Concise Characteristic Function Representations in Coalitional Games Based on Agent Types

(Extended Abstract)

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ABSTRACT

Forming effective coalitions is a major research challenge in AI and multi-agent systems. Thus, coalitional games, including coalition structure generation, have been attracting considerable attention from the AI research community. Traditionally, the input of a coalitional game is a black-box function called a characteristic function. In this paper, we develop a new concise representation scheme for a characteristic function, which is based on the idea of *agent types*. This representation can be exponentially more concise than existing concise representation schemes. Furthermore, this idea can be used in conjunction with existing schemes to further reduce the representation size.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*

General Terms

Algorithms, Theory

Keywords

Coalitional game, Coalition structure generation, Concise representation scheme

1. INTRODUCTION

Forming effective coalitions is a major research challenge in AI and multi-agent systems (MAS). A coalition of agents can sometimes accomplish things that individual agents cannot or can do things more efficiently. There are two major research topics in coalitional games. The first topic involves partitioning a set of agents into coalitions so that the sum of the rewards of all coalitions is maximized. This problem is called the Coalition Structure Generation (CSG) problem [4]. The second topic involves how to divide the value of the coalition among agents. The theory of coalitional games provides a number of solution concepts.

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A range of previous studies have found that many problems in coalitional games, including CSG, tend to be computationally intractable. Traditionally, the input of a coalitional game is a black-box function called a characteristic function, which takes a coalition as an input and returns the value of the coalition (or a coalition structure as a whole). Recently, several concise representation schemes for a characteristic function have been proposed, e.g., synergy coalition group (SCG) [1] and marginal contribution nets (MC-nets) [2]. These schemes represent a characteristic function as a set of rules rather than as a single black-box function and can effectively reduce the representation size. However, most problems are still computationally intractable.

In this paper, we develop a new concise representation scheme for a characteristic function, which is based on the idea of *agent types*. Intuitively, a type represents a set of agents, which are recognized as having the same contribution. Most of the hardness results in existing works are obtained by assuming that all agents are different types. In practice, however, in many MAS application problems, while the number of agents grows, the number of different types of agents remains small. This type-based representation can be exponentially more concise than existing concise representation schemes. Furthermore, this idea can be used in conjunction with existing schemes, i.e., SCG and MC-nets, for further reducing the representation size. We show that most of the problems in coalitional games, including CSG, can be solved in polynomial time in the number of participating agents, assuming the number of possible types t is fixed.

Our idea of using agent types is inspired by the recent innovative work of Shrot *et al.* [5]. They assume that a game is already represented in some concise representation, e.g., SCG. The goal of their work is first to identify agent types and then to efficiently solve problems in coalitional games by utilizing the knowledge of agent types. This approach becomes infeasible when a standard characteristic function representation is used, since there exists no efficient way for identifying agent types. In contrast to their study, we assume that agent types are explicitly used for describing a characteristic function in the first place. Also, we consider a wider range of problems including CSG. As a result, the overlap between our work and that of [5] is very small. Core non-empty and the Shapley value for SCG might be considered as somewhat overlapping, while other topics are not discussed in [5].

2. MODEL

Let $A = \{1, 2, \dots, n\}$ be a set of all agents. The value of a coalition S is given by a characteristic function v. A characteristic function $v : 2^A \to \mathbb{R}$ assigns a value to each set of agents (coalition) $S \subseteq A$. We assume that each coalition's value is non-negative.

A coalition structure CS is a partition of A, into disjoint, exhaustive coalitions. More precisely, $CS = \{S_1, S_2, \dots\}$ satisfies the following conditions: $\forall i, j\ (i \neq j),\ S_i \cap S_j = \phi,\ \bigcup_{S_i \in CS} S_i = A$. In other words, in CS, each agent belongs to exactly one coalition, and some agents may be alone in their coalitions.

The value of a coalition structure CS, denoted as $V(CS)$, is given by: $V(CS) = \sum_{S_i \in CS} v(S_i)$. An optimal coalition structure CS^* is a coalition structure that satisfies the following condition: $\forall CS, V(CS^*) \geq V(CS)$. We say a characteristic function is super-additive, if for any disjoint sets $S_i, S_j, v(S_i \cup S_j) \geq v(S_i) + v(S_j)$ holds. If the characteristic function is super-additive, solving CSG becomes trivial, i.e., the grand coalition is optimal. In this paper, we assume a characteristic function can be non-super-additive.

3. TYPE-BASED CHARACTERISTIC FUNCTION REPRESENTATION

Shrot *et al.* [5] introduced the idea of using *agent types* to reduce the computational complexity of coalition formation problems. If two agents have the same type, their marginal contributions are the same. They introduced two different notions of agent types, i.e., *strategic types* and *representational types*. The former defines types based on the strategic power of the agents, and the latter defines them based on the representation of the game.

In this paper, we propose an alternate approach. We assume the person who is describing a game has some prior information about the equivalence of agents. Then the person will describe the game by explicitly using the information of the agent types of which he/she is aware. We need another notion of agent types. This is because (i) the information of the person can be partial and he/she is not necessarily aware of all strategic equivalence, and (ii) the equivalence that he/she is aware of is representation-independent. Therefore, we introduce another notion called *recognizable types*. If two agents are recognizably equivalent, they have the same type.

Definition 1 *Agents $i, j \in A$ are recognizably equivalent if the person who is describing the game (either by a characteristic function or by a concise representation) knows that for any coalition S, such that $i, j \notin S : v(S \cup \{i\}) = v(S \cup \{j\})$.*

Let $T = \{1, 2, \dots, t\}$ be the set of all recognizable types and n_A^i be the number of agents of type $i \in T$ in the set of all agents A. Also, $n_A = \langle n_A^1, n_A^2, \dots, n_A^t \rangle$ denotes a vector, where each element represents the number of agents of each type in A.

We represent a characteristic function as follows:

Definition 2 *For a coalition S, the coalition type of S is a vector $n_S = \langle n_S^1, n_S^2, \dots, n_S^t \rangle$, where each n_S^i is the number of type i agents in S. We denote the set of all possible coalition types as $A^t = \{\langle n^1, n^2, \dots, n^t \rangle \mid 0 \leq n^i \leq n_A^i\}$. A type-based characteristic function is defined as $v_t : A^t \to \mathbb{R}$.*

From the definition of recognizable equivalence, $\forall S$ and its type n_S, $v(S) = v_t(n_S)$ holds.

Theorem 1 *A type-based characteristic function requires $O(n^t)$ space.*

A type-based characteristic function representation can be used in conjunction with SCG and MC-nets. If the number of agent types t is fixed, by using type-based representations, most of the problems in coalitional games, including CSG, can be solved in polynomial time in the number of agents.

4. COALITION STRUCTURE GENERATION WITH AGENT TYPES

In this section, we develop an algorithm for the CSG problem based on knapsack problems [3]. A multidimensional unbounded knapsack problem (MUKP) is the knapsack problem, where the knapsack has multidimensional constraint and multiple copies exist for each item. For each item j, we denote the profit as p_j, the weight of the i-th constraint as w_{ij}, and the number of copies packed in the knapsack as q_j. A MUKP with m items and t constraints of knapsack $c_1, \dots, c_t$ is formalized as follows:

$$\begin{array}{ll} \text{maximize} & \sum_j p_j q_j \\ \text{subject to} & \sum_j w_{ij} q_j \leq c_i,\ i = 1, \dots, t \\ & q_j \geq 0,\ j = 1, \dots, m \end{array}$$

Theorem 2 *By using a type-based characteristic function representation, finding an optimal coalition structure can be done in $O(n^{2t})$ time.*

PROOF SKETCH. We show that a CSG problem with $m = |A^t|$ coalition types and t possible agent types can be formalized as a MUKP with m items and t constraints. Let us assume that one possible coalition type $n_{S_j} \in A^t$ corresponds to item j, where its value p_j is equal to $v_t(n_{S_j})$ and its weight for the i-th constraint is equal to $n_{S_j}^i$. The capacity constraint of knapsack c_i is determined by n_A^i.

We can construct a dynamic programming based algorithm, which takes $O(n^t \times |A^t|) = O(n^{2t})$ steps (see Section 9.3.2 in [3]). Thus, for any fixed t, finding an optimal coalition structure can be done in $O(n^{2t})$ time. □

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