Reformulation of Cooperative Game Theory based on Concise Representation Scheme

(Extended Abstract)

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ABSTRACT

Forming effective coalitions is a major research challenge in AI and multi-agent systems. Thus, cooperative games, including Coalition Structure Generation (CSG), have been attracting considerable attention from the AI research community. Traditionally, the input of a cooperative game is a black-box function called a characteristic function. Therefore, many problems in cooperative games, including CSG, tend to be computationally intractable. In my thesis, I develop new representation schemes, which are concise and can enable efficient computation for solving various problems related to cooperative game theory.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems*

General Terms

Algorithms, Theory

Keywords

Cooperative game theory, coalition structure generation, concise representation scheme

1. INTRODUCTION

Coalition formation is an important capability in automated negotiation among self-interested agents. As a result, cooperative game theory has attracted much attention from AI and multi-agent systems (MAS) researchers [1].

There are two major research topics in cooperative games. The first topic involves partitioning a set of agents into coalitions so that the sum of the rewards of all coalitions is maximized. This problem is called the Coalition Structure Generation problem (CSG) [6]. The second topic involves how to divide the value of the coalition among agents. The theory of cooperative games provides a number of solution concepts, such as the core, the Shapley value, and the nucleolus.

A range of previous studies have found that many problems in cooperative games, including CSG, tend to be computationally intractable. Traditionally, the input of a cooperative game is a black-box function (oracle) called a characteristic function, which takes a coalition as an input and returns the value of the coalition (or a coalition structure as a whole). This representation is too abstract. For example, if we naively represent a characteristic function as a table, we need 2^n space (where n is the number of agents).

2. CONCISE REPRESENTATION SCHEMES

We believe the main cause of the above mentioned problems is the fact that the characteristic function representation (i.e., the black box function representation) is too abstract. We need an alternative problem representation that satisfies following requirements.

- The representation should be concise, i.e., it requires exponentially less space than 2^n.
- The representation should be general, i.e., it can represents any characteristic function.
- The representation should enable efficient computation for solving various problems related to cooperative game theory, e.g., finding the core, Shapley value, and an optimal coalition structure.

There exist several efforts to develop concise representation schemes for cooperative game, such as the Synergy Coalition Group (SCG) [2], Marginal Contribution networks (MC-nets) [4], and the graphical representation [3]. However, we do not yet have a representation scheme that satisfies all of the above mentioned requirements. The goal of my thesis is to develop a representation scheme/language that satisfies all of these requirements.

3. CURRENT ISSUES

We developed several concise representation schemes and examined the computational complexity of various problems related to cooperative game theory. The results are summarized in Table 1.

3.1 Distributed Constraint Optimization based Representation

In our paper [8], we proposed the concise representation scheme based on the distributed constraint optimization problems (DCOP). In this representation scheme, we assume that the value of a coalition is given by an optimal solution of a DCOP among the agents of the coalition.

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Table 1: Computational complexities of coalition formation problems and CSG

	DCOP based representation [8]	Type-based representation schemes [9]		
		Characteristic function	SCG	MC-nets
Core non-empty	open	polynomial	polynomial	polynomial
Core membership	open	$O(n^t)$	$O(n^{2t})$	$O(n^{2t})$
The Shapley value	open	$O(n^t)$	$O(n^{2t})$	$O(\|R\| \cdot n^{2t})$
CSG	NP-hard	$O(n^{2t})$	$O(n^{2t})$	$O(n^{2t})$

After a pioneering work by [5], DCOP has become a popular approach for modeling cooperative agents. In DCOP, each agent has a choice of actions (values). Reward/cost is determined by the combination of values. The goal is to maximize/minimize the sum of the rewards/costs. This framework is quite general and can represent various application domains in cooperative MAS.

Although a DCOP is a very general and powerful framework, the required computational costs might be too expensive, because we need to solve an NP-hard problem just to obtain the value of a single coalition. To optimally solve a CSG, we might need to solve $O(2^n)$ DCOP problem instances, where n is the number of agents. However, quite surprisingly, we developed an approximation algorithm, which can find a coalition structure with quality guarantees and whose computational cost is about the same as finding the value of the grand coalition. Also, we experimentally show that the average approximation ratio of the algorithm is by far superior to the theoretical worst-case bound.

In this direction, we are planning to develop more efficient algorithms, applying the ideas developed in [8] to real-world application domains, and extending our formalization to handle the case *with externality*, i.e., there can be positive/negative interactions among coalitions. Also, we would like to examine the computational complexities of coalition formation problem such as finding the core, the Shapley value and the nucleolus.

3.2 Type-based Representation

In our paper [9], we developed a new concise representation scheme for a characteristic function, which is based on the idea of *agent types* [7]. Intuitively, a type represents a set of agents, which are recognized as having the same contribution. Most of the hardness results are obtained by assuming that all agents are different types. In practice, however, in many MAS application problems, while the number of agents grows, the number of different types of agents remains small. This type-based representation can be exponentially more concise than existing concise representation schemes. Furthermore, this idea can be used in conjunction with existing schemes, i.e., SCG [2] and MC-nets [4], for further reducing the representation size. We show that most of the problems in cooperative games, including CSG, can be solved in polynomial time in the number of agents, assuming the number of possible types t is fixed (Table 1).

In this direction, we are planning to examine the complexity of other solution concepts in cooperative games with or without coalition structures, such as the nucleolus, kernel, and core with coalition structures. Also, we would like to investigate our representation in conjunction with graph-based concise representation schemes including graphical representation [3] and our DCOP based representation [8]. Shrot et al. showed that utilizing agent types makes various problems in cooperative games tractable when a characteristic function is represented graphically by the formalization used in [3]. Thus, we guess that our scheme also would work well, by assuming that a graphical structure describes a relationship between agent types.

4. CONCLUSIONS

I am planning to keep working on these directions during my doctoral years. Through these researches, I would like to contribute to the development of game theory and multi-agent systems, in particular, cooperative game theory.

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