Opinion Convergence in Agent Networks (Extended Abstract)

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ABSTRACT

We empirically investigated the dynamics of opinion adaption on random networks, scale-free networks and regular lattice structures where agents adopt the opinion held by the majority of their direct neighbors only if the fraction of these exceed a certain laggard threshold [1]. We observed that either due to initial random distribution of opinion to agents or through opinion adaptation in the first few iterations, isolated pockets of agents with a different opinion than those of the surrounding population form and are sustained. Such population configurations thereafter converge to mixed or heterogeneous states. For certain values of the laggard threshold, we also observe a phase of uncertain convergence: for identical system parameters, the population will converge to homogeneous opinions whose value may be different for different random initializations. We identify the regions of consistent homogeneous convergence, heterogeneous convergence and uncertain homogeneous convergence for different values of the laggard threshold.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Management, Performance, Reliability

Keywords

Agent Networks, Opinion Formation

1. THE MODEL

Each agent i in our model represents a node in a network whose state represents its opinion on the topic of interest. We consider only binary opinions. Linked nodes are in contact with each other and know each other's opinions. The opinion formation process of node i, initially in state 0(1), is a three step process [2]:

- The state of all the neighbouring nodes to i are checked.
- If the fraction of state 1(0) nodes of i's neighbours exceeds a threshold p_u , i adopts opinion 1(0).
- Otherwise i remains in the same state 0(1).

We define *mixed-convergent graph* as one in which no agent changes its state from the previous time step but the system has not reached total consensus, i.e., all agents do not share the same opinion. Given a graph $G = (V, E)$ where agents represent nodes and edges represent neighborhood relationships, a convergent graph is reached when:

$$\forall_{v\in V} |\mathrm{Adj}_v \cap \mathrm{St}_{\neg v}| \leq P_u \cdot |\mathrm{Adj}_v|,$$

where Adj_v is the set of neighbors to v and St_v ($\mathrm{St}_{\neg v}$) is the set of nodes with the same (or opposite) state as v.

We define a *stationary configuration* to be a subset of the nodes such that they all have the same state and none of these nodes will ever change state irrespective of the state changes outside of this subset. The most stringent conditions for characterizing stationary configurations can be derived by assuming the worst case scenario of all nodes outside of the configuration adopting the opposite state. Thus a stationary configuration consists of a set of agents (nodes) $S \subset V$ such that

$$\forall_{v\in S} |\mathrm{Adj}_v \cap S| \geq (1 - P_u) \cdot |\mathrm{Adj}_v|.$$

To simplify our analysis for idenitfying stationary configurations, we consider only d-regular graphs[1] and choose $P_u = \frac{d-2}{d}$. We empirically evaluate 4-regular graphs and a special case of 4-regular graphs, the toroidal grid. The corresponding laggard threshold is $P_u = \frac{4-2}{4} = 0.5$. We assume that the likelihood of existence of the smallest cycles significantly outweighs the likelihood of existence of larger ones. For general 4-regular graphs the smallest cycles would be 3-cycles and for the toroidal grid, where there are no 3-cycles, it is 4-cycles or 2x2 squares.

We assume initial node opinions are randomly distributed. Even then, some groups of nodes may form stationary configurations at the outset, surrounded by nodes of opposite opinion. Or they may settle into a stationary configuration after one or few iterations and get stuck there forever.

We now calculate the probabilities of the occurrence of such stationary configurations. To simplify the computational complexity of computing the probabilities, we have assumed that the probability of each node to have neighbours in stationary states is independent of each other.

[1]A d-regular graph is graph where all nodes have degree d.

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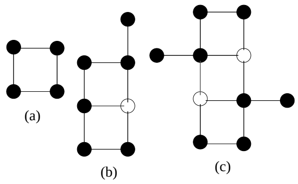

Figure 1: Some stationary configurations for toroidal grids: (a) basic stationary square configuration, (b) and (c) shows two configurations that can lead to the basic configuration in one iteration.

In Figure 1(a) we compute the probability of each of the nodes in the square to be a part of this stable configuration as the product of the probabilities of each of its neighbours to be in the stationary state (here, '1') ie, $(1-a_0)^2$, where a_0 is the initial percentage of agents of opinion 0 in the population. Hence the probability of having one such stationary square is $\mathcal{P} = ((1-a_0)^2)^4$. The maximum number of such stationary square configurations possible in a network of N nodes is $N/4$. Therefore the probability of having at least one such stationary square in the grid is $\mathcal{P}_1 = 1-(1-\mathcal{P})^{N/4}$. We can similarly calulate the probability of having a stationary configuration as in Figure 1(b) in the grid as $\mathcal{P} = a_0^3(1-a_0)^{15}$. So the probability of having at least one such configuration is again $\mathcal{P}_2 = 8(1-(1-\mathcal{P})^{N/4})$ considering 8 possible orientations of the stationary state in the grid. Similarly, for Figure 1 (c) probability of having at least one such configuration is $\mathcal{P}_3 = 4(1-(1-\mathcal{P})^{N/4})$ considering four different orientations of the configuration in the grid where $\mathcal{P} = a_0^6(1-a_0)^{17}$.Similarly we can consider cycles bigger than size 4 stuck in a particular opinion with nodes of opposite opinion filling up the whole interior of it. Hence the probabilty of having atleast one cycle of dimention sxs is $\mathcal{P}_{cycle_s} = 2(1-(1-\mathcal{P})^{N/s^2})$ where, $\mathcal{P} = (1-a_0)^2$. Now s can vary from 3 to M for a MXM grid at an increment of 2. Hence the overall probability of having such stationary configurations stuck in opinion '1' in the whole grid is $Prob('1') = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4 + \mathcal{P}_{line} + \mathcal{P}_{cycle}$ assuming independent cases. Similarly we can compute $Prob('0')$ for stationary configurations stuck in state '0'. Hence the percentage of runs where we have at least one such stationary state stuck at either state '0' or '1', i.e., where mixed convergence occurs is $Prob('0') \times Prob('1')$.

We performed similar analysis for random graphs, where we considered cycles of 3 nodes to be the simplest and most frequent stationary configuration. We do not include the corresponding expressions due to space constraints.

2. EMPIRICAL RESULTS

Because of our simplifying assumptions for computing the probabilities of stationary configurations, it is worthwhile to evaluate the accuracy of our analytical predictions using simulations. We have simulated opinion evolution in toroidal grids varying the total number of nodes from 100 to 900 , where the connectivity for each node is 4 and the laggard threshold $P_u = 0.5$.

We experimentally studied the regions of 1-convergence and 0-convergence in grid as well as in random networks for various values of a_0. For extreme values of a_0 consensus is always achieved, but for intermediate values, the network

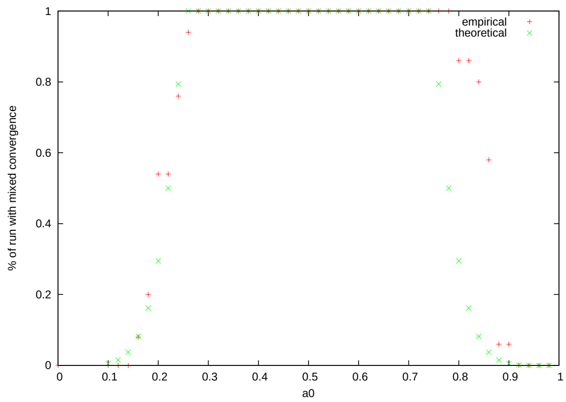

Figure 2: % runs with mixed convergence (toroidal grid: $N = 900$, $P_u = 0.5$, $\bar{k} = 4$).

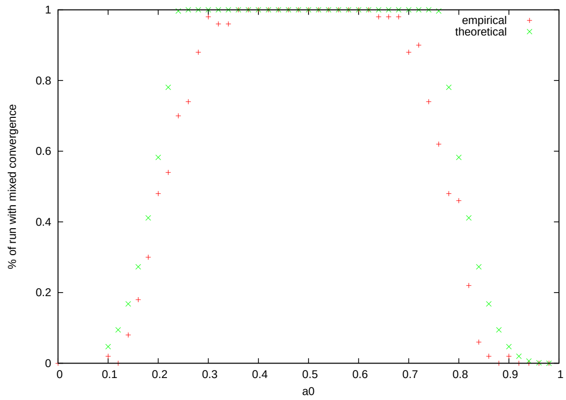

Figure 3: % runs with mixed convergence (random graph: N = 1000 , Pu = 0:5,ŕk = 4).

reaches mixed convergence. From both Figures 2 and 3 we observe that our analytical predictions closely match the empirical results despite the simplifying assumptions made to ease calculations.

An interesting observation from experimental data for toroidal grids was that as we increased the the number of nodes (N) in the grid for a given $\bar{k} = 4$, the region for mixed convergence became wider.

3. CONCLUSION

We studied the problem of opinion convergence in a society of agents situated in a fixed topological structure and identify stable subgraph configurations that will produce mixed convergence and calculated approximate probabilities for the same. Our analytical predictions approximate matched data from simulations for 4-regular graphs. We want to expand our model to cover a wider range of graphs.

4. REFERENCES

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