwise.} \end{cases} \quad (2)$$

Separable utility is *piece-wise quasi-linear* if, for a series of budget limits $b_{i,1}, b_{i,2}, \ldots, b_{i,t}$,

$$f(\theta_i, p) = \begin{cases} \alpha_1 p, & p \leq b_{i,1} \\ \alpha_j p, & b_{i,j-1} \leq p < b_{i,j} \\ \alpha_t p, & b_{i,t} < p. \end{cases} \quad (3)$$

Here, we assume $\alpha_j \geq 1$ for all j.

[1]If $f(\theta_i, p) < p$, we can change the monetary unit to satisfy this condition.

In the most general case, we can consider *inseparable utility* in which the allocated goods and the payment can interact with each other and determine the utility. For inseparable utility, we assume the following condition holds:

$$\forall k, \; \forall p \leq p' \quad u(\theta_i, k, p) - u(\theta_i, k, p') \geq p' - p. \quad (4)$$

This condition means that the utility decrease caused by the payment increase is at least (or more than) linear [2].

Now, we introduce several properties that auction protocols should satisfy. An auction protocol is *strategy-proof* if for each bidder, declaring his true type is a dominant strategy, i.e., the optimal strategy for maximizing his utility regardless of other bidders' actions. An auction protocol is *false-name-proof* if for each bidder, declaring his true type using a single identifier, while he can use false-name bids, is a dominant strategy.

3. LIMITATION OF VCG

We define the VCG for K units as follows. First, bidder i declares his type θ_i to the auctioneer. Let us denote $(v(\theta_i, 1), \cdots, v(\theta_i, K))$ as a valuation vector based on declared type. Then, the auctioneer determines an allocation and payments. For a set of feasible allocation $\mathbf{K}$ where $\mathbf{K} = \{\mathbf{k} = (k_1, \cdots, k_N) | \sum_{i \in N} k_i \leq K\}$, we define $\mathbf{k}^*$ as an optimal allocation that maximizes the sum of the declared valuations in $\mathbf{K}$, formally, $\mathbf{k}^* = \arg\max_{\mathbf{k}} \sum_{i \in N} v(\theta_i, k_i)$. Aggregated utility function V^* is defined as follows: $V^*(K, \Theta_N) = \sum_{i \in N} v(\theta_i, k_i^*)$ where $k_i^* \in \mathbf{k}^*$. Then bidder i's price for k_i^* units, $p_i(k_i^*)$, is calculated as follows:

$$p_i(k_i^*) = V^*(K, \Theta_{N \setminus \{i\}}) - V^*(K - k_i^*, \Theta_{N \setminus \{i\}}). \quad (5)$$

Borgs *et al.* showed an example where the VCG does not satisfy strategy-proofness for bidders with budget constraints. In their example, bidder i with budget limit b_i is assumed to have linear gross utility, i.e., his utility for k units is given as kc_i, where c_i is a unit value. The VCG is applied to modified valuation vector $(v'(\theta_i, 1), \cdots, v'(\theta_i, K))$ where $v'(\theta_i, k) = \min(b_i, kc_i)$.

EXAMPLE 1. *Assume* 2 *bidders take part in an auction for* 2 *units. Suppose* b_i *and* c_i *as* $(b_1, c_1) = (10, 10)$ *and* $(b_2, c_2) = (5, 3)$.

We compute the VCG payment using modified valuation vector $(v'(\theta_i, 1), v'(\theta_i, 2)) = (\min(b_i, c_i), \min(b_i, 2c_i))$. *Bidders* 1 *and* 2*'s valuation vectors are calculated as* $(10, 10)$ *and* $(3, 5)$. *Thus, bidders* 1 *and* 2 *can obtain* 1 *unit at prices* 2 *and* 0, *respectively. The utility of bidder* 1 *is calculated as* $10 - 2 = 8$.

If bidder 1 *declares his bids as* $(b_1, c_1) = (10, 5)$ *by underbidding a value per unit, he can get* 2 *units at price* 5. *As a result, bidder* 1*'s utility becomes* $20 - 5 = 15$. *This indicates that bidder* 1 *can increase his utility by underbidding.*

4. MODIFIED VCG

We show that the VCG can handle non-quasi-linear utilities with simple modification.

4.1 PORF protocol

Describing a protocol as a PORF protocol simplifies proving that it is strategy /false-name-proof [9]. The following

[2]If this condition is not satisfied, we can change the monetary unit to satisfy it.

definition of a PORF protocol covers both cases of quasi-linear and non-quasi-linear utilities. (1) bidder i declares his type $\tilde{\theta}_i$ to the auctioneer. The declared type is not necessarily his true type θ_i. (2) each price is defined for bidder i and for any number of unit $k \leq K$. These prices must be determined independently of i's declared type, while it can be dependent on the declared types of other bidders. (3) for bidder i, the optimal number of units to maximize his utility is allocated. If multiple optimal allocations exist, one of them is determined.

4.2 VCG for non-quasi-linear utility

In quasi-linear cases, the VCG can be described as a PORF protocol. The VCG price of bidder i for any k units is determined as follows:

$$p_i(k) = V^*(K, \Theta_{N \setminus \{i\}}) - V^*(K - k, \Theta_{N \setminus \{i\}}). \quad (6)$$

On the other hand, in non-quasi-linear cases, we redefine an aggregated utility function V^* as follows:

$$V^*(K, \Theta_N) = \max_{\mathbf{k} \in \mathbf{K}} \sum u(\theta_i, k_i, 0). \quad (7)$$

In the VCG for non-quasi-linear utilities, the auctioneer calculates a tentative allocation and the prices only based on the gross utility, i.e., applying formulas (6) and (7). Since the VCG is described as a PORF protocol, it is automatically strategy-proof. We only need to show that this protocol satisfies allocation feasibility.

First, we introduce one additional condition. Inseparable utility $u(\theta_i, k, p_i(k))$ satisfies *steepness condition* if the following condition holds for all $k < k'$, $p_i(k)$:

$$u(\theta_i, k, 0) - u(\theta_i, k, p_i(k)) \leq u(\theta_i, k', 0) - u(\theta_i, k', p_i(k)) \quad (8)$$

This condition means that if the allocated number of units increases, the utility decrease caused by the price increase does not become smaller. For a separable utility function, this condition is automatically satisfied, since $u(\theta_i, k, 0) - u(\theta_i, k, p_i(k))$ is equal to $f(\theta_i, p_i(k))$ for all k.

THEOREM 1. *If bidders have non-quasi-linear utilities with the steepness condition, the modified VCG protocol satisfies allocation feasibility.*

PROOF. The outline of this proof is as follows. Assume bidder i's utility is maximized by obtaining k units in quasi-linear cases. Then for all $k' > k$, the following condition must hold: $u(\theta_i, k, 0) - p_i(k) \geq u(\theta_i, k', 0) - p_i(k')$. From formulas (4), (8), and this above condition, we obtain $u(\theta_i, k, p_i(k)) \geq u(\theta_i, k', p_i(k'))$, i.e., bidder i does not want to obtain k' units, where $k' > k$. □

EXAMPLE 2. *We assume 2 bidders in an auction for 2 units, and bidders 1 and 2's gross utility $(v(\theta_i, 1), v(\theta_i, 2))$ are $(10, 20)$ and $(1, 2)$, respectively.*

As a result, the VCG prices of bidder 1 are calculated as 1 for 1 unit and 2 for 2 units. The prices of bidder 2 are calculated as 10 for 1 unit and 20 for 2 units. Here, assume that bidders 1 and 2 have a budget limit of 10 and 8, respectively. Thus, while bidder 1 can obtain 1 unit at price 10, bidder 2 can get no items because the price for 1 unit exceeds his budget.

5. NEW OPEN ASCENDING AUCTION PROTOCOL

We introduce a new false-name-proof multi-unit open ascending auction protocols for non-quasi-linear utilities.

5.1 NQ-RSA

First, we propose a false-name-proof sealed-bid auction in which a bidder declares a demand function, called the Non-Quasi-linear Residual Supply Auction (NQ-RSA) protocol. In the NQ-RSA, we use a residual supply function facing each bidder to determine the prices. We define a *demand function* $d_i(c)$ of bidder i to maximize his utility at unit price c. While a traditional definition of $d_i(c)$ assumes quasi-linear utilities, we generalize a demand function to a general case:

$$d_i(c) = \inf\{k \mid \arg\max_k u(\theta_i, k, kc)\}. \quad (9)$$

We define total demand $d(c)$ as the sum of $d_i(c)$ among a set of bidders N: $d(c) = \sum_{i \in N} d_i(c)$. Then, at price per unit c, the *residual supply function* facing bidder i denoted by $s^{\sim i}(c)$, is defined as the difference between total supply K and the number of units demanded by other bidders: $s^{\sim i}(c) = \max\{K - \sum_{l \neq i} d_l(c), 0\}$.

We can describe the NQ-RSA protocol as follows. (1) each bidder declares his type θ_i. (2) the auctioneer determines the demand function based on declared type. Next, she generates a residual supply function and calculate a price per unit $c_i(k)$ for each k units: $c_i(k) = \inf\{c \mid s^{\sim i}(c) \geq k\}$. (3) for bidder i, the optimal number of units k_i^* is determined so that it maximizes the utility: $k_i^* = \arg\max_k u(\theta_i, k, kc_i(k))$.

EXAMPLE 3. *Assume 2 units and 2 budget-constrained bidders. The minimum price unit is set to 0.1. Bidders 1 and 2's gross utility $(v(\theta_i, 1), v(\theta_i, 2))$ are $(0, 10)$ and $(6, 12)$. Also, bidders 1 and 2 have a budget b_i of 4 and 6, respectively.*

In this case, each bidder declares a demand function $d_i(c)$ as shown in Table 1.

unit price c	0	2	2.1	3	3.1
$d_1(c)$	2	2	0	0	0
$d_2(c)$	2	2	2	2	0
$\sum_{i \in \{1,2\}} d_i(c)$	4	4	2	2	0
$s^{\sim 1}(c)$	0	0	0	0	2
$s^{\sim 2}(c)$	0	0	2	2	2

Table 1: Situation of Example 3

When the unit price is 2.1, bidder 2's residual supply becomes 2. Bidder 2 can get 2 units at unit price 2.1.

Since the NQ-RSA is described as a PORF protocol, it automatically satisfies strategy-proofness. Thus, we need to show it satisfies allocation feasibility and false-name-proofness.

THEOREM 2. *The NQ-RSA satisfies allocation feasibility, formally, $\sum_{i \in N} k_i^* \leq K$.*

PROOF. By assuming $\sum_{i \in N} k_i^* > K$, we derive a contradiction. We set c_t to the minimum unit price among all c_i where $k_i^* = d_i(c_i)$. For all bidders $i \neq t$, we get $d_i(c_i) \leq d_i(c_t)$ based on $c_i \geq c_t$. Thus, we can obtain $d_t(c_t) \geq s^{\sim t}(c_t)$ based on $d_t(c_t) = k_t^* > K - \sum_{i \neq t} k_i^* \geq K - \sum_{i \neq t} d_i(c_t)$. This is inconsistent with the definition of $d_t(c_t)$. As a result, we can prove that $\sum_{i \in S} d_i(c_i) = \sum_{i \in S} k_i^* \leq K$. □

THEOREM 3. *The NQ-RSA is false-name-proof.*

PROOF. We show that bidder i cannot decrease his total payment even if the bidder uses two identifiers, i' and i'', compared with the payment when it uses one identifier, i.

Assume bidder i' can get k' units at unit price c' and bidder i'' can get k'' units at unit price c''. We also assume $c' \leq c''$. By these assumptions, we can get $d_{i'}(c') \leq K - \sum_{l \neq i'} d_l(c') \leq K - \sum_{l \neq \{i',i''\}} d_l(c') - d_{i''}(c'')$. As a result, we obtain $d_{i'}(c') + d_{i''}(c'') \leq K - \sum_{l \neq \{i',i''\}} d_l(c') \leq s^i(c')$. We can prove bidder i does not decrease his payment by submitting multiple bids. ☐

5.2 NQ-AOP

We describe a new false-name-proof open ascending auction protocol for non-quasi-linear utilities, which is called the NQ-AOP. The NQ-AOP is developed by combining the techniques of the AOP protocol and NQ-RSA.

- The auctioneer announces unit price c^l at a round $l \in \{0, \ldots, L\}$. Then each bidder declares demand based on formula (9) at a current price.

  Here, we set the conditions related to unit price and demand. First, the auctioneer cannot announce a lower price than the one called previously, i.e., $\forall l, \ c^{l-1} < c^l$. Next, bidder i cannot declare higher demand than the one declared previously, i.e., $\forall c' < c'', \ d_i(c'') \leq d_i(c')$.

- The auction is closed at a round of L when satisfying $d(c^L) \leq K < d(c^{L-1})$.

- Bidder i can choose the optimal supply to maximize utility among a set of feasible supply $\{s_i^l \mid 1 \leq l \leq L\}$:

$$s_i^l = \min\{d_i(c^l), s^{\sim i}(c^l)\}. \tag{10}$$

EXAMPLE 4. *Let us consider a situation identical to Example 3. As described in Table 1, the auctioneer raises unit price c^l. At round l when the auctioneer calls c^l, each bidder declares a demand $d_i(c^l)$. When the unit price reaches* 2.1*, the aggregate demand is equal to 2 units. Thus, the auction ends at unit price* 2.1 *and the result is identical to Example 3.*

THEOREM 4. *In the NQ-AOP, truth-telling is a weakly dominant strategy for every bidder, when he can only know whether the auction is still open.*

PROOF. We prove that bidder i cannot increase supply and improve his utility even if he declares $d_i' \neq d_i(c^l)$.

When bidder i under-declaring demand, i.e., $d_i' < d_i(c^l)$, supply s_i^l decreases by formula (10). On the other hand, when bidder i over-declaring demand, i.e., $d_i' > d_i(c^l)$, we consider two cases. If $d_i(c^l) \geq s^{\sim i}(c^l)$, bidder i's supply s_i^l remains $s^{\sim i}(c^l)$ even if he over-declares a demand. If $d_i(c^l) < s^{\sim i}(c^l)$, $d_i(c^l)$ is the optimal by formula (9). Thus, declaring d_i' cannot improve his utility. ☐

THEOREM 5. *In the NQ-AOP, truth-telling is an ex post perfect equilibrium even if every bidder can get any information about other bidders.*

Due to space limitation, we omit the rigorous proof. The same argument for proving Theorem 4 can be applied to any strategy of other bidders, as long as the strategy does not react to bidder i's action.

We can prove that the NQ-AOP satisfies allocation feasibility and false-name-proofness by applying similar argument of Theorems 2 and 3, respectively.

6. CONCLUSIONS

We developed new strategy-proof/false-name-proof auction protocols when bidders' utilities are non-quasi-linear. Studies on auction mechanism design almost universally assume quasi-linear utility of each bidder. We demonstrated that with a simple modification, the VCG can handle non-quasi-linear utilities. Furthermore, we developed a new false-name-proof open ascending auction protocol.

Our future works include a more detailed theoretical/ experimental analysis of efficiency loss and developing false-name-proof combinatorial auction protocols in non-quasi-linear utility cases.

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