Solving Strategic Bargaining with Arbitrary One–Sided Uncertainty

(Extended Abstract)

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ABSTRACT

Bilateral bargaining has received a lot of attention in the multi–agent literature and has been studied with different approaches. According to the strategic approach, bargaining is modeled as a non–cooperative game with uncertain information and infinite actions. Its resolution is a long–standing open problem and no algorithm addressing uncertainty over multiple parameters is known. In this paper, we provide an algorithm to solve bargaining with any kind of one–sided uncertainty. Our algorithm reduces a bargaining problem to a finite game, solves this last game, and then maps its strategies with the original continuous game. We prove that with multiple types the problem is hard and only small settings can be solved in exact way. In the other cases, we need to resort to concepts of approximate equilibrium and to abstractions for reducing the size of the game tree.

Categories and Subject Descriptors

I.2.11 [**Computing Methodologies**]: Distributed Artificial Intelligence

General Terms

Algorithms

Keywords

Game Theory (cooperative and non–cooperative), Bargaining, Negotiation

1. INTRODUCTION

The automation of economic transactions through negotiating software agents is receiving a large attention in the artificial intelligence community. Autonomous agents can lead to economic contracts more efficient than those drawn up by humans, saving also time and resources [10]. We focus on the main bilateral negotiation setting: the *bilateral bargaining*. This setting is characterized by the interaction of two agents, a *buyer* and a *seller*, who can cooperate to produce a utility surplus by reaching an economic agreement, but they are in conflict on what specific agreement to reach. Several approaches for bargaining are currently studied. In this paper, we focus on strategic bargaining where agents are assumed to be rational and a bargaining situation is modeled as a non–cooperative game [1]. The most expressive model is the Rubinstein's *alternating–offers* [9]: agents alternately act in turns and each agent can accept the offer made by her opponent at the previous turn or make a new offer. Agents' utility over the agreements depends on some parameters: *discount factor* (δ), *deadline* (T), *reservation price* (RP). In real–world settings, the values of these parameters are private information of the agents who have a Bayesian prior over the values of the opponent.

The game theoretic study of bargaining with uncertain information is an open challenging problem. Although it has been studied for about 30 years, no work presented in the literature so far is applicable regardless of the uncertainty *kind* (i.e., the uncertain parameters) and *degree* (i.e., the number of the parameters' possible values). The literature provides several heuristics–based approaches generally applicable to any uncertain setting, while the optimal approaches work only with very narrow uncertainty settings. In particular, no algorithm works with uncertainty over multiple parameters.

2. PROPOSED APPROACH

We consider the alternating–offers protocol [9] with deadlines in which there are two agents, a buyer $\mathbf{b}$ and a seller $\mathbf{s}$, who can play alternatively at discrete time points $t \in \mathbb{N}$. We focus on one–sided uncertain settings where the buyer's parameters are uncertain to the seller (the reverse situation is analogous). According to [3], our game is an imperfect–information game in which the buyer can be of different types, each one with different values of $RP_{\mathbf{b}}$, $\delta_{\mathbf{b}}$, and $T_{\mathbf{b}}$. Uncertainty is over the actual type of the buyer.

The appropriate solution concept is the sequential equilibrium [5]. It is a couple $a = (\mu, \sigma)$, also called assessment, in which μ is a belief system that specifies how agents must update their beliefs during the game and σ is the agents' strategy profile that specifies how they must act. μ must be *consistent* with σ and σ must be *sequentially rational* given μ.

Since bargaining with uncertainty may not admit any equilibrium in pure strategies, as shown in [2], we directly search for equilibria in mixed strategies. The basic idea behind our work is to solve the bargaining problem by reducing it to a

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finite game, deriving equilibrium strategies such that on the equilibrium path the agents can act only a finite set of actions, and then by searching for the agents' optimal strategies on the path. Our work is structured in the following three steps.

1. We analytically derive an assessment $\overline{a} = (\overline{\mu}, \overline{\sigma})$ in which the randomization probabilities of the agents are parameters and such that, when the parameters' values satisfy some conditions, $\overline{a}$ is a sequential equilibrium.
2. We formulate the problem of finding the values of the agents' randomization probabilities in $\overline{a}$ as the problem of finding a sequential equilibrium in a reduced bargaining game with finite actions, and we prove that there always exist values such that $\overline{a}$ is a sequential equilibrium.
3. We develop an algorithm based on linear complementarity mathematical programming to solve the case with multiple types.

3. SOLUTION WITH MULTIPLE TYPES

Due to space limitation, we report only how the game tree is constructed and how the equilibrium strategy can be found.

The construction of the game tree is accomplished according to the following rules:

1. no buyer's types makes offer strictly weaker than her optimal offer in the complete–information game;
2. at time $t > 0$, no agent (buyer and seller) makes offers strictly weaker (w.r.t. her utility function) than the one made by the opponent at the previous time point $t - 1$;
3. at time $t > 0$, no agent (buyer and seller) makes offers that, if accepted at $t + 1$, provide her the same utility she receives by accepting the offer made by the opponent at $t - 1$;
4. no buyer's type makes offers besides $\min\{T_{\mathbf{b}_i}, T_{\mathbf{s}}\}$ and the seller does not make offer besides $\min\{\max\{T_{\mathbf{b}_i}\}, T_{\mathbf{s}}\}$;
5. at time $t > 0$, an offer x_i is not made if the buyer's type $\mathbf{b}_i$ is out of the game (i.e., $t >= T_{\mathbf{b}_i}$ or type $\mathbf{b}_i$ has been excluded because the buyer has previously made an offer strictly weaker than the optimal complete–information offer of $\mathbf{b}_i$).

It can be easily observed that the size of the tree rises exponentially in the length of the deadlines.

To compute an equilibrium, at first we represent the game in the sequence form [4] where agents' actions are sequences in the game tree. The computation of Nash equilibria in a game in sequence-form can be accomplished by applied different algorithms presented in the literature. To find sequential equilibria, such algorithms should be extended by introducing perturbations in their mathematical programming formulation, as is shown in [7].

We implemented an *ad hoc* version of the Lemke's algorithm with perturbation as described in [7] to compute a sequential equilibrium. The algorithm is based on pivoting (similarly to the simplex algorithm) where perturbation affects only the choice of the leaving variable. We coded the algorithm in C language by using integer pivoting and the same approach of the revised simplex (to save time during the update of the rows of the tableau). We executed our algorithm with a 2.33 GHz 8 GB RAM UNIX computer. We produced several bargaining instances characterized by the number of buyer's types (from 2 up to 6) and the deadline $T = \min\{\max\{T_{\mathbf{b}_i}\}, T_{\mathbf{s}}\}$ (from 6 up to 500). Tab. 1 reports the average computational times over 10 different bargaining instances; we denote by '–' when execution exceeds one hour.

T	number of buyer's types				
	2	3	4	5	6
6	< 0.01 s	0.06 s	0.29 s	3.47 s	929.73 s
8	< 0.01 s	1.32 s	32.94 s	1890.96 s	–
10	< 0.01 s	15.16 s	2734.29 s	–	–
12	< 0.01 s	211.11 s	–	–	–
14	< 0.01 s	3146.20 s	–	–	–
50	0.22 s	–	–	–	–
100	1.55 s	–	–	–	–
500	175.90 s	–	–	–	–

Table 1: Computational times for solving a bargaining game with linear complementarity mathematical programming ($T = \min\{\max\{T_{\mathbf{b}_i}\}, T_{\mathbf{s}}\}$).

As it can be observed, the computational times are exponential in the bargaining length and have the number of types as basis and only small settings can be solved by using linear–complementarity mathematical programming.

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