Epistemic Coalition Logic: Completeness and Complexity

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ABSTRACT

Coalition logic is currently one of the most popular logics for multi-agent systems. While logics combining coalitional and epistemic operators have received considerable attention, completeness results for epistemic extensions of coalition logic have so far been missing. In this paper we provide several such results and proofs. We prove completeness for epistemic coalition logic with common knowledge, with distributed knowledge, and with both common and distributed knowledge, respectively. Furthermore, we completely characterise the complexity of the satisfiability problem for each of the three logics.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent Systems; I.2.4 [**Knowledge representation formalisms and methods**]

General Terms

Theory

Keywords

Epistemic logic, coalition logic, completeness, computational complexity

1. INTRODUCTION

Logics of coalitional ability such as *Coalition Logic* ($\mathcal{CL}$) [17], *Alternating-time Temporal Logic* ($\mathcal{ATL}$) [1], and STiT logics [2], are arguably one of the most popular types of logics in multi-agent systems research in recent years. Many different variants of these logics have been proposed and studied. Most of the obtained meta-logical results have been about computational complexity and expressive power. Completeness results have been harder to obtain, with Goranko's and van Drimmelen's completeness proof for $\mathcal{ATL}$ [8], Pauly's completeness proof for $\mathcal{CL}$ [17] and Broersen and colleagues' completeness proofs for different variants of STiT logic [4, 3, 12] being notable exceptions.

The main construction in coalitional ability logics is of the form $[G]\phi$, where G is a set of agents and ϕ a formula, intuitively meaning that G is *effective* for ϕ, or that G can make ϕ come true no matter what the other agents do. One of the most studied extensions of basic coalitional ability logics is adding *knowledge* operators of the type found in *epistemic logic* [5, 16]: both *individual* knowledge operators K_i where i is an agent, and different types of *group* knowledge operators E_G, C_G and D_G where G is a group of agents, standing for everybody-knows, common knowledge and distributed knowledge, respectively. Combining coalitional ability operators and epistemic operators in general and group knowledge operators in particular lets us express many potentially interesting properties of multi-agent systems, such as [19]:

- $K_i\phi \rightarrow [\{i\}]K_j\phi$: i can communicate her knowledge of ϕ to j
- $C_G\phi \rightarrow [G]\psi$: common knowledge in G of ϕ is sufficient for G to ensure that ψ
- $[G]\psi \rightarrow D_G\phi$: distributed knowledge in G of ϕ is necessary for G to ensure that ψ
- $D_G\phi \rightarrow [G]E_G\phi$: G can cooperate to make distributed knowledge explicit

In this paper we study axiomatisation and complexity of variants of *epistemic coalition logic* ($\mathcal{ECL}$), extensions of coalition logic with individual knowledge and different combinations of common knowledge and distributed knowledge. Coalition logic, the next-time fragment of $\mathcal{ATL}$, is one of the most studied coalitional ability logics, and this paper settles a key problem: completeness of its standard epistemic extensions with group knowledge. We furthermore completely characterise the computational complexity of the satisfiability problem for these extensions.

The combinations of coalitional ability operators and epistemic operators in the logics we study in this paper are *independent*; the original semantics of the operators is not changed. It is well known [14, 13] that there are several interesting variants of "ability" under imperfect knowledge; e.g., being able to achieve something without knowing it, vs. knowing *that* one is able to achieve something but not necessarily knowing *how*, vs. knowing *how* one can achieve something. While the two former examples can be expressed with combinations of operators with standard semantics ($[\{i\}]\phi \wedge \neg K_i[\{i\}]\phi$ and $K_i[\{i\}]\phi$ respectively, in the case of a single agent), in order to be able to express the latter (knowledge of ability "de re"), operators with alternative semantics are needed [14, 18, 11, 13]. We do not consider such operators in the current paper. Even though $\mathcal{ECL}$ with standard semantics cannot express knowledge of ability "de re", it can

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express many other interesting properties (including the examples above as well as the other "variants" of ability under imperfect knowledge).

While epistemic coalitional ability logics have been studied to a great extent, we are not aware of any published completeness results for such logics with all epistemic operators. [19] gives some axioms of $\mathcal{ATEL}$, $\mathcal{ATL}$ extended with epistemic operators, but does not attempt to prove completeness[1]. Broersen and colleagues [3, 12] prove completeness of variants of STiT logic that include individual knowledge operators, but not group knowledge operators, and [12] concludes that adding group operators is an important challenge.

The rest of the paper is organised as follows. In the next section we first give a brief review of coalition logic, and how it is extended with epistemic operators. We then, in each of the three following sections, consider basic epistemic coalition logic with individual knowledge operators extended with common knowledge, with distributed knowledge, and with both common and distributed knowledge, respectively. For each of these cases we show a completeness result. The reason that we consider each of these three systems separately, rather than only the most expressive logic with both common and distributed knowledge, is first, that we want to carefully chart the results for different combinations of operators (a common practice, also in epistemic logic), and, second, that separate proofs for the common and distributed knowledge cases are useful for further extensions for logics with only these epistemic operators. In Section 6 we consider the computational complexity of the three systems. We conclude in Section 7.

2. BACKGROUND

We will define several extensions of propositional logic, and the usual derived connectives, such as $\phi \to \psi$ for $\neg\phi \vee \psi$, will be used. We will also define a number of axiomatic systems S, and by $\vdash_S \phi$ we mean that the formula ϕ is derivable in system S.

2.1 Coalition Logic

We give a brief overview of Coalition Logic ($\mathcal{CL}$) [17]. Assume a set Θ of atomic propositions, and a finite set N of agents. A *coalition* is a set $G \subseteq N$ of agents. We sometimes abuse notation and write a singleton coalition $\{i\}$ as i.

The language of $\mathcal{CL}$ is defined by the following grammar:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid [G]\phi$$

where $p \in \Theta$ and $G \subseteq N$.

A *coalition model* is a tuple

$$M = \langle S, E, V \rangle$$

where

- S is a non-empty set of *states*;
- V is a *valuation function*, assigning a set $V(s) \subseteq \Theta$ to each state $s \in S$;
- E assigns a *truly playable effectivity function* (see below) $E(s)$ over N and S to each state $s \in S$.

[1] In an unpublished abstract of a talk given at the LOFT workshop in 2004 [7], the authors propose an axiomatisation of $\mathcal{ATEL}$ with individual knowledge and common knowledge operators. However, a completeness result or proof has not been published (personal communication, Valentin Goranko).

An *effectivity function* [17] over N and a set of states S is a function E that maps any coalition $G \subseteq N$ to a set of sets of states $E(G) \subseteq 2^S$. An effectivity function is *truly playable* [17, 6] iff it satisfies the following conditions (when $X \subseteq S$, $\overline{X}$ denotes the complement $S \setminus X$):

E1 $\forall s \in S \forall G \subseteq N : \emptyset \notin E(G)$ (Liveness)

E2 $\forall s \in S \forall G \subseteq N : S \in E(G)$ (Safety)

E3 $\forall s \in S \forall X \subseteq S : \overline{X} \notin E(\emptyset) \Rightarrow X \in E(N)$ (N-maximality)

E4 $\forall s \in S \forall G \subseteq N \forall X \subseteq Y \subseteq S : X \in E(G) \Rightarrow Y \in E(G)$ (outcome monotonicity)

E5 $\forall s \in S \forall G_1, G_2 \subseteq N \forall X, Y \subseteq S : X \in E(G_1)$ and $Y \in E(G_2) \Rightarrow X \cap Y \in E(G_1 \cup G_2)$, where $G_1 \cap G_2 = \emptyset$ (superadditivity)

E6 $E^{nc}(\emptyset) \neq \emptyset$, where $E^{nc}(\emptyset)$ is the *non-monotonic core* of the empty coalition, namely

$$E^{nc}(\emptyset) = \{X \in E(\emptyset) : \neg\exists Y (Y \in E(\emptyset) \text{and } Y \subset X)\}$$

An effectivity function that only satisfies E1-E5 is called *playable*. On finite domains an effectivity function is playable iff it is truly playable [6], because on finite domains E6 follows from E1-E5.

A $\mathcal{CL}$ formula is interpreted in a state s in a coalition model M as follows:

$M, s \models p$ iff $p \in V(s)$

$M, s \models \neg\phi$ iff $M, s \not\models \phi$

$M, s \models (\phi_1 \wedge \phi_2)$ iff ($M, s \models \phi_1$ and $M, s \models \phi_2$)

$M, s \models [G]\phi$ iff $\phi^M \in E(s)(G)$

where $\phi^M = \{t \in S : M, t \models \phi\}$.

The axiomatisation CL of coalition logic consist of the following axioms and rules:

Prop Substitution instances of propositional tautologies

G1 $\neg[G]\bot$

G2 $[G]\top$

G3 $\neg[\emptyset]\neg\phi \to [N]\phi$

G4 $[G](\phi \wedge \psi) \to [G]\psi$

G5 $[G_1]\phi \wedge [G_2]\psi \to [G_1 \cup G_2](\phi \wedge \psi)$, if $G_1 \cap G_2 = \emptyset$

MP $\vdash_{CL} \phi, \phi \to \psi \Rightarrow \vdash_{CL} \psi$

RG $\vdash_{CL} \phi \leftrightarrow \psi \Rightarrow \vdash_{CL} [G]\phi \leftrightarrow [G]\psi$

CL is sound and complete wrt. all coalition models [17].

The following *monotonicity rule* is derivable [17], and will be useful later:

Mon $\vdash_{CL} \phi \to \psi \Rightarrow \vdash_{CL} [G]\phi \to [G]\psi$

2.2 Adding Knowledge Operators

Epistemic extensions of coalition logic were first proposed in [19][2]. They are obtained by extending the language with *epistemic operators*, and the models with *epistemic accessibility relations*.

An epistemic accessibility relation for agent i over a set of states S is a binary equivalence relation $\sim_i \subseteq S \times S$. An *epistemic coalition model*, henceforth often simply called a *model*, is a tuple

$$M = \langle S, E, \sim_1, \ldots, \sim_n, V \rangle$$

where $\langle S, E, V \rangle$ is a coalition model and $\sim_i$ is an epistemic accessibility relation over S for each agent i.

Epistemic operators come in two types: individual knowledge operators K_i, where i is an agent, and group knowledge operators C_G and D_G where G is a coalition for expressing *common knowledge* and *distributed knowledge*, respectively. Formally, the language of $\mathcal{CLCD}$ (*coalition logic with common and distributed knowledge*), is defined by extending coalition logic with all of these operators:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid [H]\phi \mid K_i\phi \mid C_G\phi \mid D_G\phi$$

where $p \in \Theta$, $i \in N$, $H \subseteq N$ and $\emptyset \neq G \subseteq N$. When G is a coalition, we write $E_G\phi$ as a shorthand for $\bigwedge_{i\in G} K_i\phi$ (everyone in G knows ϕ).

The languages of the logics $\mathcal{CLK}$, $\mathcal{CLC}$ and $\mathcal{CLD}$ are the restrictions of this language with no C_G and no D_G operators, no D_G operators, and no C_G operators, respectively.

The interpretation of these languages in an (epistemic coalition) model M is defined by adding the following clauses to the definition for $\mathcal{CL}$:

$M, s \models K_i\phi$ iff $\forall t \in S, (s,t) \in \sim_i \Rightarrow M, t \models \phi$

$M, s \models C_G\phi$ iff $\forall t \in S, (s,t) \in (\bigcup_{i\in G} \sim_i)^* \Rightarrow M, t \models \phi$

$M, s \models D_G\phi$ iff $\forall t \in S, (s,t) \in (\bigcap_{i\in G} \sim_i) \Rightarrow M, t \models \phi$

where R^* denotes the transitive closure of the relation R. We use $\models \phi$ to denote the fact that ϕ is *valid*, i.e., that $M, s \models \phi$ for all M and states s in M.

2.2.1 Some Auxiliary Definitions

The following are some auxiliary concepts that will be useful in the following.

Intuitively, a *pseudomodel* is like a model except that distributed knowledge is "not quite" the intersection of individual knowledge. Formally, a pseudomodel is a tuple $M = (S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, E, V)$ where $(S, \{\sim_i : i \in N\}, E, V)$ is a model and:

- $R_G \subseteq S \times S$ is an equivalence relation for each $G \subseteq N$, $G \neq \emptyset$
- For any $i \in N$, $R_i = \sim_i$
- For any G, H, $G \subseteq H$ implies that $R_H \subseteq R_G$

The interpretation of a $\mathcal{CLCD}$ formula in a state of a pseudomodel is defined as for a model, except for the case for D_G which is interpreted by the R_G relation:

$M, s \models D_G\phi$ iff $\forall t \in S, (s,t) \in R_G \Rightarrow M, t \models \phi$

[2]In that paper for $\mathcal{ATL}$; $\mathcal{CL}$ is a fragment of $\mathcal{ATL}$.

An *epistemic model* is a model without the E function, i.e., a tuple $\langle S, \sim_1, \ldots, \sim_n, V \rangle$. An *epistemic pseudomodel* is a pseudomodel without the E function, i.e., a tuple $\langle S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, V \rangle$ (where R_G has the properties above). When $M = \langle S, E, \sim_1, \ldots, \sim_n, V \rangle$ is a model or $M = \langle S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, E, V \rangle$ is a pseudomodel, we refer to $\langle S, \sim_1, \ldots, \sim_n, V \rangle$ as M's *underlying* epistemic model.

Finally, a *playable (pseudo)model* is like a (pseudo)model, except that E is not requried to satisfy the E6 property.

We say that a formula ϕ is *satisfied in* a (playable) (pseudo)model M, if $M, s \models \phi$ for some state s in M.

3. COALITION LOGIC WITH COMMON KNOWLEDGE

In this section we consider the logic $\mathcal{CLC}$, extending coalition logic with individual knowledge operators and common knowledge. The axiomatisation CLC is the result of extending CL with the following standard axioms and rules for individual and common knowledge (see, e.g., [5]):

K $K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$

T $K_i\phi \to \phi$

4 $K_i\phi \to K_iK_i\phi$

5 $\neg K_i\phi \to K_i\neg K_i\phi$

C $C_G\phi \to E_G(\phi \wedge C_G\phi)$

RN $\vdash_{CLC} \phi \Rightarrow \vdash_{CLC} K_i\phi$

RC $\vdash_{CLC} \phi \to E_G(\phi \wedge \psi) \Rightarrow \vdash_{CLC} \phi \to C_G\psi$

It is easy to show that CLC is sound wrt. all models.

LEMMA 1 (SOUNDNESS). *For any CLC-formula ϕ, $\vdash_{CLC} \phi \Rightarrow \models \phi$.*

Outline of completeness proof.

In the remainder of this section we show that $\mathcal{CLC}$ also is complete. Before giving all the details, we describe the outline of the proof. We first construct a canonical playable model M^c, using standard definitions of the canonical epistemic accessibility relations [9] and Pauly's definition of the canonical effectivity functions [17]. There are two potential problems with M^c: first, it is not necessarily truly playable (i.e., it is not necessarily a model), and, second, the truth lemma does not necessarily work for the case $C_G\phi$. To take care of these problems we filtrate M^c through an appropriately defined closure of a given consistent formula, to obtain a finite model M^f. This is a standard technique for dealing with transitive closure operators such as the Kleene star in $\mathcal{PDL}$ [10] and indeed common knowledge. In our case the standard technique must be extended to deal with the effectivity functions. For us the technique has the convenient side effect that playability and true playability coincides (E1-E5 implies E6) on the resulting model, since it is finite. However, it remains to be shown that filtration does not break the playability properties E1-E5, and that M^f satisfies the truth lemma for the combined (epistemic-coalitonal) language.

Completeness proof.

Theorem 1. *Any CLC-consistent formula is satisfied in some playable model.*

Proof. We define a canonical playable model $M^c = (S^c, \{\sim_i^c: i \in N\}, E^c, V^c)$ as follows:

S^c is the set of all maximally CLC consistent sets of formulas

$s \sim_i^c t$ iff $\{\psi : K_i\psi \in s\} = \{\psi : K_i\psi \in t\}$

$X \in E^c(s)(G)$ (for $G \neq N$) iff there exists ψ such that $\{t \in S^c : \psi \in t\} \subseteq X$ and $[G]\psi \in s$.

$X \in E^c(s)(N)$ iff $S^c \setminus X \notin E^c(s)(\emptyset)$.

V^c: $p \in V^c(s)$ iff $p \in s$

That $\sim_i^c$ is an equivalence relation is immediate. That $E^c(s)$ is playable (satisfies E1-E5) can be shown in exactly the same way as in the completeness proof for CL [17]. The idea behind the model construction of course is that a formula belongs to a state s in a model iff it is true there (the truth lemma). However, the canonical model is in general *not* guaranteed to satisfy every consistent formula in the $\mathcal{CLC}$ language; the case of C_G in the truth lemma does not necessarily hold. Therefore we are going to transform M^c by filtration into a finite model for a given CLC consistent formula ϕ. Note that since ϕ is consistent, it will belong to at least one s in M^c.

Let $cl(\phi)$ be the set of subformulas of ϕ closed under single negations and the condition that $C_G\psi \in cl(\phi) \Rightarrow K_iC_G\psi \in cl(\phi)$ for all $i \in G$. We are going to filtrate M^c through $cl(\phi)$. The resulting model $M^f = (S^f, \{\sim_i^f: i \in N\}, E^f, V^f)$ is constructed as follows:

S^f is $\{[s]_{cl(\phi)} : s \in S^c\}$ where $[s]_{cl(\phi)} = s \cap cl(\phi)$. We will omit the subscript $cl(\phi)$ in what follows for readability.

$[s] \sim_i^f [t]$ iff $\{\psi : K_i\psi \in [s]\} = \{\psi : K_i\psi \in [t]\}$

$V^f(s) = \{p : p \in [s]\}$. Again we will omit the subscript for readability.

$X \in E^f([s])(G)$ iff $\{s' : \phi_X \in s'\} \in E^c(s)(G)$ where $\phi_X = \vee_{[t]\in X}\phi_{[t]}$ and $\phi_{[t]}$ is a conjunction of all formulas in $[t]$.

We now prove by induction on the size of θ that for every $\theta \in cl(\phi)$, $M^f, [s] \models \theta$ iff $\theta \in [s]$.

case $\theta = p$ trivial

case booleans trivial

case $\theta = K_i\psi$ assume $M^f, [s] \not\models K_i\psi$. The latter means there is a $[s']$ such that $[s] \sim_i^f [s']$ and $M^f, [s'] \not\models \psi$. By the inductive hypothesis $\psi \notin [s']$. Since $[s']$ is deductively closed wrt $cl(\phi)$ and $K_i\psi \in cl(\phi)$, also $K_i\psi \notin [s']$. $[s] \sim_i^f [s']$ means that $[s]$ and $[s']$ contain the same K_i formulas from $cl(\phi)$, hence $K_i\psi \notin [s]$.

Assume $M^f, [s] \models K_i\psi$. Then for all $[s']$ such that $[s] \sim_i^f [s']$, $M^f, [s'] \models \psi$. This means by the IH that $\psi \in [s']$ for all $[s'] \sim_i^f [s]$. Assume by contradiction that $K_i\psi \notin [s]$. Then $\phi_{[s]}$, where $\phi_{[s]}$ is the conjunction of all formulas in $[s]$, is consistent with $\neg K_i\psi$. If we write $\langle K_i\rangle$ for the dual of the K_i modality, this is equivalent to: $\phi_{[s]} \wedge \langle K_i\rangle\neg\psi$ is consistent. By forcing choices,

$$\phi_{[s]} \wedge \langle K_i\rangle \bigvee_{\neg\psi\in[t]} \phi_{[t]}$$

is consistent. By the distributivity of $\langle K_i\rangle$ over $\vee$,

$$\bigvee_{\neg\psi\in[t]} (\phi_{[s]} \wedge \langle K_i\rangle\phi_{[t]})$$

is consistent. So for some $[t]$ with $\neg\psi \in [t]$, $\phi_{[s]} \wedge \langle K_i\rangle\phi_{[t]}$ is consistent. We claim that $[s] \sim_i^f [t]$. If this is the case, we have a contradiction, since we assumed that $\psi \in [s']$ for all $[s'] \sim_i^f [s]$.

Proof of the claim: if $\phi_{[s]} \wedge \langle K_i\rangle\phi_{[t]}$ is consistent, then $[s] \sim_i^f [t]$. Suppose not $[s] \sim_i^f [t]$, that is there is a formula χ such that $K_i\chi \in [s]$ and $\neg K_i\chi \in [t]$ or vice versa. Then we have $K_i\chi \wedge \phi_{[s]} \wedge \langle K_i\rangle(\neg K_i\chi \wedge \phi_{[t]})$ is consistent, but since K_i is an S5 modality, this is impossible. Same for the case when $\neg K_i\chi \in [s]$ and $K_i\chi \in [t]$.

case $\theta = [G]\psi$

$M^f, [s] \models [G]\psi$ iff $\psi^{M^f} \in E^f([s])(G)$ iff $\{s' : (\vee_{[t]\in\psi^{M^f}}\phi_{[t]}) \in s'\} \in E^c(s)(G)$ iff (by the IH) $\{s' : (\vee_{\psi\in[t]}\phi_{[t]}) \in s'\} \in E^c(s)(G)$ iff(*) $\{s' : \psi \in s'\} \in E^c(s)(G)$ iff(**) $[G]\psi \in s$ iff (since $[G]\psi \in cl(\phi)$) $[G]\psi \in [s]$.

Proof of (*): assume S^f contains $n+k$ states, $[t_1], \ldots, [t_n]$ contain ψ and $[s_1], \ldots, [s_k]$ contain $\neg\psi$. Clearly, $\phi_{[t_1]} \vee \ldots \vee \phi_{[t_n]} \vee \phi_{[s_1]} \ldots \vee \phi_{[s_k]}$ is provably equivalent to $\top$. Consider $\vee_{\psi\in[t]}\phi_{[t]}$. It is provably equivalent to $(\psi \wedge \phi_{[t_1]}) \vee \ldots \vee (\psi \wedge \phi_{[t_n]})$. Since for every $[s_i]$ such that $\neg\psi \in [s_i]$, $(\psi \wedge \phi_{[s_i]})$ is provably equivalent to $\perp$,

$$(\psi \wedge \phi_{[t_1]}) \vee \ldots \vee (\psi \wedge \phi_{[t_n]})$$

is provably equivalent to

$$(\psi\wedge\phi_{[t_1]})\vee\ldots\vee(\psi\wedge\phi_{[t_n]})\vee(\psi\wedge\phi_{[s_1]})\vee\ldots\vee(\psi\wedge\phi_{[s_k]})$$

which in turn is provably equivalent to

$$\psi \wedge (\phi_{[t_1]} \vee \ldots \vee \phi_{[s_k]})$$

which in turn is equivalent to $\psi \wedge \top$ hence to ψ. So in M^c, $\{s' : (\vee_{\psi\in[t]}\phi_{[t]}) \in s'\} = \{s' : \psi \in s'\}$.

Proof of (**): since we defined $X \in E^c(s)(N)$ to hold iff $S^c \setminus X \notin E^c(s)(\emptyset)$, it suffices to show the case that $G \neq N$. The direction to the left is immediate: if $[G]\psi \in s$ then $\{s' \in S^c : \psi \in s'\} \in E^c(s)(G)$ by definition. For the other direction assume that $\{s' \in S^c : \psi \in s'\} \in E^c(s)(G)$, i.e., there is some γ such that $\{s' \in S^c : \gamma \in s'\} \subseteq \{s' \in S^c : \psi \in s'\}$ and $[G]\gamma \in s$. It is easy to see that $\{s' \in S^c : \gamma \in s'\} \subseteq \{s' \in S^c : \psi \in s'\}$ implies that $\vdash \gamma \to \psi$, and by the monotonicity rule it follows that $[G]\psi \in s$.

case $\theta = C_G\psi$ The proof is similar to in [20]. First we show that in M^f, if $C_G\psi \in cl(\phi)$, then $C_G\psi \in [s]$ iff every state on every $\sup_{i\in G} \sim_i^f$ path from $[s]$ contains ψ.

Suppose $C_G\psi \in [s]$. The proof is by induction on the length of the path. If the path is of 0 length, then clearly by deductive closure and by $\psi \in cl(\phi)$ we have $\psi \in [s]$. We also have $C_G\psi \in [s]$ by the assumption. IH: if $C_G\psi \in [s]$, then every state on every $\cup_{i\in G} \sim_i^f$ path of length n from $[s]$ contains ψ *and* $C_G\psi$. Inductive step: let us prove this for paths of length $n+1$. Suppose we have a path $[s] \sim_{i_1}^f [s_1] \ldots \sim_{i_n}^f [s_n] \sim_{i_{n+1}}^f [s_{n+1}]$. By the IH, $\psi, C_G\psi \in [s_n]$. Since s_n is deductively closed and $K_{i_{n+1}} C_G\psi \in cl(\phi)$, we have $K_{i_{n+1}} C_G\psi \in [s_n]$. Since $[s_n] \sim_{i_{n+1}}^f [s_{n+1}]$ and the definition of $\sim_{i_{n+1}}^f$, $C_G\psi \in [s_{n+1}]$ and hence by reflexivity $\psi \in [s_{n+1}]$.

For the other direction, suppose that every state on every $\cup_{i\in G} \sim^f$ path from $[s]$ contains ψ. Prove that $C_G\psi \in [s]$. Let $S_{G,\psi}$ be the set of all $[t]$ such that every state on every $\cup_{i\in G} \sim^f$ path from $[t]$ contains ψ. Note that each $[t]$ is/corresponds to a finite set of formulas so we can write its conjunction $\phi_{[t]}$. Consider a formula

$$\chi = \bigvee_{[t]\in S_{G,\psi}} \phi_{[t]}$$

Similarly to [20] it can be proved that $\vdash_{CLC} \phi_{[s]} \to \chi$, $\vdash_{CLC} \chi \to \psi$ and $\vdash_{CLC} \chi \to E_G\chi$. And from that follows that $\vdash_{CLC} \phi_{[s]} \to C_G\psi$ hence $C_G\psi \in [s]$.

Now we prove that $M^f, [s] \models C_G\psi$ iff $C_G\psi \in [s]$. $C_G\psi \in [s]$ iff every state on every $\cup_{i\in G} \sim_i^f$ path from $[s]$ contains ψ iff for every $[t]$ reachable from $[s]$ by a $\cup_{i\in G} \sim_i^f$ path, $M^f, [t] \models \psi$ iff $M^f, [s] \models C_G\psi$.

□

It is obvious that in M^f, $\sim_i$ are equivalence relations. So what remains to be proved is that E^f satisfies E1-E6. Since S^f is finite, it suffices to show E1-E5, which for finite sets of states entail E6.

Proposition 1. *M^f satisfies E1-E5.*

Proof.

E1 Note that $\phi_\emptyset$ is the empty disjunction, $\bot$.
$\emptyset \in E^f([s])(G)$ iff (by definition of E^f) $\{s' : \bot \in s'\} \in E^c(s)(G)$ iff $\emptyset \in E^c(s)(G)$. Since E^c satisfies E1, $\emptyset \notin E^f([s])(G)$.

E2 $S^f \in E^f([s])(G)$ iff $\{s' : \bigvee_{[t]\in S^f} \in s'\} \in E^c(s)(G)$ iff $S^c \in E^c(s)(G)$. Since E^c satisfies E2, $S^f \in E^f([s])(G)$.

E3 Let $\overline{X} \notin E^f([s])(\emptyset)$. Then $\{s' : \phi_{\overline{X}} \in s'\} \notin E^c(s)(\emptyset)$. Note that $\{s' : \phi_{\overline{X}} \in s'\}$ is the complement of $\{s' : \phi_X \in s'\}$, since $\phi_{\overline{X}} = \neg\phi_X$. Since E^c satisfies E3, this means that $\{s' : \phi_X \in s'\} \in E^c(s)(N)$. Hence $X \in E^f([s])(N)$.

E4 Let $X \subseteq Y \subseteq S^f$ and $X \in E^f([s])(G)$. Clearly $\vdash_{CLC} \phi_X \to \phi_Y$. Hence $\{s' : \phi_X \in s'\} \subseteq \{s' : \phi_Y \in s'\}$. Since $X \in E^f([s])(G)$, we have $\{s' : \phi_X \in s'\} \in E^c(s)(G)$. Since E^c satisfies E4, $\{s' : \phi_Y \in s'\} \in E^c(s)(G)$ so $Y \in E^f([s])(G)$.

E5 Let $X \in E^f([s])(G_1)$ and $Y \in E^f([s])(G_2)$ and $G_1 \cap G_2 = \emptyset$. So $\{s' : \phi_X \in s'\} \in E^c(s)(G_1)$ and $\{s' : \phi_Y \in s'\} \in E^c(s)(G_2)$ and since E^c satisfies E5, $\{s' : \phi_X \in s'\} \cap \{s' : \phi_Y \in s'\} \in E^c(s)(G_2)$. Note that $\{s' : \phi_X \in s'\} \cap \{s' : \phi_Y \in s'\} = \{s' : (\vee_{[t]\in X} \phi_{[t]}) \in s'$ and $(\vee_{[t]\in Y} \phi_{[t]}) \in s'\}$ which is in turn the same as

$$\{s' : (\vee_{[t]\in X\cap Y} \phi_{[t]}) \in s'\}$$

since $\{s' : (\vee_{[t]\in X\cap Y} \phi_{[t]}) \in s'\} \in E^c(s)(G_2)$, $X \cap Y \in E^f([s])(G_1)$.

□

Corollary 1. *For any CLC-formula ϕ, $\vdash_{CLC} \phi$ iff $\models \phi$.*

4. EPISTEMIC COALITION LOGIC WITH DISTRIBUTED KNOWLEDGE

In this section we consider the logic $\mathcal{CLD}$, extending coalition logic with individual knowledge operators and distributed knowledge.

The axiomatisation CLD is obtained by extending CL with the following standard axioms and rules for individual and distributed knowledge (see, e.g., [5]):

K $K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$

T $K_i\phi \to \phi$

4 $K_i\phi \to K_iK_i\phi$

5 $\neg K_i\phi \to K_i\neg K_i\phi$

RN $\vdash_{CLD} \phi \Rightarrow \vdash_{CLD} K_i\phi$

DK $D_G(\phi \to \psi) \to (D_G\phi \to D_G\psi)$

DT $D_G\phi \to \phi$

D4 $D_G\phi \to D_GD_G\phi$

D5 $\neg D_G\phi \to D_G\neg D_G\phi$

D1 $K_i\phi \leftrightarrow D_i\phi$

D2 $D_G\phi \to D_H\phi$, if $G \subseteq H$

As usual, soundness can easily be shown.

Lemma 2 (Soundness). *For any CLD-formula ϕ, $\vdash_{CLD} \phi \Rightarrow \models \phi$.*

Outline of completeness proof.

In the remainder of this section we show that CLD also is complete. An outline of the proof is as follows. As in the case of CLC, we start with the canonical model construction. However, rather than constructing a playable model, we construct a playable *pseudo*model M^c. The truth lemma for the combined epistemic-coalitional language holds for M^c, but the relations interpreting distributed knowledge are not necessarily the intersections of the individual epistemic accessibility relations. The idea is to transform M^c into a proper model, which has the E1-E6 propeties, without breaking the truth lemma. This is done in two additional steps. First, M^c is transformed into a finite pseudomodel M^f, exactly like in the case of CLC. The transformation preserves satisfaction, as well as the playability properties (and E6 follows from finiteness). Using pseudomodels that are then transformed into proper models is a common way to deal with intersection in general and distributed knowledge in particular [22]. We can in fact now make directly use of an existing completeness result and proof for epistemic logic with distributed knowledge [5], by taking the (finite) epistemic pseudomodel underlying M^f and transform it into a proper (not necessarily finite) epistemic model which is used as the underying epistemic model of the final model M'. It remains to be shown that the transformation did not break the true playability properties, nor satisfaction of formulae in the closure.

Completeness proof.

For a set of formulae s, let $K_a s = \{K_a\phi : K_a\phi \in s\}$ and $D_G s = \{D_G\phi : D_G\phi \in s\}$.

Definition 1 (Canonical Playable Pseudomodel). *The canonical playable pseudomodel $M^c = (S^c, \{\sim_i^c : i \in N\}, \{R_G^c : \emptyset \neq G \subseteq N\}, E^c, V^c)$ for $\mathcal{CLD}$ is defined as follows:*

- *S^c is the set of maximal consistent sets.*
- *$s \sim_i^c t$ iff $K_i s = K_i t$*
- *$sR_G t$ iff $D_H s = D_H t$ whenever $H \subseteq G$*
- *$X \in E^c(s)(G)$ (for $G \neq N$) iff there exists ψ such that $\{t \in S^c : \psi \in t\} \subseteq X$ and $[G]\psi \in s$.*
- *$X \in E^c(s)(N)$ iff $S^c \setminus X \notin E^c(s)(\emptyset)$.*
- *$V^c(s) = \{p : p \in s\}$*

Lemma 3 (Pseudo Truth Lemma). *$M^c, s \models \phi \Leftrightarrow \phi \in s$.*

Proof. The proof is by induction on ϕ. The epistemic cases are exactly as for standard normal modal logic. The case for coalition operators is exactly as in [17]. □

It is easy to check that $\sim_i^c$ are equivalence relations and E1-E5 hold for E^c.

Lemma 4 (Finite Pseudomodel). *Every CLD-consistent formula ϕ is satisfied in a finite pseudomodel where E1-E6 hold.*

Proof. The proof is exactly as in Theorem 1, namely the construction of M^f, but starting with a Canonical Playable Pseudomodel rather than Canonical Playable Model; the definition of M^c contains the clause

$$\Gamma R_G \Delta \text{ iff } \forall H \subseteq G \{\psi : D_H\psi \in \Gamma\} = \{\psi : D_H\psi \in \Delta\}$$

We add the following condition to the closure: $D_i\psi \in cl(\phi)$ iff $K_i\psi \in cl(\phi)$.

We define M^f to be a pseudomodel instead of a model, by adding the clause:

$$[s]R_G^f[s'] \text{ iff } \forall H \subseteq G \{\psi : D_H\psi \in [s]\} = \{\psi : D_H\psi \in [s']\}$$

We show that M^f is indeed a pseudomodel:

- $R_i^f = \sim_i^f$: this follows from the fact that $K_i\phi \in [s]$ iff $D_i\phi \in [s]$ for any i, ϕ and s, which holds because of the $K_i\phi \to D_i\phi$ axiom and the new closure condition above.
- $G \subseteq H \Rightarrow R_H^f \subseteq R_G^f$: this holds by definition.

We add a case for $\theta = D_G\psi$ to the inductive proof. This case is proven in exactly the same way as the $\theta = K_i\psi$ case: the definitions of $\sim_i^f$ and R_G^f are of exactly the same form (in particular, R_G^f is also an $S5$ modality). The proof that E1-E6 hold in the resulting pseudomodel is the same as in the proof of Theorem 1 for E^f. □

We are now going to transform the pseudomodel into a proper model; it is a well-known technique for dealing with distributed knowledge. In fact, we can make direct use of a corresponding existing result for epistemic logic with distributed knowledge, and extend it with the coalition operators/effectivity functions. We here give the more general result for the language with also common knowledge, which will be useful later.

Theorem 2 ([5]). *If $M_p = (S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, V)$ is an epistemic pseudomodel, then there is an epistemic model $M'_p = (S', \{\sim'_i : i \in N\}, V')$ and a surjective (onto) function $\mathbf{f} : S' \to S$ such that for every $s' \in S'$ and formula $\phi \in \mathcal{ELCD}$, $M_p, \mathbf{f}(s') \models \phi$ iff $M'_p, s' \models \phi$.*

Proof. This result is directly obtained from the completeness proof for $\mathcal{ELCD}$ sketched in [5, p. 70]. For a more detailed proof (for a more general language), see [22, Theorem 9]. □

Theorem 3. *If a formula is satisfied in some finite pseudomodel, then it is satisfied in some model.*

Proof. Let $M = (S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, E, V)$ be a finite pseudomodel such that $M, s \models \phi$. Let $M_p = (S, \{\sim_i : i \in N\}, \{R_G : \emptyset \neq G \subseteq N\}, V)$ be the epistemic pseudomodel underlying M, and let $M'_p = (S', \{\sim'_i : i \in N\}, V')$ and $\mathbf{f} : S' \to S$ be as in Theorem 2. Let $\mathbf{f}^{-1}(X) = \{s' \in S' : \mathbf{f}(s') \in X\}$ for any set $X \subseteq S$. Finally, let $M' = (S', \{\sim'_i : i \in N\}, E', V')$ where E' is defined as follows:

- For $G \neq N$: $Y \in E'(u)(G) \Leftrightarrow \exists X \subseteq S, (Y \supseteq \mathbf{f}^{-1}(X)$ and $X \in E(\mathbf{f}(u))(G))$
- for $G = N$: $Y \in E'(u)(G) \Leftrightarrow \overline{Y} \notin E'(u)(\emptyset)$

Two things must be shown: that M' is a proper model, and that it satisfies ϕ.

Since M'_p is an epistemic model, to show that M' is a model all that remains to be shown is that E' is truly playable. We now show that that follows from true playability of E.

E1 Note that $\mathbf{f}^{-1}(X) = \emptyset$ iff $X = \emptyset$.

For $G \neq N$, $\emptyset \in E'(u)(G)$ iff (by definition of E') $\exists X \subseteq S, (\emptyset \supseteq \mathbf{f}^{-1}(X)$and $X \in E(\mathbf{f}(u))(G))$ iff $\emptyset \in E(\mathbf{f}(u))(G))$ which is impossible since M satisfies E1. Note that in particular this proves $\emptyset \notin E'(u)(\emptyset)$, which we will use in the E2 case below.

For $G = N$, $\emptyset \in E'(u)(G)$ iff $S' \notin E'(u)(\emptyset)$ and we'll see that this is impossible in the E2 case below.

E2 Note that $\mathbf{f}^{-1}(S) = S'$.

For $G \neq N$, $S' \in E'(u)(G)$ iff (by definition of E') $\exists X \subseteq S$, $(S' \supseteq \mathbf{f}^{-1}(X)$and $X \in E(\mathbf{f}(u))(G))$, and by taking $X = S$ we get that $S' \in E'(u)(G)$ holds since $S' \supseteq \mathbf{f}^{-1}(S)$ and $S \in E(\mathbf{f}(u))(G)$. Note that in particular this proves $S' \in E'(u)(\emptyset)$, which we needed in the E1 case above.

For $G = N$, $S' \in E'(u)(G)$ iff $\emptyset \notin E'(u)(\emptyset)$ and this was proved in the E1 case above.

E3 $\forall u \in S' \forall Y \subseteq S'$ $\overline{Y} \notin E'(u)(\emptyset) \Rightarrow Y \in E'(u)(N)$ follows immediately from the definition for $E'(u)(N)$.

E4 E' is monotonic by definition for $G \neq N$.

For N, assume $X \subseteq Y$ and $X \in E'(u)(N)$. Then $\overline{X} \notin E'(u)(\emptyset)$. Since we already know that E' is monotonic for $G = \emptyset$ and $\overline{Y} \subseteq \overline{X}$, $\overline{Y} \notin E'(u)(\emptyset)$. So $Y \in E'(u)(N)$.

E5 Let $u \in S'$, $f(u) = s$, $G_1, G_2 \subseteq N$ such that $G_1 \cap G_2 = \emptyset$, $X', Y' \subseteq S'$, $X' \in E'(u)(G_1)$ and $Y' \in E'(u)(G_2)$. We must show that $X' \cap Y' \in E'(u)(G_1 \cup G_2)$. We reason by cases for G_1 and G_2.

First consider the case that $G_1 \cup G_2 \neq N$. We must show that there is a Z such that $\mathbf{f}^{-1}(Z) \subseteq X' \cap Y'$ and $Z \in E(s)(G_1 \cup G_2)$. We have that there are X, Y such that $\mathbf{f}^{-1}(X) \subseteq X'$ and $X \in E(s)(G_1)$ and $\mathbf{f}^{-1}(Y) \subseteq Y'$ and $Y \in E(s)(G_2)$. Take $Z = X \cap Y$. It is easy to see that $\mathbf{f}^{-1}(X \cap Y) = \mathbf{f}^{-1}(X) \cap \mathbf{f}^{-1}(Y)$ (from the defintion of $\mathbf{f}^{-1}(\cdot)$), and we thus get that $\mathbf{f}^{-1}(Z) = \mathbf{f}^{-1}(X) \cap \mathbf{f}^{-1}(Y) \subseteq X' \cap Y'$. From $X \in E(s)(G_1)$ and $Y \in E(s)(G_2)$ and superadditivity of E we get that $Z \in\in E(s)(G_1 \cup G_2)$.

Second consider the case that $G_1 = N$ or $G_2 = N$. Wlog. assume the former. That implies that $G_2 = \emptyset$. We must show that $X' \cap Y' \in E'(u)(N)$, i.e., that $\overline{X' \cap Y'} \notin E'(u)(\emptyset)$. Assume otherwise, i.e., that $\overline{X' \cap Y'} \in E'(u)(\emptyset)$, in other words that $\overline{X'} \cup \overline{Y'} \in E'(u)(\emptyset)$. As $G_2 = \emptyset$ we also have that $Y' \in E'(u)(\emptyset)$, and by E5 for E' for the case that $G_1 = G_2 = \emptyset \neq N$ (proven above) we get that $(\overline{X'} \cup \overline{Y'}) \cap Y' \in E'(u)(\emptyset)$. I.e., $\overline{X'} \cap Y' \in E'(u)(\emptyset)$. By E4 for E' (proven above), we get that $\overline{X'} \in E'(u)(\emptyset)$. But that contradicts the fact that $X' \in E(u)(G_1)$ with $G_1 = N$.

Finally, consider the case that $G_1 \cup G_2 = N$ and $G_1 \neq N$ and $G_2 \neq N$. We must show that $X' \cap Y' \in E'(u)(N)$, i.e., that $\overline{X' \cap Y'} \notin E'(u)(\emptyset)$, i.e., that there does not exist a Z such that $\mathbf{f}^{-1}(Z) \subseteq \overline{X' \cap Y'}$ and $Z \in E(s)(\emptyset)$. Assume otherwise, that such a Z exists. Let X, Y be such that

$$\begin{array}{l}\mathbf{f}^{-1}(X) \subseteq X' \text{ and } X \in E(s)(G_1) \\ \mathbf{f}^{-1}(Y) \subseteq Y' \text{ and } Y \in E(s)(G_1)\end{array}$$

which exist because $X' \in E'(u)(G_1)$ and $Y' \in E'(u)(G_2)$. From superadditivity of E we get that

$$X \cap Y \in E(s)(N) \tag{1}$$

It follows that

$$\overline{X \cap Y} \notin E(s)(\emptyset) \tag{2}$$

because otherwise $\emptyset = (X \cap Y) \cap (\overline{X \cap Y}) \in E(s)(N)$ by E5 for E, which contradicts E1 for E. We furthermore have that

$$\begin{array}{l}\overline{X'} \subseteq \overline{\mathbf{f}^{-1}(X)} \subseteq \mathbf{f}^{-1}(\overline{X}) \\ \overline{Y'} \subseteq \overline{\mathbf{f}^{-1}(Y)} \subseteq \mathbf{f}^{-1}(\overline{Y})\end{array} \tag{3}$$

which follow immediately from the facts that $\mathbf{f}^{-1}(X) \subseteq X'$ and $\mathbf{f}^{-1}(Y) \subseteq Y'$ and the definition of $\mathbf{f}^{-1}(\cdot)$. From (3) it follows that

$$\overline{X'} \cup \overline{Y'} \subseteq \mathbf{f}^{-1}(\overline{X} \cup \overline{Y}) \tag{4}$$

From (4) and the assumption that $Z \in E(s)(\emptyset)$ we get that $\mathbf{f}^{-1}(Z) \subseteq \mathbf{f}^{-1}(\overline{X} \cup \overline{Y})$, and it follows, by surjectivity of $\mathbf{f}$, that

$$Z \subseteq \overline{X \cap Y} \tag{5}$$

By (5) and the assumption that $Z \in E(s)(\emptyset)$ we get that $\overline{X \cap Y} \in E(s)(\emptyset)$. But this contradicts (2).

E6 We must show that $E'^{nc}(u)(\emptyset) \neq \emptyset$, for any u. Let $s = f(u)$, and let $X \in E^{nc}(s)(\emptyset)$ (exists because of E6 for E). We show that $\mathbf{f}^{-1}(X) \in E'^{nc}(u)(\emptyset)$. First, we have that $\mathbf{f}^{-1}(X) \in E'(u)(\emptyset)$; this follows from the fact that $X \in E(s)(\emptyset)$ and the definition of E'. Second, assume, towards a contradiction, that there exists a $Y \subset \mathbf{f}^{-1}(X)$ such that $Y \in E'(u)(\emptyset)$. By the definition of E', this means that there is a Z such that $\mathbf{f}^{-1}(Z) \subseteq Y$ and $Z \in E(s)(\emptyset)$. Since $Y \subset \mathbf{f}^{-1}(X)$ and $\mathbf{f}^{-1}(Z) \subseteq Y$ it follows that $\mathbf{f}^{-1}(Z) \subset \mathbf{f}^{-1}(X)$. It is easy to see (from surjectivity of $\mathbf{f}$) that it follows that $Z \subset X$, and this contradicts the assumption that $Z \in E(s)(\emptyset)$ and $X \in E^{nc}(s)(\emptyset)$.

In order to show that M' satisfies ϕ, we show that $M, \mathbf{f}(u) \models \gamma$ iff $M', u \models \gamma$ for any $u \in S'$ and any γ, by induction in γ. All cases except $\gamma = [G]\psi$ are exactly as in the proof of Theorem 2.

For the case that $\gamma = [G]\psi$, the inductive hypothesis is that for all proper subformulae χ of $[G]\psi$, and any v, $M, \mathbf{f}(v) \models \chi$ iff $M', v \models \chi$. We can state this as $\{v : M', v \models \chi\} = \mathbf{f}^{-1}(\chi^M)$, or $\chi^{M'} = \mathbf{f}^{-1}(\chi^M)$.

First consider the case that $G \neq N$. Let $\mathbf{f}(u) = s$. $M', u \models [G]\psi$ iff $\psi^{M'} \in E'(u)(G)$ iff there is an X such that $\mathbf{f}^{-1}(X) \subseteq \psi^{M'}$ and $X \in E(s)(G)$. This holds iff $\psi^M \in E(s)(G)$ iff $M, s \models [G]\psi$. For the implication to the left take $X = \psi^M$; for the implication to the right observe that $\mathbf{f}^{-1}(X) \subseteq \mathbf{f}^{-1}(\psi^M)$ implies that $X \subseteq \psi^M$, and $\psi^M \in E(s)(G)$ follows from $X \in E(s)(G)$ by outcome monotonicity of E.

Second consider the case that $G = N$. $M, s \models [N]\psi$ iff $\psi^M \in E(s)(N)$ iff (*) $\neg\psi^M \notin E(s)(\emptyset)$ iff (as above) $\neg\psi^{M'} \notin E'(u)(\emptyset)$ iff $M', u \models [N]\psi$. (*): one direction E3, the other direction E5 and E1. □

Corollary 2. *For any CLD-formula ϕ, $\vdash_{CLD} \phi$ iff $\models \phi$.*

5. EPISTEMIC COALITION LOGIC WITH BOTH COMMON AND DISTRIBUTED KNOWLEDGE

In this section we consider the logic $\mathcal{CLCD}$, extending coalition logic with operators for individual knowledge, common knowledge and distributed knowledge.

The axiomatisation $CLCD$ is obtained by extending CL with the axioms and rules of CLC and CLD.

Lemma 5 (Soundness). *For any CLCD-formula ϕ, $\vdash_{CLCD} \phi \Rightarrow \models \phi$.*

Completeness can in fact be shown in exactly the same was as for CLD, except that there is an extra clause for $C_G\phi$ in the proof of satisfaction which is taken care of in the same way as in the proof for CLC.

Theorem 4. *Any CLCD-consistent formula is satisfied in some finite pseudomodel.*

Proof. The proof is identical to the proof of Lemma 4, starting with the canonical playable pseudomodel, with the addition of the inductive clause $\theta = C_G\psi$ as in the proof of Theorem 1. □

We can now use the same approach as in the case of $\mathcal{CLD}$.

Theorem 5. *If a $\mathcal{CLCD}$ formula is satisfied in some finite pseudomodel, it is satisfied in some model.*

Proof. The proof goes exactly like the proof of Theorem 3, using Theorem 2. The definition of the model M' is identical to the definition in Theorem 3, as is the proof that it is a proper model. For the last part of the proof, i.e., showing that M' satisfies ϕ, note that the last clause in Theorem 2 holds for epistemic logic with both distributed and common knowledge. Thus, the proof is completed by only adding the inductive clause for $[G]\phi$, which is done in exactly the same way as in Theorem 3. $\square$

COROLLARY 3. *For any CLCD-formula ϕ, $\vdash_{CLCD} \phi$ iff $\models \phi$.*

6. COMPUTATIONAL COMPLEXITY

The following complexity result is an easy consequence of the known results for other logics:

THEOREM 6. *The satisfiability problem for CLC and for CLCD is EXPTIME-complete.*

PROOF. EXPTIME-hardness follows from EXPTIME-hardness of $S5_n + C$ ([9]). EXPTIME upper bound follows from the upper bound for ATEL ([21]). $\square$

THEOREM 7. *The satisfiability problem for CLD is PSPACE-complete.*

PROOF. PSPACE-hardness follows from PSPACE-hardness of $S5_n$ [9] and also from PSPACE-hardness of CL [17].

The PSPACE upper bound can be obtained by combining the tableaux algorithm for $S5_n + D$ given in [9] with the algorithm in [17] which checks the satisfiability of a finite set of coalition logic formulas. The two algorithms need to call each other recursively. In addition, since [9] only has the D operator for the grand coalition, the rule for producing witnesses for the $\neg D_G\phi$ formulae has to be modified as in the tableaux algorithm for $K_\omega^{\cup,\cap}$ [15]. $\square$

7. CONCLUSIONS

This papers settles several hitherto unsolved problems. It proves completeness of coalition logic extended with different combinations of group knowledge operators. The axioms for the epistemic modalities are standard in epistemic logic, but the completeness proofs require non-trivial combinations of techniques. The proofs are given in detail, and can be used and extended in future work. The paper furthermore completely characterises the computational complexity of the considered logics. They are all decidable. We can conclude that adding coalition operators to epistemic logic comes "for free" without changing the complexity of the satisfiability problem: the extension of epistemic logic with distributed and common knowledge with coalition operators remains EXPTIME-complete, the extension of epistemic logic with only distributed knowledge with coalition operators remains PSPACE-complete.

8. REFERENCES

[1] R. Alur, T. A. Henzinger, and O. Kupferman. Alternating-time temporal logic. *Journal of the ACM*, 49:672–713, 2002.

[2] N. Belnap and M. Perloff. Seeing to it that: a canonical form for agentives. *Theoria*, 54:175–199, 1988.

[3] J. Broersen. A complete stit logic for knowledge and action, and some of its applications. In *Proceedings of DALT 2008*, volume 5397 of *LNCS*, pages 47–59, 2009.

[4] J. Broersen, A. Herzig, and N. Troquard. A normal simulation of coalition logic and an epistemic extension. In *Proc. of TARK 2007*, pp. 92–101, 2007.

[5] R. Fagin, J. Y. Halpern, Y. Moses, and M. Y. Vardi. *Reasoning About Knowledge*. The MIT Press, 1995.

[6] V. Goranko, W. Jamroga, and P. Turrini. Strategic games and truly playable effectivity functions. In *Proceedings of AAMAS 2011*, pages 727–734, 2011.

[7] V. Goranko, W. Jamroga, and G. van Drimmelen. Axiomatic systems for alternating time temporal epistemic logics (extended abstract). 2004.

[8] V. Goranko and G. van Drimmelen. Complete axiomatization and decidability of alternating-time temporal logic. *Theoretical Computer Science*, 353(1):93–117, 2006.

[9] J. Y. Halpern and Y. Moses. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54(2):319–379, 1992.

[10] D. Harel, D. Kozen, and J. Tiuryn. *Dynamic Logic*. MIT Press, 2000.

[11] A. Herzig and N. Troquard. Knowing how to play: Uniform choices in logics of agency. In *Proceedings of AAMAS 2006*, pages 209–216, 2006.

[12] A. H. J. Broersen and N. Troquard. What groups do, can do, and know they can do: an analysis in normal modal logics. *J. of Applied Non-Classical Logic*, 19(3):261–289, 2009.

[13] W. Jamroga and T. Ågotnes. Constructive knowledge: what agents can achieve under imperfect information. *J. of Applied Non-Classical Logics*, 17:423–475, 2007.

[14] W. Jamroga and W. van der Hoek. Agents that know how to play. *Fundam. Informaticae*, 63:185–219, 2004.

[15] C. Lutz and U. Sattler. The complexity of reasoning with boolean modal logics. In *Advances in Modal Logic*, volume 3, pages 329–348. World Scientific, 2002.

[16] J.-J. C. Meyer and W. van der Hoek. *Epistemic Logic for AI and Computer Science*. Cambridge University Press: Cambridge, England, 1995.

[17] M. Pauly. A modal logic for coalitional power in games. *Journal of Logic and Computation*, 12(1):149–166, 2002.

[18] P. Y. Schobbens. Alternating-time logic with imperfect recall. *Electronic Notes in Theoretical Computer Science*, 85(2), 2004.

[19] W. van der Hoek and M. Wooldridge. Cooperation, knowledge and time: Alternating-time temporal epistemic logic and its applications. *Studia Logica*, 75:125–157, 2003.

[20] H. van Ditmarsch, W. van der Hoek, and B. Kooi. *Dynamic Epistemic Logic*, Springer, 2007.

[21] D. Walther. ATEL with common and distributed knowledge is ExpTime-complete. In *Proc. of M4M-4*, 2005.

[22] Y. Wang and T. Ågotnes. Public announcement logic with distributed knowledge. In *Proceedings of LORI 2011*, volume 6953 of *LNCS*. Springer, 2011.