

# Extensive–form Games with Heterogeneous Populations

## (Extended Abstract)

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### ABSTRACT

The adoption of Nash equilibrium (NE) in real–world settings is often impractical due to its too restrictive assumptions. Game theory and artificial intelligence provide alternative solution concepts. When knowledge about opponents utilities and types is not available, the appropriate solution concept for extensive–form games is the *self–confirming equilibrium* (SCE), which relaxes NE allowing agents to have incorrect beliefs off the equilibrium path. In this paper, we extend SCEs to capture situations in which a two–agent extensive–form game is played by heterogeneous populations of individuals that repeatedly match (e.g., eBay).

### Categories and Subject Descriptors

I.2.11 [Artificial Intelligence]: Multi–agent systems

### General Terms

Algorithms, Economics

### Keywords

Game Theory (cooperative and non-cooperative)

## 1. INTRODUCTION

The study of strategic interactions has recently deserved a lot of attention in artificial intelligence. The design of rational agents is usually pursued by exploiting models from *microeconomics* [6] and by *algorithmic* tools to find strategies [6]. The central solution concept is *Nash equilibrium* (NE). Although in principle NE can be applied to an enormous range of situations to *prescribe* strategies to agents, it presents two drawbacks that make its prescriptive power non–satisfactory. The first drawback concerns its epistemic requirements (e.g., *common* and *complete* information over payoffs) that are rarely met in real–world situations. The second drawback is the multiplicity of equilibria: with multiple equilibria, agents cannot coordinate on which equilibrium to play unless they are somehow correlated, but in this case a *correlated equilibrium* should be played. Even when NE is used as *descriptive* tool to study what are stable states of learning agents, some problems arise, learning

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agents having non–NE stable states, and therefore NE may have a non–satisfactory descriptive power [3].

The aforementioned drawbacks of NE pushed researchers to design alternative solution concepts taking also into account learning dynamics, e.g., the CURB set [1] is a non–equilibrium concept defined as a set of strategies that contains the best responses to any mixture over itself and any best–response dynamics will stay within it [5]. With extensive–form games, relaxing the epistemic requirements of NE, it is possible to have stable states that are not NEs [3]. These equilibria, called *self–confirming equilibria* (SCEs), require that each agent plays best response strategies to her beliefs, but the beliefs can be *wrong* off the equilibrium path (while they are *confirmed on*). This concept is perfectly suitable for learning agents: if agents can entirely explore the strategy profile space, they would have correct beliefs everywhere on the game tree and they would play a *subgame perfect equilibrium* (SPE) or a *sequential equilibrium* (SE), but, in practice, learning agents cannot explore the whole space and therefore they can have wrong beliefs over some portion of the tree and the strategy profile they play is an SCE [4].

## 2. EXTENSIVE–FORM GAMES WITH POPULATIONS

We focus on two–agent extensive–form games that are repeatedly played by different individuals as described in [3]. More precisely, for each agent (representing a *role*), there is a population of *individuals* and, at each repetition of the game, one individual is drawn from each population and the two drawn individuals are matched and then play the game. At each repetition, different individuals may play. A common example is a market in which bilateral negotiations are carried out: there are two agents/roles (i.e., buyer and seller), but different buyers and different sellers can match. Other economic examples are given by auctions.

The game model proposed in [3] presents two main limitations that we remove in this paper: infinite populations and identical individuals. In our model, we allow populations to be finite or infinite and to be heterogeneous including individuals of different types, where types differentiate for their preferences. We denote by  $\Theta = (\Theta_1, \dots, \Theta_{|N|})$  the set of all the types ( $\theta$  denotes a generic type), where  $\Theta_i$  is the set of types of agent  $i$ . We assume that the number of types is finite. We denote by  $\Theta_{-i} = \times_{j \neq i} \Theta_j$  the set of all the possible profiles of types of all the agents except agent  $i$ . Utility functions are defined also over types (in addition to the strategies of the agents). When types are *non–interdependent*, we have that utility function of agent  $i$  is defined as  $u_i = u_i(\theta, \sigma_i, \sigma_{-i})$  where  $\theta \in \Theta_i$  and  $\sigma_j$  is the

strategy of agent  $j$ ; instead when types are *interdependent* we have  $u_i = u_i(\theta, \theta', \sigma_i, \sigma_{-i})$  where  $\theta \in \Theta_i$  and  $\theta' \in \Theta_{-i}$ . For each type  $\theta \in \Theta_i$ , there is a (possibly infinite) population of individuals  $\Lambda_\theta$  (we denote by  $\lambda$  a generic individual and by  $\Lambda_i$  the set of all the individuals of agent  $i$ ). Each type  $\theta \in \Theta_i$  is associated with a probability  $\omega_{i,\theta}$  with which an individual of the pertinent population is drawn. Obviously,  $\sum_{\theta \in \Theta_i} \omega_{i,\theta} = 1$ . Similarly, each individual  $\lambda \in \Lambda_\theta$  is associated with a probability  $\omega_{i,\theta,\lambda}$  with which the individual is drawn such that  $\sum_{\lambda \in \Lambda_\theta} \omega_{i,\theta,\lambda} = \omega_{i,\theta}$ . For the sake of presentation, we assume that each individual has a different index  $\lambda$  and therefore we can refer to an individual  $\lambda \in \Lambda_\theta$  by using  $\lambda$  in place of  $\langle i, \theta, \lambda \rangle$ . Similarly, we assume each type has a different  $\theta$ , therefore we can refer to a type  $\theta \in \Theta_i$  by using  $\theta$  in place of  $\langle i, \theta \rangle$ .

Different individuals may adopt different strategies.  $\sigma_\lambda$  denotes the strategy of individual  $\lambda$ . The *aggregate strategy* of the individuals of type  $\theta \in \Theta_i$  is  $\sigma_\theta = \sum_{\lambda \in \Lambda_\theta} \sigma_\lambda \cdot \omega_\lambda$ , and the one of agent  $i$  is  $\sigma_i = \sum_{\theta \in \Theta_i} \sigma_\theta \cdot \omega_\theta$ .

As in [3], we assume that agents have no information about the opponents. Specifically, we assume:

- each individual has no information about the utility functions of the other agents;
- each individual has no information about the individuals of the other agents;
- when utilities are interdependent, each individual knows the types of the opponents, but she does not know the pertinent probabilities and utilities;
- when utilities are not interdependent, no assumption about the knowledge of the opponents' types is made. This is because it can be proved, with a simple extension of [2], that knowing or not the opponents' types (without knowing the utilities and probabilities) leads to the same set of equilibria.

Customarily, a game with types is said Bayesian. Each individual forms a belief over the opponents and adjusts it during the play. More precisely, when utilities are not interdependent, each individual  $\lambda$  of agent  $i$  has a (potentially different) belief  $\hat{\sigma}_\lambda^j$  over the aggregate strategy of agent  $j$  for every  $j \neq i$ . This is because each individual  $\lambda$  does not know the types and the individuals of the opponents and therefore she cannot form any belief over the single individual or type of the opponents. When utilities are interdependent, each individual  $\lambda$  of agent  $i$  has a (potentially different) belief  $\hat{\sigma}_\lambda^\theta$  over the aggregate strategy of type  $\theta \in \Theta_j$  for every  $j \neq i$  and a (potentially different) belief  $\hat{\omega}_\lambda^\theta$  over the pertinent probability  $\omega_\theta$  (also in this case individuals have not beliefs over the single individuals of the opponents).

### 3. SOLUTION CONCEPTS

Different solution concepts extending the SCE can be provided according to each specific situation, see Tab. 1.

		individuals per type		
		1	$n$	$\infty$
types per agent	1	USCE	FHSCE	IHSCE
	$n$	BUSCE	BFHSCE	BIHSCE

**Table 1: Extensions of the SCE concept.**

The basic solution concept, introduced in [3], is the *unitary* SCE (USCE) that captures situations with a unique type per agent and a unique individual per type. A USCE constrains the agent's strategy to be best response to the

belief over opponent's strategy and constrains beliefs to be correct (w.r.t. the strategies) on the equilibrium path.

DEFINITION 3.1. A *USCE* is a pair  $(\sigma, \hat{\sigma})$  such that:

- each agent  $i$  has single type  $\theta$  and single individual  $\lambda$ ;
- for every agent  $i$ ,  $\sigma_\lambda$  is a best response to  $\hat{\sigma}_\lambda^{-i}$ , where  $\lambda$  is the agent  $i$ 's individual;
- for every agent  $i$ ,  $\hat{\sigma}_\lambda^{-i}$  is equal to  $\sigma_{\lambda'}$  on the equilibrium path, where  $\lambda$  and  $\lambda'$  are the individuals of agent  $i$  and agent  $-i$ , respectively.

Upon the USCE concept, we build the solution concepts for other (more complex) situations. We consider the situation with one type per agent and multiple finite individuals: the solution concept is *finite heterogeneous* SCE (FHSCE).

DEFINITION 3.2. An *FHSCE* is a pair  $(\sigma, \hat{\sigma})$  such that:

- each agent  $i$  has a single type  $\theta$  and a finite number of individuals  $\lambda$ ;
- for every agent  $i$ ,  $\sigma_\lambda$  is a best response to  $\hat{\sigma}_\lambda^{-i}$ , where  $\lambda$  is an agent  $i$ 's individual;
- for every agent  $i$ ,  $\hat{\sigma}_\lambda^{-i}$  is equal to  $\sigma_{-i}$  on the equilibrium path identified by  $\sigma_\lambda$  and  $\sigma_{-i}$ , where  $\lambda$  is an individual of agent  $i$ ;
- for every agent  $i$ ,  $\sigma_i$  is the aggregate strategy of all the agent  $i$ 's individuals.

Notice that the constraints over the beliefs of an individual  $\lambda$  and the equilibrium path she observes depend on  $\sigma_\lambda$ . Thus, different individuals may have different strategies, each supported by different beliefs. The above solution concepts extend to the case of infinite individuals (IHSCE), and to the case of Bayesian information (BUSCE, BFHSCE, BIHSCE).

### 4. FUTURE WORKS

In future, we will study three main problems associated with SCEs:

- *Solution concept characterization*: we will characterize the relationships between the different solutions concepts.
- *Equilibrium computation*: we will study the problem of computing an equilibrium given the agents' utilities and of enumerating all the equilibria.
- *Learning dynamics*: we will study the dynamics of learning agents, analyzing how replicator dynamics with mutation (simulating Q-learning) is affected (in terms of attraction of repulsion) by the presence of SCEs.

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