Maximizing Matching in Double-sided Auctions

(Extended Abstract)

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ABSTRACT

Traditionally in double auctions, offers are cleared at the equilibrium price. In this paper, we introduce a novel, non-recursive, matching algorithm for double auctions, which aims to maximize the amount of commodities to be traded. Our algorithm has lower time and space complexities than existing algorithms.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, design, experimentation, measurement, performance

Keywords

Matching algorithm, double auctions, auction mechanism design

1. INTRODUCTION

Double auctions [2] are widely used in financial markets, and automated control [1], as well as in solving, for example, environmental problems [4] for their desirable properties, e.g., high *allocative efficiency* in particular. But there are other metrics that may carry more weight in certain scenarios, and *trading volume* is one such metric. It measures the total amount of commodity transferred from sellers to buyers in an auction, and in marketplaces where traders are charged a flat fee for each transaction it is clearly advantageous for the operators of those marketplaces to maximize trading volume. (Although this may be at odds with the best interests of the traders, who benefit the most if allocative efficiency is maximized.)

Classic double auctions like the continuous double auction (CDA) or the clearing houes (CH) clear at the equilibrium price based on the reported demand and supply curves, a matching algorithm called *equilibrium matching*, or simply ME. With ME, all matched asks are priced no higher than the matched bids, and given this constraint ME actually realizes the maximal trading volume that is possible. Without this constraint on prices, there is the potential to further increase the trading volume. The idea is simply not to match between intra-marginal asks and intra-marginal bids as ME does, but to match extra-marginal asks with intra-marginal bids that are priced no lower than the asks, and match extra-marginal

bids with intra-marginal asks that are priced no higher than the bids. Given any particular demand and supply schedules, this can potentially double the trading volume realized by ME. We use the term *maximal-volume matching* to describe algorithms that maximize trading volume.

This idea is not new. Indeed, we are aware of two pieces of prior work [4, 5] on this topic. However neither of the algorithms proposed before are both computationally efficient and achieve the highest allocative efficiency possible while realizing the maximal trading volume. Rich *et al.* [4] illustrated a scheme to do the matching, but depending upon the shape of the supply and demand curves in the market, the set of matching shouts that their algorithm produces may exclude more competitive shouts and include less competitive ones from the same side, failing in the sense of *fairness* and leading to lower allocative efficiency. Zhao *et al.* [5], on the other hand, presented an algorithm that correctly produces the set of matching shouts, but their algorithm is both computationally inefficient and obscure. Our main contribution in this work is to introduce a computationally efficient, maximal-volume matching algorithm, which we call MV. The MV algorithm was actually made public in 2007, released in the JCAT software package [3] (the server platform for CAT tournaments on market design), about three years before the publication of [5], the authors of which were among those who designed the jackaroo entry in the CAT tournaments.

Due to the space constraint, we omit many details in this extended abstract. A full-length version of this paper is available at http://web.cs.gc.cuny.edu/~jniu/research/publications/.

2. MAXIMAL-VOLUME MATCHING

The MV algorithm consists of two steps. The first step calculates the maximal trading volume between the demand and supply schedules, and the second step determines the bid-ask pairs to form the matching set. The idea of the first step can be intuitively illustrated. Suppose the demand and supply schedules are those in bold as shown in Figure 1. Then the supply schedule is flipped horizontally before being shifted to the right towards the demand schedule until the two 'touch'. The distance that the supply schedule moves is the minimal horizontal distance between the flipped supply curve and the demand curve, the very trading volume that the MV algorithm computes as the maximal volume. As the distance between the demand curve and the supply curve at price p, shown along the horizontal dashed line in Figure 1, is exactly the sum of the demand and the supply at p, the minimal horizontal distance or the maximal trading volume can be presented as

$$Q_{mv} = \min_{p}\bigl(S(p) + D(p)\bigr) \qquad (1)$$

where $S(p)$ and $D(p)$ are respectively the supply and demand at price p. To prove the soundness of this approach, we have the
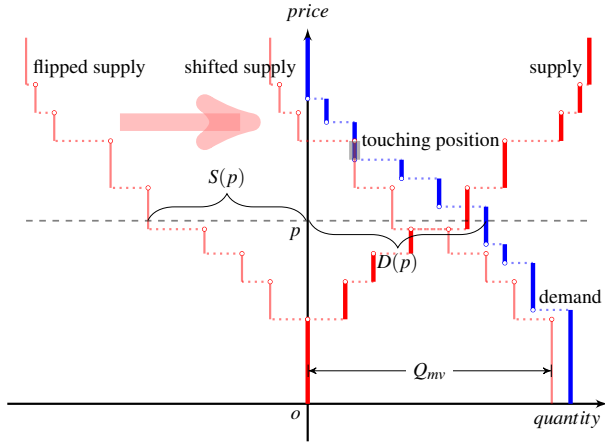
Figure 1: Supply schedule flipped horizontally and then shifted right towards demand schedule. The distance that the supply curve can move before touching the demand curve is exactly the maximal trading volume, Q_{mv}.

Algorithm 1: The MV-GETQ function calculating Q_{mv}.

Input: B (A) — queue of bids (asks) in the order of ascending price
Output: q_{min}

```
1  begin
2  |   q_min ← 0
3  |   a ← poll(A)
4  |   if a is not NULL then
5  |   |   b ← poll(B)
6  |   |   while b is not NULL and p(b) < p(a) do
7  |   |   |   b ← poll(B)
   |   |
   |   |   // q_d:  will be the demand at price 0.
8  |   |   q_d ← 0          q ← 0
9  |   |   while b is not NULL do
10 |   |   |   if a is not NULL and p(a) ≤ p(b) then
11 |   |   |   |   q ← q + q(a)        a ← poll(A)
12 |   |   |   else
13 |   |   |   |   q ← q − q(b)        q_min ← min(q_min, q)
14 |   |   |   |   q_d ← q_d + q(b)    b ← poll(B)
   |   |
15 |   |   q_min ← q_min + q_d
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following lemma:

LEMMA 1. *Given a set of matching bid-ask pairs, M, the trading volume achieved by M, $\|M\|$, is no higher than the sum of the demand and the supply of the market at any given price, i.e.,*

$$\|M\| \leq S(p) + D(p)$$

for any price $p \geq 0$.

Due to the space constraint, we omit the proof here. This lemma basically gives an upper-bound of trading volume for given demand and supply schedules. It is not difficult to see that the procedure we describe above and illustrate in Figure 1 obtains exactly that upper-bound, hence maximizing trading volume. We show the formal procedure that calculates Q_{mv} in Algorithms 1, the main part of the MV algorithm. This procedure processes the input, two queues of bids and asks in the order of ascending price, in a one-pass scan. With the sorted shouts and Q_{mv} available, pairing up matchable

shouts requires just another pass. So the total time complexity of the MV algorithm is no more than the complexity required to sort shouts, $O(\max(n,m) \cdot \log \max(n,m))$, where n and m are respectively the number of bids and asks in the market. In addition, the MV algorithm consumes no more space than that needed for storing the sorted bids and asks, so its space complexity is $O(n+m)$. Both complexities are better than those of [5]'s approach to generating the same set of matching shouts.

Note that the MV algorithm, identical to [5], is based on a snapshot of the demand and supply schedules in a market. We carried out experiments using JCAT to examine whether the MV algorithm helps to increase trading volume even when the demand and supply schedules dynamically change over time as in a real market. In each experiment, the market is populated by a certain number of homogeneous traders, i.e., traders using the same trading strategy. Strategies we considered are common in the literature, including truth-telling, PS, which bids at a fixed profit margin, ZI-C, which bids randomly with budget constraint, and GD, which bids in a way to optimize the expected profit based on information about shouts and transactions in the market. Traders are allowed to place new shouts or adjust their existing offers multiple times during an auction. We consider the MV market, a CH mechanism but with the stock ME algorithm replaced with MV, and evaluate it in comparison with the classic CH and CDA markets that are configured otherwise the same way. Experimental results show that the MV market generates higher trading volumes than the CH and the CDA across all the trader types that we tested. So, it can be argued that the MV algorithm, despite being designed by considering unchanging supply and demand curves, holds up when the supply and demand curves change as a result of trades and price adjustments made over time. Furthermore, for certain traders — the zero intelligence traders ZI-C and the sophisticated GD traders — the MV market can achieve the same high levels of allocative efficiency that are obtainable in the CDA and the CH. Together, these two results suggest that the MV algorithm has the potential to be adopted in real-world markets. This is because it balances the objectives of the main parties in those markets, providing high efficiency, which is what traders want, while pushing up volume and thus allowing those who operate the markets to increase their profits.

3. REFERENCES

[1] S. H. Clearwater, editor. *Market-Based Control: A Paradigm for Distributed Resource Allocation*. World Scientific Publishing Co., Inc., River Edge, NJ, USA, 1996.

[2] D. Friedman. The double auction institution: A survey. In D. Friedman and J. Rust, editors, *The Double Auction Market: Institutions, Theories and Evidence*, pages 3–25. Westview Press, Perseus Books Group, Cambridge, MA, 1993.

[3] J. Niu, K. Cai, S. Parsons, E. Gerding, P. McBurney, T. Moyaux, S. Phelps, and D. Shield. JCAT: A platform for the TAC Market Design Competition. In Padgham, Parkes, Müller, and Parsons, editors, *Seventh International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 1649–1650, Estoril, Portugal, May 2008. Demo Paper.

[4] C. S. Rich and D. Friedman. The matching market institution: A laboratory investigation. *American Economic Review*, 88(5):1311–1322, December 1998.

[5] D. Zhao, D. Zhang, M. Khan, and L. Perrussel. Maximal matching for double auction. In J. Li, editor, *AI 2010: Advances in Artificial Intelligence*, pages 516–525. Springer Berlin / Heidelberg, 2011.