False-name-proofness in Online Mechanisms

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ABSTRACT

In real electronic markets, each bidder arrives and departs over time. Thus, such a mechanism that must make decisions dynamically without knowledge of the future is called an *online mechanism*. In an online mechanism, it is very unlikely that the mechanism designer knows the number of bidders beforehand or can verify the identity of all of them. Thus, a bidder can easily submit multiple bids (*false-name bids*) using different identifiers (e.g., different e-mail addresses). In this paper, we formalize false-name manipulations in online mechanisms and identify a simple property called (value, time, identifier)-monotonicity that characterizes the allocation rules of false-name-proof online auction mechanisms. To the best of our knowledge, this is the first work on false-name-proof online mechanisms. Furthermore, we develop a new false-name-proof online auction mechanism for k identical items. When $k = 1$, this mechanism corresponds to the optimal stopping rule of the secretary problem where the number of candidates is unknown. We show that the competitive ratio of this mechanism for efficiency is 4 and independent from k by assuming that only the distribution of bidders' arrival times is known and that the bidders are impatient.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*; J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

Algorithms, Economics, Theory

Keywords

Auctions, mechanism design, game theory, online algorithms

1. INTRODUCTION

Auctions have become an integral part of electronic commerce and a promising application field of game theory and mechanism design theory. Traditionally, mechanism design of auctions has mainly considered static (offline) environments where all bidders arrive and depart simultaneously and the mechanism makes a decision only one time. In real electronic markets, however, each bidder may arrive and depart over time, so a mechanism must make decisions dynamically without knowledge of the future. This uncertainty often makes traditional works inapplicable to such online environments. Therefore, designing mechanisms for dynamic environments (i.e., *online mechanism design*) has lately attracted considerable attention in the algorithmic game-theory field [14].

One desirable characteristic of a mechanism is *strategy-proofness*. A mechanism is strategy-proof if for each bidder, truthfully reporting her type (private information) is a dominant strategy. Unlike traditional mechanism design environments, in online mechanisms, the private information of each bidder consists not only of his valuation but also of her arrival/departure times, and bidders can misreport them to maximize their utility. For these reasons, designing a strategy-proof mechanism is much more challenging in online environments than in traditional static environments.

Designing a strategy-proof online mechanism is strongly connected to the optimal stopping theory, in particular, the secretary problem. In fact, from the perspective of this theory, Hajiaghayi et al. [9] developed an online mechanism for a single item where an auction with a single item is held in finite periods. Hereafter, we refer to it as Mechanism 1. Consider there are n bidders. Each bidder $i \in N$ values the item at r_i and stays in the auction at interval $[a_i, d_i]$.

MECHANISM 1. *Let n denote the number of bidders and a^* be the arrival time of $\lfloor n/e \rfloor$-th bidder, where e is the base of the natural logarithm.*

1. *(learning phase): At period a^*, let $r_{(1)}, r_{(2)}$ be the first and second highest bidding values received so far.*
2. *(transition): If a bidder whose bidding value is $r_{(1)}$ remains present at period a^*, then sell an item to that bidder (breaking ties deterministically, e.g., based on the lexicographic order of the identifiers) at price $r_{(2)}$.*
3. *(accepting phase): Otherwise, sell an item to the next bidder whose bidding value is at least $r_{(1)}$ (breaking ties deterministically) at price $r_{(1)}$.*

Mechanism 1 is strategy-proof assuming no bidder can be present longer than her true stay. A winner cannot decrease her payment by making her stay shorter or by misreporting her valuation. A loser cannot win unless she pays more than her true valuation or stays longer.

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Table 1: False-name-proofness fails in Mechanism 1. a_i, d_i, and r_i indicate arrival period, departure period, and the valuation of bidder i.

	a_i	d_i	r_i		a_i	d_i	r_i
bidder 1	1	3	6	bidder 4	(later)		7
bidder 2	4	5	2	bidder 5	(later)		4
bidder 3	4	4	8	bidder 6	(later)		1

However, untruthfully declaring private information is only one way to manipulate the outcome. Another way is for one bidder to pretend to be multiple bidders. Such *false-name bids* [16], i.e., bids submitted under fictitious names such as multiple e-mail addresses, are especially feasible in Internet auctions due to their relative anonymity. Unfortunately, Mechanism 1 is not false-name-proof; a bidder can profit by pretending to be multiple bidders.

EXAMPLE 1. *Consider a single item online auction with six bidders, each of whom has a preference, as shown in Table 1. Since $n = 6$, the mechanism waits for the second bidder ($\lfloor 6/e \rfloor = 2$). When each bidder reports truthfully, the item is not sold until arrival period 4 of bidder 2. In this case, bidder 1 cannot win even if her bidding value is very high, because she is not present when the winner is determined. Next, consider the case when bidder 1 uses two identifiers, $1'$ and $1''$. Identifier $1'$ keeps her bid, and identifier $1''$ reports $(2, 2, \epsilon)$. In this case, the transition to the accepting phase occurs after $\lfloor 7/e \rfloor = 2$ bids. Bidder 1 wins at period 2 and pays ϵ.*

The example shows that false-name bids are profitable for bidder 1. In fact, bidder 1 can also win if she once departs from the auction at period 2 and arrives again at period 3 using another identifier $1''$.

Furthermore, in such environments where bidders can use multiple identifiers, the number of participating bidders (identifiers) n depends on the strategies of the bidders. This means that a mechanism cannot observe correct information about the number of participating bidders; it can only observe the number of identifiers used by the bidders. Thus, it is impractical to design an online mechanism that is based on the fact that the mechanism knows the number of participating identifiers n in advance. This difficulty was also pointed out by [10].

Readers might think that if a market can use some personal identification method (e.g., checking the participant's credit card number or social security number), the problem resulting from false-name bids disappears. Introducing such a method can indeed slightly increase the cost of using false-name bids, but it cannot completely solve the problem. A person can ask his/her family, friends, or employers to submit bids on her/his behalf. False-name manipulations can be considered as a very restricted subclass of collusions, where a person can only collude with other participants when they were initially not interested in participating in the mechanism, but they agree to work on behalf of the person by obtaining a small side-payment. Such manipulations cannot be prevented by a simple personal identification method. Conitzer and Yokoo [4] provided a more detailed discussion why false-name-proof mechanisms matter.

Our Results.

To the best of our knowledge, this is the first work that deals with false-name manipulations in online mechanisms. This paper formalizes false-name manipulations in online mechanisms and proposes a simple property called *(value, time, identifier)-monotonicity*, which characterizes false-name-proof online auction mechanisms in single-valued domains. Then it introduces two non-trivial false-name-proof mechanisms for k identical items. Furthermore, the competitive analysis revealed that for sufficiently large k, one of them is 4-competitive for efficiency by introducing a different adversarial model from the traditional one under the assumption that all bidders are impatient. We assume here that a mechanism has no information about the number of bidders; it does know the distribution of their arrival times, since it is quite natural that their real number is unknown and unpredictable in situations where false-name bids are possible.

Related Work.

Lavi and Nisan [12] was the first work on mechanism design of auctions in dynamic environments. Hajiaghayi et al. [9] proposed a strategy-proof online mechanism in limited-supply environments, based on the optimal stopping rule of the secretary problem. Hajiaghayi et al. [8] proposed a strategy-proof online mechanism for selling expiring items. In Hajiaghayi et al. [10], a technique called automated mechanism design was applied to construct online auction mechanisms. Furthermore, Parkes [14] showed that the revelation principle can fail in online mechanisms when the no-early arrival, no-late departure property does not hold. Gerding et al. [5] introduced two procedures for item burning into online mechanisms to achieve truthfulness.

Yokoo et al. [16] pointed out the effects of false-name manipulations in combinatorial auctions and showed that even the Vickrey-Clarke-Groves (VCG) mechanism is vulnerable against false-name manipulations. Besides combinatorial auctions, the notion of false-name-proofness have been discussed in other application fields of game theory, such as resource allocation [7] and coalitional games [1].

Myerson [13] proposed the *monotonicity* property of allocation rules, which characterizes strategy-proof auction mechanisms in single-parameter settings. Bikhchandani et al. [2] extended the property to such multi-dimensional settings as combinatorial auctions and proposed a property called *weak-monotonicity* that characterizes strategy-proof mechanisms. Todo et al. [15] proposed the *sub-additivity* property as a full characterization of false-name-proof combinatorial auction mechanisms. For online auction mechanisms, Hajiaghayi et al. [8] and Parkes [14] introduced a property called *monotonicity* [1] and showed that it characterizes strategy-proof online auction mechanisms.

2. PRELIMINARIES

Let $N = \{1, 2, \dots n\}$ denote a set of bidders and $\mathbb{T} = \{1, \dots, T\}$ a set of finite and discrete time periods in which an auction is held. Each bidder $i \in N$ has private information, or a type, $\theta_i = (a_i, d_i, r_i)$ drawn from Θ_i. The type of bidder defines its value for the allocations of an online mechanism. Θ_i is a type space, or a domain of types, defined as

[1] To distinguish this property from the original monotonicity introduced by Myerson [13], we refer to it as *(value, time)-monotonicity*.

$\Theta_i = \mathbb{T} \times \mathbb{T} \times \mathbb{R}_{\geq 0}$. Let a_i and d_i be *arrival* and *departure* times. In the interval of a_i and d_i, a bidder has a valuation r_i on the auctioned item. Define $x = (x^1, \ldots, x^T) \in X$ as a possible allocation in a mechanism. Each $x^t = (x_1^t, \ldots, x_n^t)$ represents the allocation at period $t \in \mathbb{T}$, where x_i^t is the allocation to bidder i at period t; if bidder i is allocated an item at period t, then $x_i^t = 1$ holds; otherwise $x_i^t = 0$. We represent the gross utility of bidder i whose type is θ_i for an allocation x as $v(\theta_i, x)$.

We restrict the domain of types Θ_i to *single-valued* domains [14], in which each $\theta_i \in \Theta_i$ is defined as a triple (a_i, d_i, r_i), where the gross utility of bidder i whose type is θ_i is defined as follows:

$$v(\theta_i, x) = \begin{cases} r_i & \text{if } x_i^t = 1 \text{ holds for some } t \in [a_i, d_i] \\ 0 & \text{otherwise.} \end{cases}$$

We also assume a *quasi-linear* utility; the net utility of bidder i who obtains at least one item during her stay and pays p is represented as $v(\theta_i, x) - p = r_i - p$.

An online mechanism $\mathcal{M}(f,p)$ consists of an allocation rule f and a payment rule p. An allocation rule f is defined as $f = \{f^t | t \in \mathbb{T}\}$. Here, $\theta = (\theta_1, \ldots, \theta_n)$ denotes a type profile reported by a set of bidders N, and $\Theta = \times_{i \in N} \Theta_i$ denotes a set of possible type profiles. Each $f^t : \Theta \to \{0,1\}^n$ is a mapping from a set of reported type profiles to a set of possible allocations. Let $f_i(\theta_i, \theta_{-i})$ denote the allocation to bidder i where θ_i is the declared type of bidder i and θ_{-i} is the declared type profile of other bidders. A payment rule p is defined as $p = (p_1, \ldots, p_n)$. Each $p_i : \Theta \to \mathbb{R}_{\geq 0}$ is a mapping from a set of type profiles to a set of non-negative real numbers. Notice that bidder i's reported type $\theta'_i = (a'_i.d'_i, r'_i)$ is not necessarily the same as her true type $\theta_i = (a_i, d_i, r_i)$. However, we assume no bidder can be present longer than her true stay, i.e., the *no-early arrival, no-late departure* property holds; a reported type θ'_i satisfies $a'_i \geq a_i$ and $d'_i \leq d_i$.

In this paper, we restrict our attention to *direct-revelation, deterministic* online mechanisms. Also, we assume that a mechanism is *almost anonymous* and *individually rational*. A mechanism is almost anonymous if the obtained results are invariant under the permutation of identifiers, except for ties where several bidders have an identical type but their allocations are different (e.g., only one winner). We assume the net utilities of bidders involved in tie-breaking must be the same. Individual rationality means that no participant suffers any loss in a dominant strategy equilibrium; i.e., the payment never exceeds the gross utility of the allocated items. Thus, a mechanism does not collect any payment from losers.

Now, let us define *strategy-proofness*.

Definition 1 (Strategy-proofness). *An online mechanism $\mathcal{M}(f,p)$ is strategy-proof if $\forall i, \theta_{-i}, \theta_i, \theta'_i$,*

$$v(\theta_i, f_i(\theta_{-i})) - p_i(\theta_i, \theta_{-i}) \geq v(\theta_i, f_i(\theta'_i, \theta_{-i})) - p_i(\theta'_i, \theta_{-i}).$$

A strategy-proof allocation rule is fully characterized by a simple property called *(value, time)-monotonicity* in the single-valued domain [8]. To define the monotonicity property, let us first introduce a concept of *critical value*, which plays an important role for guaranteeing strategy-proofness. In words, a critical value cv is the minimal (threshold) value for a bidder to be a winner.

$$\mathrm{cv}(a_i, d_i, \theta_{-i}) = \begin{cases} \inf r_i \text{ s.t. } f_i((a_i, d_i, r_i), \theta_{-i}) = 1 \\ \infty, \text{ if no such } r_i \text{ exists.} \end{cases} \quad (1)$$

Definition 2 ((value, time)-monotonicity). *An allocation rule f is (value, time)-monotonic if $\forall i, \theta_{-i}, \theta_i = (a_i, d_i, r_i)$, $\theta'_i = (a'_i, d'_i, r'_i)$, the following condition holds:*

$$\begin{aligned} &\text{if } f_i(\theta'_i, \theta_{-i}) = 1 \wedge\ r'_i > \mathrm{cv}(a'_i, d'_i, \theta_{-i}) \\ &\wedge\ a_i \leq a'_i \leq d'_i \leq d_i \wedge\ r_i \geq r'_i \\ &\text{then } f_i(\theta_i, \theta_{-i}) = 1. \end{aligned}$$

Note that the condition $r'_i > \mathrm{cv}(a'_i, d'_i, \theta_{-i})$ is necessary to prevent inconsistent allocations due to tie-breaking, e.g., bidder i and j have the same type. If valuation r_i is strictly greater than r'_i, we do not need this condition.

Hajiaghayi et al. [8] proved that if and only if an allocation rule is (value, time)-monotonic, we can find an appropriate payment rule that truthfully implements it in a dominant strategy equilibrium. In addition, it is straightforward to derive such an appropriate payment rule so that an online mechanism $\mathcal{M}(f,p)$ is strategy-proof:

$$p_i(\theta_i, \theta_{-i}) = \begin{cases} \mathrm{cv}(a_i, d_i, \theta_{-i}), & \text{if } f_i(\theta_i, \theta_{-i}) = 1 \\ 0, & \text{otherwise.} \end{cases}$$

When bidder i reports a shorter stay, her payment does not decrease, since bidder i's critical value does not decrease if an allocation rule is (value, time)-monotonic.

In this paper, we focus on a worst-case analysis (*competitive analysis*) to consider the performance of mechanisms. Such analysis is commonly used in recent mechanism design literature, especially by computer scientists. Let us define the competitive ratios for efficiency and revenue. Here, $z \in Z$ denotes the set of inputs available to the adversary and θ^z the corresponding type profile.

Definition 3 (Competitive Ratio for Efficiency). *An online mechanism $\mathcal{M}(f,p)$ is c-competitive for efficiency if for some constant c,*

$$\min_{z \in \mathcal{Z}} \mathbb{E}\{\mathrm{Val}(f(\theta^z))/V^*(\theta^z)\} \geq 1/c.$$

For efficiency, $\mathrm{Val}(f(\theta^z))$ indicates the social surplus of the decision made by an allocation rule f given input θ^z. $V^*(\theta^z)$ indicates the surplus of the best possible allocation obtained by an offline mechanism. This expectation is taken with respect to the random choice derived from the model of an adversary.

Definition 4 (Competitive Ratio for Revenue). *An online mechanism $\mathcal{M}(X,p)$ is c-competitive for revenue if for some constant c,*

$$\min_{z \in \mathcal{Z}} \mathbb{E}\{\mathrm{Rev}(p(\theta^z))/R^*(\theta^z)\} \geq 1/c.$$

For revenue, $R^*(\theta^z)$ indicates the revenue achieved by $\mathcal{F}^{(2,k)}$ auction with k items [6]. The revenue as a benchmark is used in [8]. Here, if $k \geq 2$, $R^*(\theta^z) = \max_{2 \leq m \leq k} m \cdot r_{(m)}$, where $r_{(m)}$ denotes the m-th highest value among all bidders. If $k = 1$, we use VCG revenue of $r_{(2)}$ as the benchmark.

Next, we introduce several notations for discussing false-name-proofness in online auction mechanisms. Let ϕ_i denote the set of identifiers owned by bidder i. Let $\mathbb{N}$ denote a set of identifiers, i.e., $\mathbb{N} = \bigcup_{i \in N} \phi_i$, where N denotes a set of real bidders. Let us re-define θ as the type profile reported by all identifiers. Here, $\mathbf{0}$ indicates that the identifier is not used by its owner. Furthermore, let θ_{ϕ_i} denote a type profile reported by a set of identifiers ϕ_i and $\theta_{-\phi_i}$ a type profile reported by identifiers except for ϕ_i. Using these notations, the allocation to an identifier j when the set of identifiers ϕ_i reports θ_{ϕ_i} and the other identifiers reports $\theta_{-\phi_i}$ is represented as $f_j(\theta_{\phi_i}, \theta_{-\phi_i})$.

DEFINITION 5 (FALSE-NAME-PROOFNESS). *An online mechanism $\mathcal{M}(f,p)$ is false-name-proof if $\forall i$, ϕ_i, $\theta_{-\phi_i}$, θ_i, θ_{ϕ_i}, the following inequality holds:*

$$v(\theta_i, f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i})) - p_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) \\ \geq v(\theta_i, \sum_{j\in\phi_i} f_j(\theta_{\phi_i}, \theta_{-\phi_i})) - \sum_{j\in\phi_i} p_j(\theta_{\phi_i}, \theta_{-\phi_i})$$

A mechanism is *false-name-proof* if it is a dominant strategy for each bidder to report her true type using a single identifier (although the bidder can use multiple identifiers). When $|\phi_i| = 1$, this definition is identical to Definition 1.

3. CHARACTERIZATION OF FALSE-NAME-PROOFNESS

In this section, we propose a simple property called *(value, time, identifier)-monotonicity* that characterizes false-name-proof allocation rules in online auction mechanisms.

DEFINITION 6 ((VALUE, TIME, IDENTIFIER)-MONOTONICITY). *An allocation rule f is (value, time, identifier)-monotonic if for any i, ϕ_i, θ_i, $\theta_{-\phi_i}$, θ_{ϕ_i}, the following holds:*

$$\begin{aligned}&\textit{if } \exists j' \in \phi_i \textit{ s.t.},\\ &\quad \big(f_{j'}(\theta_{\phi_i}, \theta_{-\phi_i}) = 1 \wedge\ r_{j'} > \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i})\big)\\ &\wedge \big(\forall j \in \phi_i, a_i \leq a_j \leq d_j \leq d_i\big)\ \wedge\ r_i \geq \textstyle\sum_{j'\in\phi_i:j' \textit{ wins}} r_{j'}\\ &\text{then}\ \ f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 1.\end{aligned} \tag{2}$$

Note that $\theta_{\phi_i\setminus\{j'\}}$ denote the type profile by the set of identifiers $\phi_i \setminus \{j'\}$. Thus, $\mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i})$ indicates the critical value of a bidder that stays $[a_{j'}, d_{j'}]$ when the other identifiers reports $\theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i}$.

Now let us provide an illustrative example of an allocation rule that satisfies Def. 6. Assume that the set of identifiers ϕ_i surrounded by the dashed rectangle in Fig. 1 (a) is owned by bidder i and that the identifier j' with value $r_{j'}$ wins an item. If θ_{ϕ_i} in Fig. 1 (a) is replaced by one type $\theta_i = (a_i, d_i, r_{j'} + \epsilon)$ in Fig. 1 (b), then the allocation rule that satisfies (value, time, identifier)-monotonicity must choose θ_i as a winner, as long as θ_i satisfies the following conditions: (i) the interval $[a_i, d_i]$ includes $[a_j, d_j]$ for all $j \in \phi_i$, (ii) the value $r_{j'} + \epsilon$ exceeds the winner's value $r_{j'}$.

Intuitively, in the inequality $r_i \geq \sum_{j'\in\phi_i:j' \text{ wins}} r_{j'}$ in Def. 6, each term $r_{j'}$ on the right-hand side corresponds to a payment of each winning identifier j' that may be owned by i. To avoid false-name manipulations, bidder i with value r_i that exceeds the sum of $r_{j'}$ must win an item. Otherwise, bidder i has an incentive to manipulate using the set of identifiers ϕ_i. We remark that this property is inspired by two characterizations: (value, time)-monotonicity by Hajiaghayi et al. and sub-additivity by Todo et al. In fact, the property becomes equivalent to (value, time)-monotonicity when $|\phi_i| = 1$ for all i and to sub-additivity when $T = 1$, i.e., in offline auction settings.

Before showing our characterization theorem, let us provide the following lemma:

LEMMA 1. *Given an allocation rule that satisfies (value, time, identifier)-monotonicity, the critical value of bidder i is independent of her valuation r_i and weakly increasing in shorter stay and more identifiers.*

PROOF. In Parkes [14], it was shown that the critical value is weakly increasing in shorter stay. Then we can show that the critical value is weakly increasing in a larger number of rivals. Now assume that there exists an additional identifier $\theta_{i'} = (a_{i'}, d_{i'}, r_{i'})$ such that $a_i \leq a_{i'} \leq d_{i'} \leq d_i$ and $\mathrm{cv}(a_i, d_i, \theta_{-i}) > \mathrm{cv}(a_i, d_i, \theta_{-i} \cup \theta_{i'})$ to derive a contradiction.

Here, modify the valuation of type θ_i such that $r_i = \mathrm{cv}(a_i, d_i, \theta_{-i} \cup \theta_{i'})$. Then, from the definition of cv when the other bidders (identifiers) report $\theta_{-i} \cup \theta_{i'}$, $f_i(\theta_i, \theta_{-i} \cup \theta_{i'}) = 1$. Also, from the definition of cv when the other bidders (identifiers) report θ_{-i}, $f_i(\theta_i, \theta_{-i}) = 0$. Thus, by setting $\theta_{\phi_i} = \{\theta_i, \theta_{i'}\}$, $\theta_{-\phi_i} = \theta_{-i}$ and $\theta_{j'} = \theta_i$, we have

$$\begin{aligned}&\exists j' \in \phi_i \text{ s.t.},\\ &\quad \big(f_{j'}(\theta_{\phi_i}, \theta_{-\phi_i}) = 1 \wedge\ r_{j'} > \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i})\big)\\ &\wedge \big(\forall j \in \phi_i, a_i \leq a_j \leq d_j \leq d_i\big) \wedge\ r_i \geq \textstyle\sum_{j'\in\phi_i:j' \text{ wins}} r_{j'}\\ &\text{and } f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 0,\end{aligned}$$

which violates (value, time, identifier)-monotonicity. □

THEOREM 1. *On a single-valued domain, there always exists an appropriate payment rule p so that an online mechanism $\mathcal{M}(f,p)$ is false-name-proof if and only if the allocation rule f satisfies (value, time, identifier)-monotonicity.*

PROOF. **(only if part)** We first prove that if an online mechanism $\mathcal{M}(f,p)$ is false-name-proof, then the allocation rule f satisfies (value, time, identifier)-monotonicity. Parkes [14] proved that if $\mathcal{M}$ is strategy-proof, then f satisfies (value, time)-monotonicity. Since the definition of false-name-proofness is a generalization of strategy-proofness, if $\mathcal{M}$ is false-name-proof, then it is also strategy-proof. Thus, we can assume that f satisfies (value, time)-monotonicity and that p is determined based on the *critical* values in Eq. 1.

We derive a contradiction by assuming that the allocation rule f does not satisfy (value, time, identifier)-monotonicity. More specifically, we assume for bidder i (with type θ_i), who owns the set of identifiers ϕ_i, the following condition holds:

$$\begin{aligned}&\exists j' \in \phi_i \text{ s.t.},\\ &\quad \big(f_{j'}(\theta_{\phi_i}, \theta_{-\phi_i}) = 1 \wedge\ r_{j'} > \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i})\big)\\ &\wedge \big(\forall j \in \phi_i, a_i \leq a_j \leq d_j \leq d_i\big) \wedge\ r_i \geq \textstyle\sum_{j'\in\phi_i:j' \text{ wins}} r_{j'}\\ &\text{and } f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 0.\end{aligned} \tag{3}$$

When Eq. 3 holds, bidder i with type θ_i cannot win the item by truthfully reporting her type. Thus,

$$v(\theta_i, f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i})) - p_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 0.$$

Also, Eq. 3 implies that if bidder i reports θ_{ϕ_i} using false identifiers, she wins at least one item. Note that since there exists at least one winning identifier j' in ϕ_i such that $r_{j'} > \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i\setminus\{j'\}} \cup \theta_{-\phi_i})$, we have $\sum_{j'\in\phi_i:j' \text{ wins}} r_{j'} > \sum_{j'\in\phi_i:j' \text{ wins}} p_{j'}(\theta_{\phi_i}, \theta_{-\phi_i})$. Thus,

$$v(\theta_i, \sum_{j\in\phi_i} f_j(\theta_{\phi_i}, \theta_{-\phi_i})) - \sum_{j\in\phi_i} p_j(\theta_{\phi_i}, \theta_{-\phi_i}) \\ > r_i - \sum_{j'\in\phi_i:j' \text{ wins}} r_{j'} \geq 0.$$

This bidder can increase her utility by using false identifiers, contradicting the assumption of false-name-proofness.

(if part) Next we prove that if an allocation rule f satisfies (value, time, identifier)-monotonicity, then there exists an appropriate payment rule p such that $\mathcal{M}(f,p)$ is false-name-proof. We derive a contradiction by assuming that

$$\begin{aligned}&\forall p, \exists \theta_i, \theta_{\phi_i},\\ &v(\theta_i, f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i})) - p_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i})\\ &< v(\theta_i, \textstyle\sum_{j\in\phi_i} f_j(\theta_{\phi_i}, \theta_{-\phi_i})) - \sum_{j\in\phi_i} p_j(\theta_{\phi_i}, \theta_{-\phi_i})\end{aligned} \tag{4}$$

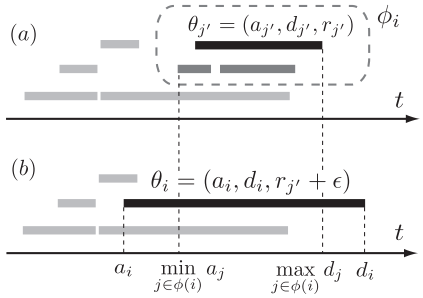

Figure 1: Example of allocation rule that satisfies (value, time, identifier)-monotonicity

holds. More specifically, we show that if a payment rule p is defined by a critical value (as Eq. 1), Eq. 4 does not hold in the following two cases: (I) bidder i is winning when she reports truthfully, and (II) bidder i is losing.

Case I: $f_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) = 1$ holds. Since we assume a single-valued domain, the two terms $v(\theta_i, \cdot)$ in Eq. 4 are equivalent. Thus, from Eq. 4, $p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) > \sum_{j \in \phi_i} p_j(\theta_{\phi_i}, \theta_{-\phi_i})$. Furthermore, since we assume the mechanism does not collect payments from losers, we obtain

$$p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) > \sum_{j' \in \phi_i : j' \text{ wins}} p_{j'}(\theta_{\phi_i}, \theta_{-\phi_i}). \quad (5)$$

On the other hand, from Lemma 1, the critical value of bidder i with stay $[a_i, d_i]$ weakly increases in shorter stay and more identifiers. Thus, for all winners $j' \in \phi_i$, $\mathrm{cv}(a_i, d_i, \theta_{-\phi_i}) \le \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i \setminus \{j'\}} \cup \theta_{-\phi_i})$, and therefore

$$\mathrm{cv}(a_i, d_i, \theta_{-\phi_i}) \le \sum_{j' \in \phi_i : j' \text{ wins}} \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i \setminus \{j'\}} \cup \theta_{-\phi_i}). \quad (6)$$

Thus, with a payment rule p defined by Eq. 1, we obtain

$$p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) \le \sum_{j' \in \phi_i : j' \text{ wins}} p_{j'}(\theta_{\phi_i}, \theta_{-\phi_i}), \quad (7)$$

and this contradicts Eq. 5.

Case II: $f_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) = 0$ holds. Assume that bidder i, whose true type is $\theta_i = (a_i, d_i, r_i)$, cannot win when she reports truthfully. That is, $r_i < \mathrm{cv}(a_i, d_i, \theta_{-\phi_i})$, and when she reports truthfully her utility is zero.

Here, applying the same argument as in Eq. 6, we have

$$r_i < \sum_{j' \in \phi_i : j' \text{ wins}} \mathrm{cv}(a_{j'}, d_{j'}, \theta_{\phi_i \setminus \{j'\}} \cup \theta_{-\phi_i}).$$

The right-hand side corresponds to the total payment of bidder i when she uses false identifiers θ_{ϕ_i}. This implies that her utility with this manipulation is negative. Thus,

$$v(\theta_i, f_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i})) - p_i((\theta_i, \mathbf{0}, \dots, \mathbf{0}), \theta_{-\phi_i}) \\ = 0 > v(\theta_i, \textstyle\sum_{j \in \phi_i} f_j(\theta_{\phi_i}, \theta_{-\phi_i})) - \sum_{j \in \phi_i} p_j(\theta_{\phi_i}, \theta_{-\phi_i})$$

holds, which contradicts Eq. 5. □

To show that the allocation rule of Mechanism 1 does not satisfy (value, time, identifier)-monotonicity, consider Example 1 again. When bidder 1 uses two identifiers, $1'$ and $1''$, seven identifiers participate in the auction ($n = 7$). Since Mechanism 1 waits for the second bidder, identifier $1'$ with $(1, 3, 6)$ obtains the item using identifier $1''$ with $(2, 2, \epsilon)$. However, when bidder 1 uses only one identifier, six identifiers participate in the auction ($n = 6$). In this case, the situation becomes identical to Table 1 and bidder 1 no longer obtains the item. Thus, this allocation rule does not satisfy (value, time, identifier)-monotonicity.

Note that our characterization is constructed on a single-valued domain and thus can be applied to any environment on the domain. For example, (value, time, identifier)-monotonicity is applicable to *expiring-item* environments, where a mechanism allocates a single indivisible item to a bidder in each period, e.g., the right to use a shared computer or a network resource. We then show a representative strategy-proof mechanism for expiring-item environments, called *greedy auction*, and verify whether the allocation rule satisfies (value, time, identifier)-monotonicity.

MECHANISM 2 (GREEDY AUCTION [14]). *In each period $t \in \mathbb{T}$, allocate the item to a bidder who has the highest value at t and who has not been assigned an item yet (breaking ties deterministically). Every allocated bidder pays its critical value, which is collected upon its reported departure.*

CLAIM 1. *The allocation rule of Mechanism 2 satisfies (value, time, identifier)-monotonicity.*

PROOF. Let us describe the allocation rule f_i^t for bidder i at period t when θ_{-i} is fixed:

$$f_i^t(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if } a_i \le t \le d_i \text{ and unallocated in } t' < t \\ & \quad \text{and } r_i \ge r_l \ \forall l (\in \mathbb{N} \setminus \{i\}) \text{ s.t.} \\ & \quad\ a_l \le t \le d_l \text{ and unallocated in } t' < t \\ 0 & \text{otherwise.} \end{cases}$$

We derive a contradiction by assuming that, when there exist at least one winner j' in ϕ_i for some $\phi_i, \theta_{-\phi_i}, \theta_{\phi_i}$, there also exists type θ_i that satisfies Eq. (3).

Choose winner $j' \in \phi_i$ whose arrival period $a_{j'}$ is the earliest among the identifiers in ϕ_i. Let $t_{j'}$ denote the period in which j' wins. At period $t_{j'}$, θ_i is present, since $a_i \le a_{j'} \le d_{j'} \le d_i$ holds from Eq. (3). The bid $r_{j'}$ is the highest one at $t_{j'}$ in the presence of ϕ_i. Consider the absence of ϕ_i. Since ϕ_i is replaced with $\mathbf{0}$, the highest bid changes from $r_{j'}$ to r_i at $t_{j'}$. $r_i \ge r_l$ for all $l \in \mathbb{N} \setminus \phi_i$ such that bidder l is present at $t_{j'}$ ($a_l \le t_{j'} \le d_l$) and is unallocated in $t' < t_{j'}$. Thus, bidder i with type θ_i is chosen as a winner at period $t_{j'}$ (or before $t_{j'}$) if θ_{ϕ_i} is replaced with $(\theta_i, \mathbf{0}, \dots, \mathbf{0})$. Accordingly, this contradicts the assumption. □

We can easily verify that the payment rule of Mechanism 2 is defined appropriately such that the mechanism is false-name-proof. We omit the proof due to space limitation.

4. NON-TRIVIAL FALSE-NAME-PROOF MECHANISMS

In this section, we present two non-trivial false-name-proof online mechanisms for k identical items. Before introducing them, we recall the intuitive reason why Mechanism 1 is not false-name-proof. The number of bidders, probably including false identifiers, determines when Mechanism 1 transits to the accepting phase. More precisely, a bidder can manipulate the transition period a^*, since it is set as the arrival period of the $\lfloor n/e \rfloor$-th bidder. To avoid such a manipulation, we must determine the transition period independently from the the number of bidders. The basic idea of our mechanisms is that they transit in a predefined constant period τ. The following is our first mechanism.

Mechanism 3. *Let k be the number of items for sale and τ be a predefined period s.t. $0 \leq \tau \leq T$.*

1. *(learning phase): At period τ, sort the bidding values observed so far in descending order and denote them as $r_{(1)}, r_{(2)}, \ldots, r_{(k)}, r_{(k+1)}, \ldots$. If there exist only $k'(< k+1)$ bids, we assume $r_{(k'+1)}, \ldots$ are 0.*
2. *(transition): If any bidder who bids $r_{(1)}, \ldots, r_{(k)}$ is still present at period τ, then sell to that bidder at price $r_{(k+1)}$.*
3. *(accepting phase): As long as there exists a remaining item, sell to the next bidder whose bid is at least $r_{(k)}$ at price $r_{(k)}$.*

When $k = 1$, Mechanism 3 can be considered an application of the optimal stopping rule for a class of secretary problems, where the number of candidates n is unknown [3].

Example 2. *Now let us describe the behavior of Mechanism 3 based on Table 1, assuming $\tau = 3$ and $k = 1$. First, consider the case where bidder 1 reports truthfully. From the definition, Mechanism 3 does not allocate the item to any bidder until period 3. At period 3, it allocates the item to bidder 1 at price 0. Next, consider another case where she uses two identifiers, $1'$ and $1''$, and reports $(1, 3, 6)$ and $(2, 2, \epsilon)$, respectively. Again, the mechanism does not allocate the item to any bidder until period 3. At period 3, it allocates the item to identifier $1'$ at price ϵ. Clearly that bidder 1 cannot increase her utility even if she uses false identifiers in Mechanism 3. Furthermore, losing bidders 2-6 cannot be winners even if they use false identifiers, since we assume the no-early arrival, no-late departure property.*

Although Mechanism 3 is false-name-proof, it requires a predefined transition period. In general, it is difficult to determine an appropriate transition period with respect to efficiency and revenue. However, we show the competitive ratio below in this section, assuming the mechanism knows the distribution of bidder arrival times.

On the other hand, one might think that the bidders who depart before the transition period do not have an incentive to join the auction, since they know that they have no chance to win. One possible remedy is to keep the information about the transition period τ private by not announcing it beforehand. Another remedy is to use a random timing device to determine the transition period. It can ring with small enough probability in each period before the default transition period τ and must ring in τ at the latest.

Utilizing our characterization, we show the next theorem.

Theorem 2. *Mechanism 3 is false-name-proof.*

Proof. We first prove that the allocation rule of this mechanism satisfies (value, time, identifier)-monotonicity and then show that the payment rule is defined by critical values. When θ_{-i} is fixed, the allocation rule f_i for bidder i can be described as follows. We denote the k-th highest value observed until τ except i's bid as $r^{-i}_{(k)}$.

$$f_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if either (i) } a_i \leq \tau \wedge d_i \geq \tau \wedge r_i \geq r^{-i}_{(k)}, \\ & \text{or (ii) } a_i > \tau \wedge r_i \geq r^{-i}_{(k)} \wedge |W| < k, \\ & \text{where } W = \{w \mid w \neq i \wedge a_w \leq a_i \\ & \qquad \wedge d_w \geq \tau \wedge r_w \geq r^{-i}_{(k)}\} \\ 0 & \text{otherwise.} \end{cases}$$

We are going to derive a contradiction by assuming that the allocation rule does not satisfy (value, time, identifier)-monotonicity. More specifically, we assume that when at least one winner l exists in ϕ_i, for some $\phi_i, \theta_{-\phi_i}, \theta_{\phi_i}$, there exists type $\theta_i = (a_i, d_i, r_i)$ such that

$$\big(\forall j \in \phi_i, a_i \leq a_j \leq d_j \leq d_i\big) \wedge r_i \geq \textstyle\sum_{j' \in \phi_i : j' \text{ wins}} r_{j'}$$
$$\text{and } f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 0.$$

Choose j' as the winner in ϕ_i and its arrival period $a_{j'}$ is earliest. Note that j' is a false identifier owned by i.

First, consider the case where $a_i \leq \tau$. Since j' is a winner, regardless whether $a_{j'}$ is before or after τ, $d_{j'} \geq \tau$ and $r_{j'} \geq r^{-j'}_{(k)}$. Also, $r^{-i}_{(k)} \leq r^{-j'}_{(k)}$. This is because $r^{-i}_{(k)}$ is the k-th highest valuation observed until τ except the bid of i, and $r^{-j'}_{(k)}$ is the k-th highest valuation observed until τ, including ϕ_i except j'. >From the assumption, $a_i \leq a_{j'} \leq \tau$, $d_i \geq d_{j'} \geq \tau$, and $r_i \geq r_{j'}$. Thus, we obtain $r_i \geq r^{-j'}_{(k)} \geq r^{-i}_{(k)}$, and condition (i) of the allocation rule holds. This contradicts the assumption that $f_i((\theta_i, \mathbf{0}, \ldots, \mathbf{0}), \theta_{-\phi_i}) = 0$.

Next, consider the case where $a_i > \tau$. For all $j \in \phi_i$, $a_i \leq a_j$ holds; no bidder in ϕ_i arrives before τ. Thus, $r^{-i}_{(k)} = r^{-j'}_{(k)}$ holds. Since j' is the winner, $r_{j'} \geq r^{-j'}_{(k)}$ also holds. From the assumption, $r_i \geq r_{j'}$ holds. Thus, we obtain $r_i \geq r^{-i}_{(k)}$. Also, since j' is the winner in ϕ_i and its arrival period is the earliest. Thus, for $W_{j'} = \{w \mid w \in \mathbb{N} \setminus \{j'\}$ and $a_w \leq a_{j'}$ and $d_w \geq \tau$ and $r_w \geq r^{-j'}_{(k)}\}$ and $W_i = \{w \mid w \in \mathbb{N} \setminus \phi_i$ and $a_w \leq a_i$ and $d_w \geq \tau$ and $r_w \geq r^{-i}_{(k)}\}$, since $a_i \leq a_{j'}$, $W_i \subseteq W_{j'}$ holds. Thus, $|W_i| \leq |W_{j'}|$ holds. Since j' is a winner, $|W_{j'}| < k$ holds. Thus, $|W_i| < k$ holds. Therefore, condition (ii) of the allocation rule holds, but this contradicts the assumption.

Critical value cv of bidder i is defined as follows:

$$\mathrm{cv}(a_i, d_i, \theta_{-i}) = \begin{cases} r^{-i}_{(k)} & \text{if either (i) } a_i \leq \tau \wedge d_i \geq \tau, \\ & \text{or (ii) } a_i > \tau \wedge |W| < k, \\ & \text{where } W = \{w \mid w \neq i \\ & \qquad \wedge a_w \leq a_i \wedge d_w \geq \tau \\ & \qquad \wedge r_w \geq r^{-i}_{(k)}\} \\ \infty & \text{otherwise.} \end{cases}$$

Also, the appropriate payment rule p is derived as follows:

$$p_i(\theta_i, \theta_{-i}) = \begin{cases} \mathrm{cv}(a_i, d_i, \theta_{-i}) & \text{if } f_i(\theta_i, \theta_{-i}) = 1 \\ 0 & \text{otherwise.} \end{cases}$$

This payment rule is identical to Mechanism 1. □

Competitive analysis for online mechanisms requires to assume an adversarial model as well as the optimal stopping theory. A representative model is the random-ordering model used in [9], which requires a mechanism to observe the exact number of bidders beforehand. Therefore, we cannot apply the model to our situation where a mechanism can not certainly observe the number of real bidders. Thus, we introduce another adversarial model from [3]. Unlike the random-ordering model, the model requires mechanisms to observe only the distribution of arrival times of bidders. It is quite natural that a mechanism has knowledge about the distribution of arrival times in such real-world economic environments as Internet auctions. For example, an auctioneer can usually obtain trends about the density of bids, e.g., the number of bids on weekends exceeds those in the daytime on weekdays. Focusing on the model where only the distribution of bidder arrival times is known, we apply Bruss's adversarial model to our competitive analysis. Notice that Hajiaghayi et al. [10] deal with a situation where the number of bidders is unknown to a mechanism. Instead, although they assume that the distribution of valuations of bidders is known, we will investigate this model in future work.

In our model, the auction is performed within finite continuous interval $[0, T]$ and all bidders are *impatient*; $\forall i \in N$, $a_i = d_i$. This continuous model makes the analysis much simpler; in a discrete time interval model, there might be no transition period τ s.t. $G(\tau) = \frac{1}{2}$ in Theorem 3. We assume that for n bidders, an adversary specifies its valuations. We also restrict our attention to cases where all valuations are unique to ignore ties. In addition, we let a mechanism know a distribution function G, from which bidder arrival times are drawn i.i.d. However, the mechanism has neither information about the number of bidders n nor their valuations.

For $k = 1$, our model becomes almost identical to that of the secretary problem discussed in [3]. Thus, we can easily see that for any distribution G, Mechanism 3 is e-competitive for efficiency by defining $\tau = G^{-1}(e^{-1})$. Although we strongly believe this stopping rule is optimal for efficiency, we cannot directly use the result in [3], since the mechanism can observe richer information, i.e., the bids of bidders, than for the secretary problem. However, as discussed in [9], it is very unlikely that a mechanism can capitalize on this numerical information, since we are making absolutely no assumptions about the distribution of bids.

We now show a more general result for arbitrary k. The next theorem shows that the competitive ratio of Mechanism 3 for efficiency is independent of the number of items k, if the transition period τ satisfies $F(\tau) = \frac{1}{2}$.

THEOREM 3. *In our model, Mechanism 3 with constant stopping time $\tau_{1/2}$ such that $G(\tau_{1/2}) = \frac{1}{2}$ is 4-competitive for efficiency as $n \to \infty$ when k is sufficiently large and all bidders are impatient in finite continuous interval $[0, T]$.*

PROOF (SKETCH). In the worst case, each of the top k bidders has a high value (e.g., 1) and the others have a low value (e.g., 0). The probability that k-th highest bidder, who arrives before $\tau_{1/2}$, is $k + s + 1$-st highest overall, is given as $\binom{k+s}{k-1} \cdot (\frac{1}{2})^{k+s+1}$. Possible winners are bidders $1, \ldots, k + s$. A winner must arrive after $\tau_{1/2}$ and before k items are sold out. The actual value of efficiency (i.e., the expected number of winners $1, \ldots, k$) is given as $SS = \sum_{s=0}^{\infty} g(s) \cdot \min(s + 1, k)$, where $g(s) = \binom{k+s}{k-1} \cdot (\frac{1}{2})^{k+s+1} \cdot \frac{k}{k+s}$. Clearly, this is smaller than $SS' = \sum_{s=0}^{\infty} g(s) \cdot k$, which equals $\frac{k^2}{k-1}(\frac{1}{2} - \frac{1}{2^k})$ by multinomial coefficient. Thus, for sufficiently larger k, $SS' > k/2$ holds. Furthermore, we can prove that $SS' \leq 2SS$ holds; SS' is an over-estimation of SS but SS' is at most twice as large as SS. More specifically, the amount of over-estimation, i.e., $SS' - SS$ is given as $\sum_{s=0}^{k-2} g(s) \cdot (k - s - 1)$. We can show that this is smaller than SS, i.e., $SS' - SS \leq SS$ holds, since $g(s)$ is basically an increasing function of s (where s is smaller than $k-2$). Thus, $SS > k/4$ holds. Since the optimal social surplus is k, we obtain the competitive ratio of 4. □

In contrast to efficiency, the competitive ratio of Mechanism 3 for revenue is 0, which occurs in the same valuations above. To achieve better revenue, we introduce another mechanism.

MECHANISM 4. *Let k be the number of items for sale and $\tau_1, \ldots, \tau_k$ ($\tau_1 < \ldots < \tau_k$) be a sequence of predefined periods.*

1. *(learning phase): At period τ_m ($1 \leq m \leq k$), sort bidding values observed so far in descending order and denote them as $r^m_{(1)}, r^m_{(2)}, \ldots$. If there exist no bids, we assume $r^m_{(1)}, r^m_{(2)}, \ldots$ are 0.*
2. *(transition): If the bidder of $r^m_{(1)}$ is still present at period τ_m, then sell to him at price $r^m_{(2)}$.*
3. *(accepting phase): As long as the item remains and current time t satisfies $t < \tau_{m+1}$, sell to the next bidder whose bid is at least $r^m_{(1)}$ at price $r^m_{(1)}$.*

Intuitively, Mechanism 4 is false-name-proof, since the prices at transition periods $\tau_1, \ldots, \tau_k$ never decrease, and an unsold item will not be carried forward to the next period.

THEOREM 4. *In our model, Mechanism 4 with a sequence of stopping times $\tau_1, \ldots, \tau_k$ s.t., $G(\tau_m) = \frac{mT}{k+1}$ $\forall m \in \{1, \ldots, k\}$ is $\frac{k}{\log k}$-competitive for revenue as $n \to \infty$ when k is sufficiently large and all bidders are impatient in finite continuous interval $[0, T]$.*

PROOF. An adversary chooses a set of valuations so that all bidders $i \in \{1, \ldots, k\}$ have $1 - i \cdot \epsilon$ and all other $n - k$ bidders $i \in k+1, \ldots, n$ have $(n - i + 1) \cdot \epsilon$ as the worst case.

The probability that a particular pair of bidders arrives within the same period is $\frac{1}{k}$. For sufficiently large k, the probability becomes small enough to be ignored. The probability that bidder 1 wins an item and pays a high value is given by the summation of the probabilities that bidder 1 arrives (a) after bidder 2, (b) before bidder 2 and after bidder 3, (c) before bidders 2 and 3 and after bidder 4, ..., i.e., $\frac{1}{2} + \frac{1}{3!} + \frac{2!}{4!} + \cdots + \frac{(k-2)!}{k!} = 1 - \frac{1}{k}$. In general, the probability that bidder i wins and pays a high value is $\frac{(i-1)!}{(i+1)!} + \cdots + \frac{(k-2)!}{k!} = \frac{1}{i} - \frac{1}{k}$. Thus, the expected revenue is calculated as $\sum_{i=1}^{k-1} \frac{1}{i} - \frac{k-1}{k}$. Since the first term $\sum_{i=1}^{k-1} \frac{1}{i}$ is a harmonic series, we have $\sum_{i=1}^{k-1} \frac{1}{i} - \frac{k-1}{k} \geq \log k - \frac{k-1}{k}$ and for large k, Mechanism 4 is $\frac{k}{\log k}$-competitive. □

Using a similar argument to the above proof, we can also show that Mechanism 4 is $\frac{k}{\log(k+1)}$-competitive for efficiency.

The competitive ratios shown in Theorems 4 and 5 are not tight since, we have not yet obtained theoretical lower bounds. However, even in one-shot mechanisms, there have been very few results on the competitive ratios of false-name-proof mechanisms, except for those by [11, 7]. Thus, we believe the results in this paper are an important first step to clarify the bounds in online false-name-proof mechanisms.

5. EXPERIMENTAL ANALYSIS

In addition to the worst-case analysis in Section 4, we experimentally evaluated Mechanism 3 when $k = 1$. We set discrete time periods $\{1, \ldots, 20\}$ ($T = 20$), varying the number of bidders from 10 to 100 by 10. Each bidder's type $\theta_i = (a_i, d_i, r_i)$ is generated as follows. The valuation r_i is drawn from a uniform distribution over $[0, \bar{r}]$. The arrival time a_i is drawn from a uniform distribution over $[0, T]$, and the departure time d_i is drawn from a uniform distribution over $[a_i, T]$. Notice that, although we run our simulation with a variety of values $\bar{r}$, the performance does not depend on $\bar{r}$. Thus, we show the results in the case of $\bar{r} = 100$. We set the stopping strategy τ of Mechanism 3 to $\lfloor T/e \rfloor = 7$.

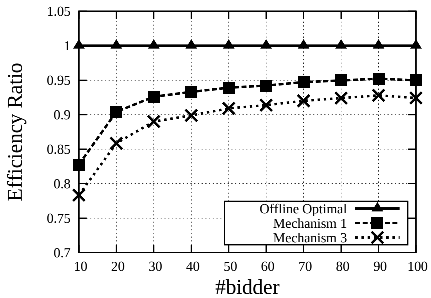

Figure 2: Efficiency ratios in average case with respect to Offline Optimal Mechanism

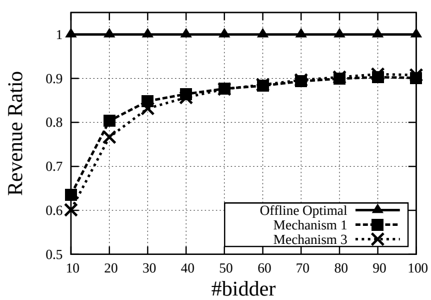

Figure 3: Revenue ratios in average case with respect to Offline Optimal Mechanism

From Theorem 3, this is the stopping strategy to achieve a competitive ratio of e for efficiency if the bidders are impatient. We then averaged the ratios of efficiency and revenue, generating 10000 instances for each number of bidders.

Figures 2 and 3 illustrate the average ratios of efficiency and revenue, respectively, achieved by Mechanism 1 and Mechanism 3, varying the number of bidders. Note that the result of Mechanism 1 is provided to show an ideal ratio, where the mechanism can set the optimal learning period by knowing the number of bidders n beforehand, and bidders do not use false-name bids. In Fig. 2, we can see that in terms of efficiency, Mechanism 3 achieves 93% of the offline optimal mechanism as the number of bidders grows and it is slightly outperformed by Mechanism 1. Furthermore, in terms of revenue, Fig. 3 shows that Mechanism 3 performs almost equivalently to Mechanism 1.

6. CONCLUSIONS AND FUTURE WORKS

In this paper, we characterized false-name-proof online mechanisms and proposed two non-trivial ones for k identical items. When $k = 1$, Mechanism 3 corresponds to the optimal stopping rule of a class of secretary problems [3], where the number of candidates n is unknown to the employer who only knows the distribution of the candidate arrival times. We further revealed that Mechanism 3 is 4-competitive for efficiency, which is independent on the number of items k. Also, Mechanism 4 is $\frac{k}{\log k}$-competitive for revenue.

One open problem is obtaining a lower bound of the competitive ratio of false-name-proof online mechanisms for efficiency and revenue. For efficiency, we strongly believe that the lower bound is e when $k = 1$, although it remains unproved. We would like to relax several assumptions we introduced for competitive analysis, e.g., impatient bidders. Furthermore, we would like to extend our results beyond single-valued domains (e.g., dynamic multi-unit auctions [5]). Considering the case that a bidder can only use a limited number of fake identifiers might also be interesting. This restriction would weaken bidders in the market, and help us design false-name-proof mechanisms.

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