State and Path Coalition Effectivity Models for Logics of Multi-Player Games

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ABSTRACT

We consider models of multi-player games where abilities of players and coalitions are defined in terms of sets of outcomes which they can effectively enforce. We extend the well studied state effectivity models of one-step games in two different ways. On the one hand, we develop multiple state effectivity functions associated with different long-term temporal operators. On the other hand, we define and study coalitional path effectivity models where the outcomes of strategic plays are infinite paths. For both extensions we obtain representation results with respect to concrete models arising from concurrent game structures. We also apply state and path coalitional effectivity models to provide alternative, arguably more natural and elegant semantics to the alternating-time temporal logic ATL*, and discuss their technical and conceptual advantages.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*; I.2.4 [**Artificial Intelligence**]: Knowledge Representation Formalisms and Methods—*Modal logic*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Theory

Keywords

Games models, effectivity, strategic logic

1. INTRODUCTION

A wide variety of multi-player games can be modeled by so called 'multi-player game models' a.k.a. 'concurrent game structures' [9, 3] which can be seen as a generalization of extensive form games or of repeated normal form (strategic) games. Here, we view them as general models of (qualitative) *multi-step games*. Intuitively, such game is based on a labelled transition system where every state is associated with a normal form game and the transitions between states are labelled by tuples of actions,[1] one for each player. Thus, the outcome of playing a normal form game at a given state is a transition to a new state, respectively to a new normal form game. In the quantitative version of such games, the outcome states are also associated with payoff vectors, while in the version that we consider here, the payoffs are *qualitative* – defined by properties of the outcome states, possibly expressed in a logical language. The players' objectives in multi-step games can simply be about reaching a desired ('winning') state, or they can be more involved, such as forcing a desired *long-term behaviour* (transition path, run).

Various logics for reasoning about coalitional abilities in multi-player games have been proposed and studied in the last two decades – most notably, Coalition Logic (CL) [9] and Alternating-time Temporal Logic (ATL* and its fragment ATL) [3]. Coalition Logic can be seen as a logic for reasoning about abilities of coalitions in one-step (strategic) games to bring about an outcome *state* with desired properties by means of single actions, while ATL* allows to express statements about multi-step scenarios. For example, the ATL formula $\langle\langle C\rangle\rangle F\varphi$ says that the coalition of players (or agents) C can ensure that φ will become true at *some* future moment, no matter what the other players do; likewise, $\langle\langle C\rangle\rangle G\varphi$ expresses that the coalition C can enforce φ to be *always* the case. More generally, the ATL* formula $\langle\langle C\rangle\rangle\gamma$ holds true iff C has a strategy to ensure that any resulting behavior of the system (i.e., any play of the game) will satisfy the temporal property γ.

In this paper we study how multi-step games can be modeled and characterized in terms of *effectivity of coalitions* with respect to possible outcome states or behaviours, and how such models can be used to provide conceptually simple and technically elegant semantics for logics of multi-player games such as ATL*. The paper has three main objectives:

(i) To extend the semantics for CL based on one-step coalitional effectivity to semantics for ATL over state-based coalitional effectivity models;
(ii) To develop the analogous notion of *coalitional path effectivity* representing the powers of coalitions in multi-step games to ensure long-term behaviors, and to provide semantics for ATL* based on it;
(iii) To obtain characterizations of multi-player game models in terms of abstract state and path coalitional effectivity models, analogous to Pauly's representation theorem [9, 5].

[1]Such actions are also called 'strategies' in normal form games, but we reserve the use of the term 'strategy' for a *global conditional plan* in a multi-step scenario.

Appears in: *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012)*, Conitzer, Winikoff, Padgham, and van der Hoek (eds.), 4-8 June 2012, Valencia, Spain.

We argue that characterizing effectivity of coalitions in multi-step games in terms of paths (cf. points (ii) and (iii) above) is conceptually more natural and elegant than in terms of outcome states, in several respects. First, collective strategies in such games generate *outcome paths* (plays), not just outcome states. Second, *one* path effectivity function is sufficient to define the powers of coalitions in a multi-step game for all kinds of temporal patterns, through the standard semantics of temporal operators. This point is further supported by the fact that path effectivity models provide a straightforward semantics for the whole language of ATL* (which is not definable by alternation-free fixpoint operators on the one-step ability). Finally, we argue that path effectivity can just as well be applied to variants of ATL(*) with imperfect information, where even simple modalities do not have fixpoint characterizations [6]. Still, also in that case, executing a strategy 'cuts out' a set of possible paths, just like in the perfect information case.

The paper is structured as follows. We begin by introducing basic notions in Section 2. In Section 3 we develop state-based effectivity models that suffice to define semantics of ATL. The models include three different effectivity functions, one for each basic modality $X, G, \mathcal{U}$. Then, in Section 4 we develop and study effectivity models based on paths. We show how they provide semantics to ATL*, and identify appropriate "playability" conditions, which we use to establish correspondences between powers of coalitions in the abstract models and strategic abilities of coalitions in concurrent game models. Finally, we briefly discuss how the path-oriented view can be used to facilitate reasoning about games with imperfect information in Section 5.

2. PRELIMINARIES

We begin by introducing some basic game-theoretic and logical notions. In all definitions hereafter, the sets of players, game (outcome) states, and actions available to players are assumed non-empty. Moreover, the set of players is always assumed finite.

2.1 Concurrent game models

Strategic games (a.k.a. normal form games) are basic models of non-cooperative game theory [8]. Following the tradition in the qualitative study of games we focus on abstract game modes, where the effect of strategic interaction between players is represented by abstract outcomes from a given set and players' preferences are not specified.

Definition 1 (Strategic game). *A* strategic game *is a tuple $G = (\mathbb{A}\mathrm{gt}, St, \{Act_i | i \in \mathbb{A}\mathrm{gt}\}, o)$ consisting of a set of players (agents) $\mathbb{A}\mathrm{gt}$, a set of outcome states St, a set of actions (atomic strategies) Act_i for each player $i \in \mathbb{A}\mathrm{gt}$, and an outcome function $o : \prod_{i\in\mathbb{A}\mathrm{gt}} Act_i \to St$ which associates an outcome with every action profile.*

We define coalitional strategies α_C in G as tuples of individual strategies α_i for $i \in C$, i.e., $Act_C = \prod_{i\in C} Act_i$.

Strategic games are one-step encounters. They can be generalized to multi-step scenarios, in which every state is associated with a strategic game. Such games are also known as *concurrent game structures* [3].

Definition 2 (Concurrent game structures). *A* concurrent game structure *(CGS) is a tuple*

$$F = (\mathbb{A}\mathrm{gt}, St, Act, d, o)$$

which consists of a set of players $\mathbb{A}\mathrm{gt} = \{1, \dots, k\}$, a set of states St, a set of (atomic) actions Act, a function $d : \mathbb{A}\mathrm{gt} \times St \to \mathcal{P}(Act)$ that assigns a sets of actions available to players at each state, and a (deterministic) transition function o that assigns the outcome state $o(q, \alpha_1, \dots, \alpha_k)$ to every starting state q and a tuple of actions $\langle\alpha_1, \dots, \alpha_k\rangle$, $\alpha_i \in d(i, q)$, that can be executed by $\mathbb{A}\mathrm{gt}$ in q.

A concurrent game model *(CGM) is a CGS endowed with a valuation $V : St \to \mathcal{P}(Prop)$ for some fixed set of atomic propositions $Prop$.*

Note that in a CGS all players execute their actions synchronously and the combination of the actions, together with the current state, determines the transition in the CGS.

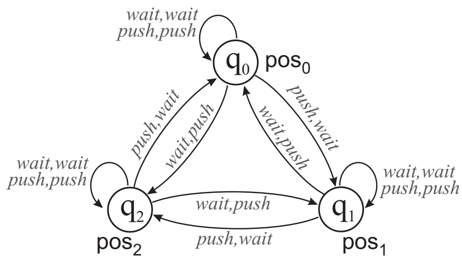

Figure 1: Repeated matching pennies: a concurrent game model M_1.

Example 1 (Repeated matching pennies). *Two agents play matching pennies repeatedly on a triangular board in such a way that the initial state of the next game depends on what they did before. More precisely, showing the heads means that the player wants to push the token, and showing the tails means that she wants the token to be left in the same place. Moreover, player 1 can only push the token to the right, while 2 can only push it to the left. The scenario can be formalized using the CGM in Figure 1.*

Strategies in multi-step games. A *path* in a CGS/CGM is an infinite sequence of states that can result from subsequent transitions in the structure/model. A *strategy* of a player a in a CGS/CGM $\mathcal{M}$ is a conditional plan that specifies what a should do in each possible situation. Depending on the type of memory that we assume for the players, a strategy can be *memoryless*, formally represented with a function $s_a : St \to Act$, such that $s_a(q) \in d_a(q)$, or a *perfect recall strategy*, represented with a function $s_a : St^+ \to Act$ such that $s_a(\langle\dots, q\rangle) \in d_a(q)$, where St^+ is the set of *histories*, i.e., finite prefixes of paths in $\mathcal{M}$ [3, 10]. A *collective strategy* for a group of players $C = \{a_1, ..., a_r\}$ is simply a tuple of strategies $s_C = \langle s_{a_1}, ..., s_{a_r}\rangle$, one for each player from C. We denote player a's component of the collective strategy s_C by $s_C[a]$.

We define the function $out(q, s_C)$ to return the set of all paths $\lambda \in St^\omega$ that can be realised when the players in C follow the strategy s_C from state q onward. Formally, for memoryless strategies, it can be defined as below:

$out(q, s_C) = \{\lambda = q_0, q_1, q_2... \mid q_0 = q$ and for each $i = 0, 1, ...$ there exists $\langle\alpha^i_{a_1}, ..., \alpha^i_{a_k}\rangle$ such that $\alpha^i_a \in d_a(q_i)$ for every $a \in \mathbb{A}\mathrm{gt}$, and $\alpha^i_a \in s_C[a](q_i)$ for $a \in C$, and $q_{i+1} = o(q_i, \alpha^i_{a_1}, ..., \alpha^i_{a_k})\}$.

The definition for perfect recall strategies is analogous.

2.2 Abstract models of coalitional effectivity

Effectivity functions have been introduced in cooperative game theory [7] to provide an abstract representation of the powers of coalitions to influence the outcome of the game.

Definition 3 (Effectivity functions and models). *A* local effectivity function $E : \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St))$ *associates a family of sets of states with each set of players.*

A global effectivity function $E : St \times \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St))$ *assigns a local effectivity function to every state $q \in St$. We will use the notations $E(q)(C)$ and $E_q(C)$ interchangeably.*

Finally, a coalitional effectivity model *consists of a global effectivity function, plus a valuation of atomic propositions.*

Intuitively, elements of $E(C)$ are *choices* available to the coalition C: if $X \in E(C)$ then by choosing X the coalition C can force the outcome of the game to be in X. The idea to represent a choice (action) of a coalition by the set of possible outcomes which can be effected by that choice was also captured by the notion of 'alternating transition system' used originally to provide semantics for ATL in [2].

Definition 4 (True playability [9, 5]). *A local effectivity function E is* truly playable *iff the following hold:*

Outcome monotonicity: *$X \in E(C)$ and $X \subseteq Y$ implies $Y \in E(C)$;*

Liveness: *$\emptyset \notin E(C)$;*

Safety: *$St \in E(C)$;*

Superadditivity: *if $C \cap D = \emptyset$, $X \in E(C)$ and $Y \in E(D)$, then $X \cap Y \in E(C \cup D)$;*

$\mathbb{A}\mathrm{gt}$***-maximality:*** *$\overline{X} \notin E(\emptyset)$ implies $X \in E(\mathbb{A}\mathrm{gt})$;*

Determinacy: *if $X \in E(\mathbb{A}\mathrm{gt})$ then $\{x\} \in E(\mathbb{A}\mathrm{gt})$ for some $x \in X$.*

A global effectivity function is truly playable iff it consists only of local functions that are truly playable.

α-Effectivity. Each strategic game G can be canonically associated with an effectivity function, called the α-effectivity function of G and denoted with E^α_G [9].

Definition 5 (α-effectivity in strategic games). *For a strategic game G, the (coalitional)* α-effectivity function $E^\alpha_G : \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St))$ is defined as follows: $X \in E^\alpha_G(C)$ if and only if there exists σ_C such that for all $\sigma_{\overline{C}}$ we have $o(\sigma_C, \sigma_{\overline{C}}) \in X$.

Example 2. *The α-effectivity for M_1, q_0 is: $E(\{1,2\}) = \{\{q_0\},\{q_1\},\{q_2\},\{q_0,q_1\},\{q_0,q_2\},\{q_1,q_2\},\{q_0,q_1,q_2\}\}$; $E(\{1\}) = E(\{2\}) = \{\{q_0,q_1\},\{q_0,q_2\},\{q_0,q_1,q_2\}\}$; $E(\emptyset) = \{\{q_0,q_1,q_2\}\}$. Clearly, E is truly playable.*

Theorem 1 (Representation Theorem [9, 5]). *A local effectivity function E is truly playable if and only if there exists a strategic game G such that $E^\alpha_G = E$.*

2.3 Logical reasoning about multi-step games

The Alternating-time Temporal Logic ATL* [2, 3] is a multimodal logic with strategic modalities $\langle\langle C\rangle\rangle$ and temporal operators X ("at the next state"), G ("always from now on"), and $\mathcal{U}$ ("until"). There are two types of formulae of ATL*, *state formulae* and *path formulae*, respectively defined by the following grammar:

$\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle C\rangle\rangle\gamma$,
$\gamma ::= \varphi \mid \neg\gamma \mid \gamma\wedge\gamma \mid X\gamma \mid G\gamma \mid \gamma\,\mathcal{U}\,\gamma$, for $C \subseteq \mathbb{A}\mathrm{gt}, \mathsf{p} \in Prop$.
F ("sometime in the future") can be defined as $F\varphi \equiv \top\,\mathcal{U}\,\varphi$.

Let M be a CGM, q a state in M, and $\lambda = q_0, q_1, \dots$ a path in M. For every $i \in \mathbb{N}$ we denote $\lambda[i] = q_i$; $\lambda[0..i]$ is the prefix $q_0, q_1, \dots, q_i$, and $\lambda[i..\infty]$ is the respective suffix of λ.

The semantics of ATL* is given by the following clauses [3]:

- $M, q \models \mathsf{p}$ iff $q \in V(\mathsf{p})$, for $\mathsf{p} \in Prop$;
- $M, q \models \neg\varphi$ iff $M, q \not\models \varphi$;
- $M, q \models \varphi_1 \wedge \varphi_2$ iff $M, q \models \varphi_1$ and $M, q \models \varphi_2$;
- $M, q \models \langle\langle C\rangle\rangle\gamma$ iff there is a strategy s_C for the players in C such that for each path $\lambda \in out(q, s_C)$ we have $M, \lambda \models \gamma$.
- $M, \lambda \models \varphi$ iff $M, \lambda[0] \models \varphi$;
- $M, \lambda \models \neg\gamma$ iff $M, \lambda \not\models \gamma$;
- $M, \lambda \models \gamma_1 \wedge \gamma_2$ iff $M, \lambda \models \gamma_1$ and $M, \lambda \models \gamma_2$;
- $M, \lambda \models X\gamma$ iff $M, \lambda[1, \infty] \models \gamma$;
- $M, \lambda \models G\gamma$ iff $M, \lambda[i, \infty] \models \gamma$ for every $i \geq 0$; and
- $M, \lambda \models \gamma_1\,\mathcal{U}\,\gamma_2$ iff there is i such that $M, \lambda[i, \infty] \models \gamma_2$ and $M, \lambda[j, \infty] \models \gamma_1$ for all $0 \leq j < i$.

Example 3. *Consider again the repeated matching pennies from Example 1. No player can make sure that the token moves to any particular position (e.g., $M_1, q_0 \models \neg\langle\langle 1\rangle\rangle F\,\mathsf{pos}_1$). On the other hand, the player can at least make sure that the game will* avoid *particular positions: $M_1, q_0 \models \langle\langle 1\rangle\rangle G\neg\mathsf{pos}_1$. And, if the players cooperate then they control the game completely: $M_1, q_0 \models \langle\langle 1,2\rangle\rangle X\,\mathsf{pos}_0 \wedge \langle\langle 1,2\rangle\rangle X\,\mathsf{pos}_1 \wedge \langle\langle 1,2\rangle\rangle X\,\mathsf{pos}_2$.*

ATL and CL as fragments of ATL*. The most important fragment of ATL* is ATL where each strategic modality is directly followed by a single temporal operator. Thus, the semantics of ATL can be given entirely in terms of states, cf. [3] for details. We point out that for ATL the two notions of strategy (memoryless vs. perfect recall) yield the same semantics.

Furthermore, Coalition Logic (CL) [9] can be seen as the fragment of ATL involving only booleans and operators $\langle\langle C\rangle\rangle X$, and thus it inherits the semantics of ATL on CGMs.

3. STATE EFFECTIVITY IN MULTI-STEP GAMES

An alternative semantics of CL has been given in [9] in terms of the effectivity models defined in section 2.2, via the following clause, where $\varphi^M := \{s \in St \mid M, s \models \varphi\}$.

$M, q \models \langle\langle C\rangle\rangle X\varphi$ iff $\varphi^M \in E_q(C)$.

It is easy to see that the CGM-based and effectivity-based semantics of CL coincide on truly playable models.

The semantics of ATL has never been explicitly defined in terms of abstract effectivity models. An informal outline of such semantics has been suggested in [4], essentially by representation of the modalities $\langle\langle C\rangle\rangle G$ and $\langle\langle C\rangle\rangle\,\mathcal{U}$ as appropriate fixpoints of $\langle\langle C\rangle\rangle X$. The idea was based on the result from [3] showing that the alternation-free fragment of Alternating μ-Calculus is strictly more expressive that ATL. In this section, we actually extend state-based effectivity models to provide semantics for ATL. For that, as pointed out earlier, a different effectivity function will be needed for each temporal pattern.

We note that an effectivity function for the "always" modality G was already constructed in [9]. Moreover, an effectivity function for reachability, i.e. for the F modality, has been presented in [1]. Our construction here differs significantly from both approaches, and allows to cover all kinds of effectivity that can be addressed in ATL.

3.1 Operations on state effectivity functions

First, we define basic operations on effectivity functions, reflecting the meaning of these as operations on games.

Composition of effectivity functions $E, F : St \times \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St))$ is the effectivity function $E \circ F$ where $Y \in (E \circ F)_q(C)$ iff there exists a subset Z of St, such that $Z \in E_q(C)$ and $Y \in F_z(C)$ for every $z \in Z$.

Union of effectivity functions E, F is the effectivity function $E \cup F$ where $Y \in (E \cup F)_q(C)$ iff $Y \in E_q(C)$ or $Y \in F_q(C)$. *Intersection* of effectivity functions is defined analogously. Likewise, we define union and intersection of any family of effectivity functions. Hereafter, we assume that $\circ$ has a stronger binding power than $\cup$ and $\cap$.

Inclusion of effectivity functions is defined as follows: $E \subseteq F$ iff $E_q(C) \subseteq F_q(C)$ for every $q \in St$ and $C \subseteq \mathbb{A}\mathrm{gt}$.

The *idle effectivity function* $\mathbf{I}$ is defined as follows: $\mathbf{I}_q(C) = \{Y \subseteq St \mid q \in Y\}$ for every $q \in St$ and $C \subseteq \mathbb{A}\mathrm{gt}$.

PROPOSITION 2. *The following hold for any outcome monotone effectivity functions E, F, G :*

1. $E \circ \mathbf{I} = \mathbf{I} \circ E = E$.
2. *If $F_1 \subseteq F_2$ then $E \circ F_1 \subseteq E \circ F_2$.*
3. $(E \cup F) \circ G = (E \circ G) \cup (E \circ F)$.
4. $(E \cap F) \circ G = (E \circ G) \cap (E \circ F)$.

REMARK 3. *The identities $E \circ (F \cup G) = (E \circ F) \cup (E \circ G)$ and $E \circ (F \cap G) = (E \circ F) \cap (E \circ G)$ are not valid.*

DEFINITION 6. *For any effectivity function E we define inductively the effectivity functions $E^{(n)}$ and $E^{[n]}$ as follows:*
$E^{(0)} = \mathbf{I}$, $E^{(n+1)} = \mathbf{I} \cup E \circ E^{(n)}$,
$E^{[0]} = \mathbf{I}$, $E^{[n+1]} = \mathbf{I} \cap E \circ E^{[n]}$.

PROPOSITION 4. *For every $n \geq 0$: $E^{(n)} \subseteq E^{(n+1)}$ and $E^{[n+1]} \subseteq E^{[n]}$.*

PROOF. Routine, by induction on n. $\square$

DEFINITION 7. *The* weak iteration *of E is the function $E^{(*)} = \bigcup_{k=0}^{\infty} E^{(k)}$, i.e., $Y \in E_q^{(*)}(C)$ iff $\exists n.\ Y \in E_q^{(n)}(C)$.*

The strong iteration *of E is the function $E^{[*]} = \bigcap_{k=0}^{\infty} E^{[k]}$, i.e., $Y \in E_q^{[*]}(C)$ iff $\forall n.\ Y \in E_q^{[n]}(C)$.*

PROPOSITION 5. *Unions, intersections, compositions, week and strong iterations preserve outcome-monotonicity of effectivity functions.*

PROPOSITION 6. *For any effectivity function E :*

1. *$E^{(*)}$ is the least fixed point of the monotone operator $\mathfrak{F}_w$ defined by $\mathfrak{F}_w(F) = \mathbf{I} \cup E \circ F$.*
2. *$E^{[*]}$ is the greatest fixed point of the monotone operator $\mathfrak{F}_q$ defined by $\mathfrak{F}_q(F) = \mathbf{I} \cap E \circ F$.*

PROOF. **(1)** First, we show by induction on k that for every k, $E^{(k)} \subseteq \mathbf{I} \cup E \circ E^{(*)}$. Indeed, $E^{(0)} = \mathbf{I} \subseteq \mathbf{I} \cup E \circ E^{(*)}$; $E^{(k+1)} = \mathbf{I} \cup E \circ E^{(k)} \subseteq \mathbf{I} \cup E \circ E^{(*)}$ by the inductive hypothesis and proposition 2. Thus, $E^{(*)} \subseteq \mathbf{I} \cup E \circ E^{(*)}$.

For the converse inclusion, let $Y \in (\mathbf{I} \cup E \circ E^{(*)})_q(C)$. If $Y \in \mathbf{I}_q(C)$, then $Y \in E_q^{(*)}$ by definition. Suppose $Y \in (E \circ E^{(*)})_q(C)$. Then, there is $Z \in E_q(C)$ such that for every $z \in Z$, $Y \in {E^{(*)}}_z(C)$, hence $Y \in E_z^{(k_z)}(C)$ for some $k_z \geq 0$. Let $m = \max_{z \in Z} k_z$. Then, by proposition 4, $Y \in E_z^{(m)}(C)$ for every $z \in Z$. Therefore, $Y \in (E \circ E^{(m)})_q(C) \subseteq E_q^{(m+1)}(C) \subseteq E_q^{(*)}(C)$.

Thus, $E^{(*)}$ is a fixed point of the operator $\mathfrak{F}_w$.

Now, suppose that F is such that $\mathfrak{F}_w(F) = \mathbf{I} \cup E \circ F$. Then, we show by induction on k that for every k, $E^{(k)} \subseteq F$. Indeed, $E^{(0)} = \mathbf{I} \subseteq \mathbf{I} \cup E \circ F = F$. Suppose $E^{(k)} \subseteq F$. Then $E^{(k+1)} = \mathbf{I} \cup E \circ E^{(k)} \subseteq \mathbf{I} \cup E \circ F = F$ by the inductive hypothesis and proposition 2. Thus, $E^{(*)} \subseteq F$. Therefore, $E^{(*)}$ is the least fixed point of $\mathfrak{F}_w$.

(2). The argument is dually analogous. $\square$

3.2 Binary effectivity functions

DEFINITION 8. *Given a set of players $\mathbb{A}\mathrm{gt}$ and a set of states St, a* local binary effectivity function *for $\mathbb{A}\mathrm{gt}$ on St is a mapping $U : \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St) \times \mathcal{P}(St))$ associating with each set of players a family of pairs of outcome sets.*

A global binary effectivity function *associates a local binary effectivity function with each state from St.*

Now we define some basic (global) binary effectivity functions and operations on them.

Left-idle binary effectivity function $\mathbf{L} : St \times \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St) \times \mathcal{P}(St))$ is defined by $\mathbf{L}_q(C) = \{(X, Y) \mid q \in X\}$ for any $q \in St$ and $C \subseteq \mathbb{A}\mathrm{gt}$. Respectively, *right-idle* binary effectivity function $\mathbf{R}$ is defined by $\mathbf{R}_q(C) = \{(X, Y) \mid q \in Y\}$ for any $q \in St$ and $C \subseteq \mathbb{A}\mathrm{gt}$.

Union of binary effectivity functions $U, W : St \times \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St) \times \mathcal{P}(St))$ is the binary effectivity function $U \cup W$ where $(X, Y) \in (U \cup W)_q(C)$ iff $(X, Y) \in U_q(C)$ or $(X, Y) \in V_q(C)$. *Intersection* of binary effectivity functions is defined analogously. *Right projection* of U is the unary effectivity function E such that $E_q(C) = \{Y \mid (X, Y) \in U_q(C)\}$ for all q, C. Likewise, we define union, intersection, and right projection of any family of binary effectivity functions.

Composition of a unary effectivity function E with a binary effectivity function U is the binary effectivity function $E \circ U$ such that $(X, Y) \in (E \circ U)_q(C)$ iff there exists a subset Z of St, such that $Z \in E_q(C)$ and $(X, Y) \in U_z(C)$ for every $z \in Z$. *Inclusion* of binary effectivity functions: $U \subseteq W$ iff $U_q(C) \subseteq W_q(C)$ for every $q \in St$ and $C \subseteq \mathbb{A}\mathrm{gt}$.

DEFINITION 9. *For any unary effectivity function E we define the binary effectivity functions $E^{\{n\}}$, $n \geq 0$, inductively as follows: $E^{\{0\}} = \mathbf{R}; E^{\{n+1\}} = R \cup (L \cap E \circ E^{\{n\}})$.*

Then, the binary iteration *of E is defined as the binary effectivity function $E^{\{*\}} = \bigcup_{k=0}^{\infty} E^{\{k\}}$, i.e. $(X, Y) \in E_q^{\{*\}}(C)$ iff $(X, Y) \in E_q^{\{n\}}(C)$ for some n.*

DEFINITION 10. *A binary effectivity function U is* outcome-monotone *if every $U_q(C)$ is upwards closed, i e. $(X, Y) \in U_q(C)$ and $X \subseteq X', Y \subseteq Y'$ imply $(X', Y') \in U_q(C)$.*

PROPOSITION 7. *For any finite set of states St and unary effectivity function E,* $E^{\{*\}}$ *is the least fixed point of the monotone operator* $\mathfrak{F}_b$ *defined by* $\mathfrak{F}_b(U) = \mathbf{R} \cup (\mathbf{L} \cap E \circ U)$.

PROOF. Analogous to the proof of proposition 6. □

PROPOSITION 8. $E^{(*)}$, $E^{[*]}$ *and* $E^{\{*\}}$ *are outcome-monotone. Moreover,* $E^{(*)}$ *is the right projection of* $E^{\{*\}}$.

3.3 State-based effectivity models for ATL

The semantics of ATL can now be given in terms of models that are more abstract and technically simpler than CGM.

DEFINITION 11. *A* state-based effectivity frame (SEF) for ATL *is a tuple* $\mathcal{F} = \langle \mathbb{A}\mathrm{gt}, St, \mathbf{E}, \mathbf{G}, \mathbf{U}\rangle$, *where:* $\mathbb{A}\mathrm{gt}$ *is a set of players, St is a set of states,* $\mathbf{E}$ *and* $\mathbf{G}$ *are outcome-monotone effectivity functions, and* $\mathbf{U}$ *is an outcome-monotone binary effectivity function.*

A state-based effectivity model (SEM) for ATL *is a SEF plus a valuation of atomic propositions.*

DEFINITION 12. *A SEF* $\mathcal{F}$ *is* standard *iff (1)* $\mathbf{E}$ *is truly playable, (2)* $\mathbf{G} = \mathbf{E}^{[*]}$, *and (3)* $\mathbf{U} = \mathbf{E}^{\{*\}}$.

A SEM $\mathcal{M} = \langle \mathcal{F}, V\rangle$ *is* standard *if* $\mathcal{F}$ *is standard.*

Now, we define truth of an ATL formula at a state of a state-based effectivity model uniformly as follows:

$\mathcal{M}, q \models \langle\langle C\rangle\rangle X\varphi$ iff $\varphi^{\mathcal{M}} \in \mathbf{E}_q(C)$,

$\mathcal{M}, q \models \langle\langle C\rangle\rangle G\varphi$ iff $\varphi^{\mathcal{M}} \in \mathbf{G}_q(C)$,

$\mathcal{M}, q \models \langle\langle C\rangle\rangle \psi\mathcal{U}\varphi$ iff $(\psi^{\mathcal{M}}, \varphi^{\mathcal{M}}) \in \mathbf{U}_q(C)$.

Extending α-Effectivity to SEM. Given a CGM $M = (\mathbb{A}\mathrm{gt}, St, Act, d, o, V)$, we construct its corresponding SEM as follows: $\mathrm{SEM}(M) = (\mathbb{A}\mathrm{gt}, St, \mathbf{E}, \mathbf{G}, \mathbf{U})$ where $\mathbf{E}_q = E(q)^{\alpha}_{M}$ for all $q \in St$, $\mathbf{G} = \mathbf{E}^{[*]}$ and $\mathbf{U} = \mathbf{E}^{\{*\}}$.

EXAMPLE 4. *The "always" effectivity in state* q_0 *of the repeated matching pennies can be written as follows:* $\mathbf{G}_{q_0}(\{1,2\}) = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\}$, $\mathbf{G}_{q_0}(\{1\}) = \mathbf{G}_{q_0}(\{2\}) = \{\{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\}$, $\mathbf{G}_{q_0}(\emptyset) = \{\{q_0, q_1, q_2\}\}$.

The next result easily follows from Theorem 1:

THEOREM 9 (REPRESENTATION THEOREM). *A state effectivity model* $\mathcal{M}$ *for ATL is standard iff there exists a CGM* M *such that* $\mathcal{M} = \mathrm{SEM}(M)$.

Moreover, we observe that the ATL semantics in CGMs and in their associated standard SEMs coincide.

PROPOSITION 10. *For every CGM M, state q in M, and ATL formula* φ, *we have that* $M, q \models \varphi$ *iff* $\mathrm{SEM}(M), q \models \varphi$.

PROOF. Routine, by structural induction on formulae. □

COROLLARY 11. *Any ATL formula* φ *is valid (resp., satisfiable) in concurrent game models iff* φ *is valid (resp., satisfiable) in standard state-based effectivity models.*

4. COALITIONAL PATH EFFECTIVITY

State-based effectivity models for ATL partly characterize coalitional powers for achieving long-term objectives. However, the applicability of such models is limited by the fact that they characterize effectivity with respect to outcome states, while effectivity for outcome *paths (i.e., plays)* is only captured when such paths are described by the specific temporal patterns definable in ATL. Thus, in particular, state-based effectivity models are not suitable for providing semantics of the whole ATL*.

In this section we aim at getting to the core of the notion of effectivity in multi-step games, regardless of the temporal pattern that defines the winning condition, by re-defining it in terms of outcome *paths*, rather than states. The idea is natural: every collective strategy of the grand coalition in a multi-step game determines a unique path (play) through the state space of the game. Consequently, the outcome of following an individual or coalitional strategy in such game is a set of paths (plays) that can result from execution of the strategy, depending on the moves of the remaining players. Hence, powers of players and coalitions in multi-step games can be characterized by sets of sets of paths. We claim that the notion of path effectivity captures adequately the meaning of strategic operators in ATL(*). Moreover, it provides correct semantics for the whole ATL*, and not only its limited fragment ATL.

4.1 Frames, models, effectivity functions

DEFINITION 13 (PATH EFFECTIVITY FUNCTION). *Let* $\mathbb{A}\mathrm{gt}$ *be a set of players, and St a set of states. A* path *in St is any infinite sequence of states of St. The set of all paths in St is therefore denoted by* St^{ω}. *A* path effectivity function *is a mapping* $\mathcal{E} : \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St^{\omega}))$ *that assigns to each coalition a non-empty family of sets of paths.*

The intuition is analogous to that for state effectivity: a set of paths X is in $\mathcal{E}(C)$ means that the coalition C can choose a strategy that ensures that the game will develop along one of the paths in X. Note that this notion refers to global effectivity only: $X \in \mathcal{E}(C)$ can include paths starting from different states. Local effectivity is easily extractable from the global one. This is in line with the concept of a strategy as a complete conditional plan: in particular, the strategy must prescribe collective actions of the coalition from all possible initial states of the game.

Also, we will assume that $\mathcal{E}$ captures the *actual effectivity*, i.e., it collects only the actual outcome paths of choices available to C, and is not necessarily closed under upwards monotonicity. We note that the outcome-monotone notion of effectivity has a somewhat negative meaning, in the sense that $X \in E(C)$ is usually interpreted as "the coalition C can ensure that the outcome of the game *cannot be outside* X", whereas this does not mean that every element of X is a feasible outcome. This distinction is conceptual, rather than technical, but it will influence our construction of effectivity functions for concrete models. The "actuality" assumption is necessary to specify appropriate abstract playability conditions characterizing path effectivity in concrete models.

DEFINITION 14 (PATH EFFECTIVITY FRAMES/MODELS). *A* path effectivity frame (PEF) *is a structure* $\mathcal{F} = (\mathbb{A}\mathrm{gt}, St, \mathcal{E})$ *consisting of a set of players* $\mathbb{A}\mathrm{gt}$, *a set of states St and a path effectivity function* $\mathcal{E}$ *on these. A* path effectivity model (PEM) *expands a PEF by a valuation of the propositions* $V : Prop \to \mathcal{P}(St)$.

By analogy with identifying choices as sets of outcome states in state effectivity models, we will refer to sets of paths in a PEF as 'choices', with the intuition that $\mathcal{E}(C)$

defines the strategic choices of the coalition C in a PEF $\mathcal{F}$ as sets of paths in $\mathcal{F}$ that C can enforce. However, not every such path can be a *feasible* outcome in some concrete model (i.e, a CGM), but only those that follow existing transitions in the CGM. So, given a path effectivity frame $\mathcal{F}$, we define the set of 'feasible' paths in $\mathcal{F}$ as

$$Paths_{\mathcal{F}} = \bigcup_{C \subseteq \mathbb{A}\mathrm{gt}} \bigcup_{X \in \mathcal{E}(C)} X.$$

For a PEM $\mathcal{M} = (\mathcal{F}, V)$, we define $Paths_{\mathcal{M}} = Paths_{\mathcal{F}}$.

4.2 Path effectivity in concurrent games

Not every set of paths is a feasible choice for a coalition. Note that the powers of players and coalitions in a game crucially depend on their available strategies. There are different notions of strategy, e.g., depending on the amount of memory they can use, so we will parameterize the new notion of α-effectivity with a type (class) of strategies. Every class of individual strategies of players gives rise to a class of coalitional strategies obtained by freely combining the individual strategies of the participating players.[2] We say that a class Σ of individual and coalitional strategies is *feasible* if every coalition has at least one strategy in Σ. Two types of feasible strategies are especially relevant: *perfect recall* and *memoryless* strategies (introduced in Section 2.1). We will refer to them with $\mathfrak{mem}$ and $\mathfrak{nomem}$, respectively.

Definition 15 (Σ-effectivity). *Let M be a CGM and $\Sigma = \bigcup_{C \subseteq \mathbb{A}\mathrm{gt}} \Sigma_C$ be a feasible set of coalitional strategies in M. The* Σ-effectivity function of M *is defined as*

$$\mathcal{E}_M^{\Sigma}(C) = \{ \bigcup_{q \in St} out(q, s_C) \mid s_C \in \Sigma_C \}.$$

Specifically, we denote by $\mathcal{E}_M^{\mathfrak{mem}}$ and $\mathcal{E}_M^{\mathfrak{nomem}}$ the effectivity of coalitions respectively for perfect recall strategies and for memoryless strategies in M. Note that $\mathcal{E}_M^{\Sigma}$ collects only the *actual* outcome paths of *actual* choices of coalitions in M.

Example 5. *The difference between perfect recall and memoryless effectivity is most easily seen in the case of the grand coalition: $\mathcal{E}_M^{\mathfrak{mem}} = \{\{\lambda\} \mid \lambda \in \{q_0, q_1, q_2\}^\omega\}$, but $\mathcal{E}_M^{\mathfrak{nomem}} = \{\{(q_i)^\omega\} \mid i \in \{0,1,2\}\} \cup \{\{q_i(q_j)^\omega\}\} \cup \{\{(q_iq_j)^\omega\}\} \cup \{\{q_iq_j(q_k)^\omega\} \mid i \neq j\} \cup \{\{q_i(q_jq_k)^\omega\} \mid i \neq j\} \cup \{\{(q_iq_jq_k)^\omega\} \mid i \neq j\}$. That is, the players can enforce any sequence of states when they have perfect memory, but only the "periodic" ones in the memoryless case.*

For a class Σ of strategies in a CGM M we denote the set of paths feasible with respect to Σ in M by $Paths_M^{\Sigma}$. For any path effectivity function $\mathcal{E}$ we denote the set of all feasible paths in $\mathcal{E}$, i.e., appearing in any choice of $\mathcal{E}$, by $Paths_{\mathcal{E}}$.

Proposition 12. *For every CGM M and a feasible class Σ of coalitional strategies in M:*

1. *Every coalition has a collective strategy, and therefore for every state q in M it can enforce at least one, non-empty set of outcome paths starting from q.* (Safety)
2. *For any coalition C in M, every coalitional strategy produces a non-empty set of outcome paths.* (Liveness)
3. *$\mathcal{E}_M^{\Sigma}(\emptyset)$ is a singleton. More precisely, $\mathcal{E}_M^{\Sigma}(\emptyset) = \{Paths_M^{\Sigma}\}$.*
4. *$\mathcal{E}_M^{\Sigma}(\mathbb{A}\mathrm{gt})$ consists of singleton sets. More precisely, $\mathcal{E}_M^{\Sigma}(\mathbb{A}\mathrm{gt}) = \{\{\lambda\} \mid \lambda \in Paths_M^{\Sigma}\}$.* (Determinacy)
5. *Every two disjoint coalitions can join their chosen coalitional strategies to enforce an outcome set of paths that is included in the outcome set of paths enforceable by each of the coalitions following its respective strategy.* (Superadditivity)

[2]Here we adhere to the standard ATL assumption that players in a coalition execute their parts of the joint strategy independently.

To define "playability" conditions for path effectivity frames, we will need some additional notation. Let $q \in St$, $h, h' \in St^+$, $X \in \mathcal{P}(St^\omega)$, and $\mathcal{X} \in \mathcal{P}(\mathcal{P}(St^\omega))$. Then we denote:

- $h \preceq h'$ if h' is an extension of h;
- $X(q) := \{\lambda \in X \mid \lambda[0] = q\}$; $X[i] := \{\lambda[i] \mid \lambda \in X\}$
- $X(h) := \{\lambda \mid \lambda \in X, \text{ and } \lambda[0..k] = h \text{ for some } k\}$ is the set of paths in X starting with h;
- $X|h := \{\lambda[k..\infty] \mid \lambda \in X \text{ and } \lambda[0..k] = h\}$ is the set of suffixes of paths in X, extending h;
- $\mathcal{X}(q) = \{X(q) \mid X \in \mathcal{X}\}$, $\mathcal{X}(h) = \{X(h) \mid X \in \mathcal{X}\}$, and $\mathcal{X}|h = \{X|h \mid X \in \mathcal{X}\}$.

4.3 Path effectivity semantics of ATL*

Given an ATL* path formula γ and a path effectivity model M, let

$$\gamma^M = \{\lambda \in Paths_M \mid M, \lambda \models \gamma\}.$$

denote the set of paths in M that satisfy γ. Note that relation $M, \lambda \models \gamma$ is already well defined by the relevant semantic clauses in Section 2.3 (it is essentially the semantics of Linear Time Logic LTL). Then, the path effectivity semantics of ATL* is given by the clause below:

$M, q \models \langle\langle C \rangle\rangle \gamma$ iff there is $X \in \mathcal{E}(C)$ such that $X(q) \subseteq \gamma^M$.

Equivalently: $M, q \models \langle\langle C \rangle\rangle \gamma$ iff $\gamma^M(q) \in \widehat{\mathcal{E}}(C)(q)$, where $\widehat{\mathcal{E}}(C) = \{X \mid Y \subseteq X \text{ for some } Y \in \mathcal{E}(C)\}$ is the *outcome-monotone closure* of $\mathcal{E}$.

Thus, path effectivity models yield a conceptually simple and technically elegant semantics of ATL*, where one effectivity function suffices to completely describe the powers of coalitions to enforce any ATL*-definable behaviour. In particular, only one simple semantic clause is needed to define strategic ability in ATL*, because the temporal patterns are appropriately handled by LTL semantics.

Example 6. *Like before, we have $\mathcal{E}_{M_1}, q_0 \models \langle\langle 1, 2 \rangle\rangle F\mathsf{pos}_i$ for every $i = 0, 1, 2$ in both perfect recall and memoryless strategies. This can be demonstrated e.g. by the choice $\{q_0(q_i)^\omega\}$ that belongs to $\mathcal{E}_{M_1}^{\mathfrak{mem}}(\{1,2\})$ as well as $\mathcal{E}_{M_1}^{\mathfrak{nomem}}(\{1,2\})$.*

4.4 Characterizing path effectivity functions

The path effectivity semantics for ATL* defined above is too abstract to be of practical use. Here we identify the characteristic properties of path effectivity functions arising in CGMs and define an analogue of the notion of (truly) playable state effectivity functions.

Definition 16 (Playability in path effectivity). *A path effectivity function $\mathcal{E} : \mathcal{P}(\mathbb{A}\mathrm{gt}) \to \mathcal{P}(\mathcal{P}(St^\omega))$ is* actually playable *if it satisfies the following conditions:*

P-Safety: *$\mathcal{E}(C)(q)$ is non-empty for every $C \subseteq \mathbb{A}\mathrm{gt}, q \in St$.*

P-Liveness: $\emptyset \notin \mathcal{E}(C)(q)$ *for every* $C \subseteq \mathbb{A}\mathrm{gt}, q \in St$.

P-Monotonicity: *For every* $C_1 \subseteq C_2 \subseteq \mathbb{A}\mathrm{gt}$ *and for every* $X_2 \in \mathcal{E}(C_2)$ *there is a* $X_1 \in \mathcal{E}(C_1)$ *such that* $X_2 \subseteq X_1$.

P-Superadditivity: *if* $C \cap D = \emptyset$, $X \in \mathcal{E}(C)$ *and* $Y \in \mathcal{E}(D)$, *then* $Z \in \mathcal{E}(C \cup D)$ *for some* $Z \subseteq X \cap Y$.

P-$\emptyset$-Minimality: $\mathcal{E}(\emptyset)$ *is the singleton* $\{Paths_{\mathcal{E}}\}$.

P-Determinacy: $\mathcal{E}(\mathbb{A}\mathrm{gt})(q) \subseteq \{\{\lambda\} \mid \lambda \in Paths_{\mathcal{E}}\}$.[3]

We note that the playability conditions above are variants of true playability, sans outcome monotonicity, for path-oriented effectivity. These conditions can be adapted to state effectivity to provide abstract characterization for actual state effectivity in CGMs, analogous to Pauly's characterization of the outcome-monotone state effectivity in [9]. For lack of space we do not include that result here.

Besides the general conditions in Definition 16, we need additional constraints which are specific to the underlying class of strategies, and relate local choices with global strategies in path effectivity frames.

Path effectivity functions for memoryless strategies

The following definition formalizes the consistency of a family of local choices with a global memoryless strategy.

Definition 17 (nomem-consistent family of choices). *Given a PEF* $\mathcal{F} = (\mathbb{A}\mathrm{gt}, St, \mathcal{E})$ *and a coalition* $C \subseteq \mathbb{A}\mathrm{gt}$, *let* $\mathcal{X} = \{X^q\}_{q \in St}$ *be such that* $X^q \in \mathcal{E}(C)(q)$ *for every* $q \in St$. *We call* $\mathcal{X}$ nomem-consistent *if for every pair of states* $q_1, q_2 \in St$, *path* $\lambda \in X^{q_1}$, *and a prefix* $h = \lambda[0..i]$ *for some* $i \in \mathbb{N}$ *such that* $\lambda[i] = q_2$, *it holds that* $X^{q_1}|h = X^{q_2}$.

Definition 18 (nomem-realizability). *A playable path effectivity function* $\mathcal{E} : \mathcal{P}(\mathbb{A}\mathrm{gt}) \rightarrow \mathcal{P}(\mathcal{P}(St^\omega))$ *is* nomem-realizable *if it also satisfies the following conditions:*

nomem-***Regularity:*** *For every* $C \subseteq \mathbb{A}\mathrm{gt}$ *and* $X \in \mathcal{E}(C)$, *the family* $\{X(q)\}_{q \in St}$ *is* nomem-*consistent.*

nomem-***Convexity:*** *Let* $\{X^q\}_{q \in St}$ *s.t.* $X^q \in \mathcal{E}(C)(q)$ *for all* $q \in St$, *be* nomem-*consistent. Then the set of all paths generated by the relation* $\{(q, q') \mid q \in St, q' \in X^q[1]\}$ *is in* $\mathcal{E}(C)$.

LimitClosure: *Every* $X \in \mathcal{E}(C)$ *is* limit-closed, *i.e., for every path* λ, *if every* $\lambda[0..i]$, *for* $i \in \mathbb{N}$, *is a prefix of some path* $\lambda_i \in X$, *then* $\lambda \in X$.

Thus, nomem-regularity of $\mathcal{E}$ means that when coalition C follows a fixed memoryless strategy which determines a set of outcome paths X, it is effective for the same set of outcome paths starting from every occurrence of q along any path from X, viz. $X(q)$. Moreover, nomem-convexity requires that any consistent collection of 'locally applied' strategies for a given coalition C can be pieced together into a global memoryless strategy for C.

Theorem 13 (nomem-Representation theorem). *A path effectivity function* $\mathcal{E}$ *equals* $\mathcal{E}_M^{\mathrm{nomem}}$ *for some concurrent game model* M *if and only if it is* nomem-*realizable.*

[3]Unlike the case of state effectivity functions, where the determinacy constraint is only needed for infinite state games (cf. [5]), it becomes essential here, because even very simple 2-state structures can generate uncountably many paths.

Proof. (*Sketch*) Showing that for every CGM M, $\mathcal{E}_M^{\mathrm{nomem}}$ is nomem-realizable is fairly routine. For the converse implication, given a nomem-realizable path effectivity function $\mathcal{E}$ we first define a global actual state effectivity function AE by collecting for every C and q the successor states from each set of paths in $\mathcal{E}(C)(q)$ and thus producing $AE(C)(q)$. Then we produce the respective global state effectivity function E by closing AE under upwards monotonicity. It is straightforward to show that E is truly playable by the actual path playability of $\mathcal{E}$. Then using the representation theorem for truly playable state effectivity functions in [5] we construct a CGM M for the same set of agents $\mathbb{A}\mathrm{gt}$ and state space St, such that the α-effectivity function E_M^α of M coincides with E. Using M we construct the respective path effectivity function $\mathcal{E}_M^{\mathrm{nomem}}$ according to Definition 15. Finally, we show that it coincides with $\mathcal{E}$ by using the nomem-realizability of each of $\mathcal{E}$ and $\mathcal{E}_M^{\mathrm{nomem}}$. $\square$

Path effectivity functions for perfect recall strategies

Here we provide a partial characterization of path effectivity functions for perfect recall strategies.

Definition 19 (mem-consistent family of choices). *Given a PEF* $\mathcal{F} = (\mathbb{A}\mathrm{gt}, St, \mathcal{E})$ *and a coalition* $C \subseteq \mathbb{A}\mathrm{gt}$, *let* $\mathcal{X} = \{X^h\}_{h \in St^+}$ *be such that* $X^h \in \mathcal{E}(C)(h)$ *for every* $h \in St^+$. *We call* $\mathcal{X}$ mem-consistent *if for every pair* $h_1, h_2 \in St^+$ *such that* $h_1 \preceq h_2$, *we have* $X^{h_1}(h_2) = X^{h_2}$.

Definition 20 (mem-Realizable effectivity). *A playable path effectivity function* $\mathcal{E} : \mathcal{P}(\mathbb{A}\mathrm{gt}) \rightarrow \mathcal{P}(\mathcal{P}(St^\omega))$ *is* mem-realizable *if it also satisfies the following conditions for every coalition* C:

mem-***Regularity:*** *For every* $C \subseteq \mathbb{A}\mathrm{gt}$ *and* $X \in \mathcal{E}(C)$, *the family* $\{X(q)\}_{q \in St}$ *is* mem-*consistent.*

mem-***Convexity:*** *Let* $\{X^h\}_{h \in St^+}$, *where* $X^h \in \mathcal{E}(C)(h)$ *for every* $h \in St^+$, *be* mem-*consistent. Then the set of all paths generated by the relation* $\{(q, q') \mid q = X^h|h[0], q' \in X^h|h[1]\}$ *is in* $\mathcal{E}(C)$.

LimitClosure: *as in Definition 18.*

Intuition for the mem-Regularity condition: if C can be efective for $X|h$ after history h, they can obtain it right at the beginning of the game starting from $q = last(h)$. This is because every substrategy of a perfect recall strategy is also a viable perfect recall strategy. Intuition for the mem-Convexity condition: any 'consistent' collection of history-based local choices for a given coalition C can be pieced together into a global perfect recall strategy for C.

The following is easy to check.

Proposition 14. *For every CGM* M, $\mathcal{E}_M^{\mathrm{mem}}$ *is* mem-*realizable.*

For lack of space we defer the respective representation theorem for perfect recall strategies to a further work.

5. BEYOND PERFECT INFORMATION

So far, we have only been concerned with games where every player knows the global state of the system at any moment. Modeling and reasoning about imperfect information scenarios is more sophisticated. First, not all strategies are executable – even in the perfect recall case. This is because the agents cannot specify that they will execute two different actions in situations that look the same to them. Therefore, only *uniform* strategies are admissible here. Moreover, it is often important to find a uniform strategy that succeeds in *all* indistinguishable for the agent states, rather than contend that there is such a successful strategy for the current state of the system.

In this section, we briefly sketch how path effectivity models can be used to give account on powers of coalitions under imperfect information. This is by no means intended as an exhaustive analysis. Rather, we point out that the modeling power of path effectivity can be applied to more sophisticated scenarios than ones assuming complete knowledge.

We take Schobbens' ATL_{ir} [10] as the "core", minimal ATL-based language for strategic ability under imperfect information. ATL_{ir} includes the same formulae as ATL, only the cooperation modalities are presented with a subscript: $\langle\!\langle A\rangle\!\rangle_{ir}$ to indicate that they address agents with imperfect **i**nformation and imperfect **r**ecall. Models of ATL_{ir} are *imperfect information concurrent game models* (iCGM), which can be seen as concurrent game models augmented with a family of indistinguishability relations $\sim_a\subseteq St\times St$, one per agent $a\in\mathbb{A}\mathrm{gt}$. The relations describe agents' uncertainty: $q\sim_a q'$ means that, while the system is in state q, agent a considers it possible that it is in q'.

A *uniform strategy* for agent a is a function $s_a: St\to Act$, such that: (1) $s_a(q)\in d(a,q)$; (2) if $q\sim_a q'$ then $s_a(q)=s_a(q')$. A collective strategy is uniform if it contains only uniform individual strategies. Again, function $out(q,s_A)$ returns the set of all paths that may result from agents A executing strategy s_A from state q onward. The semantics of cooperation modalities in ATL^*_{ir} is defined as follows:

$M,q\models\langle\!\langle A\rangle\!\rangle_{ir}\gamma$ iff there exists a uniform collective strategy s_A such that, for each $a\in A$, q' such that $q\sim_a q'$, and path $\lambda\in out(s_A,q')$, we have $M,\lambda\models\gamma$.

First, we observe that the same type of effectivity functions can be used to model powers in imperfect information games: $\mathcal{E}:\mathcal{P}(\mathbb{A}\mathrm{gt})\to\mathcal{P}(\mathcal{P}(St^\omega))$. Moreover, the notion of Σ-effectivity does not change. Given an iCGM M and $\Sigma=\bigcup_{C\subseteq\mathbb{A}\mathrm{gt}}\Sigma_C$ be a set of (uniform) coalitional strategies in M, the Σ-*effectivity function of* M is still defined as $\mathcal{E}^\Sigma_M(C)=\{\bigcup_{q\in St} out(q,s_C)\mid s_C\in\Sigma_C\}$.

The semantics of ATL^*_{ir} is also very similar to the perfect information case:

$M,q\models\langle\!\langle C\rangle\!\rangle_{ir}\gamma$ iff there is $X\in\mathcal{E}(C)$ such that

$$\bigcup_{a\in C}\ \bigcup_{q':q\sim_a q'} X(q')\subseteq\gamma^M.$$

That is, $\langle\!\langle C\rangle\!\rangle_{ir}\gamma$ if C have a single choice satisfying γ on all outcome paths starting from states that look the same as q.

What changes is the structural properties of actual effectivity functions that are induced by iCGM's. $\mathbb{A}\mathrm{gt}$-maximality and determinacy are no longer valid since even the grand coalition cannot always enforce every possible course of events, cf. [6]. The regularity and convexity conditions must also be revised because the standard fixpoint characterizations of the temporal modalities do not hold anymore under imperfect information [6]. The detailed study of appropriate realizability conditions is subject to future research.

6. CONCLUSIONS

In this paper we have developed the idea of characterizing multi-player multi-step games in terms of what sets of outcomes – states or paths – coalitions can ensure by executing one or another collective strategy. These characterizations lead to respective notions of state-based and path-based coalition effectivity models, which provide alternative semantics for logics of such games, most notably ATL and ATL*. We find such characterizations both conceptually important and technically interesting because they extract the core game-theoretic essence from game models. They also resolve some technical issues arising in the original semantics for ATL*, particularly in the cases of incomplete and imperfect information. We believe that the understanding of abstract realizability under imperfect information can lead to satisfiability checking procedures and complete axiomatic characterization for these variants of ATL.

Acknowledgements. Wojciech Jamroga acknowledges the support of the FNR (National Research Fund) Luxembourg under project S-GAMES – C08/IS/03.

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