Security Games with Interval Uncertainty

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ABSTRACT

Security games provide a framework for allocating limited security resources in adversarial domains, and are currently used in deployed systems for LAX, the Federal Air Marshals, and the U.S. Coast Guard. One of the major challenges in security games is finding solutions that are robust to uncertainty about the game model. Bayesian game models have been used to model uncertainty, but algorithms for these games do not scale well enough for many applications. We take an alternative approach based on using intervals to model uncertainty in security games. We present a fast polynomial time algorithm for security games with interval uncertainty, which represents the first viable approach for computing robust solutions to very large security games. We also introduce a methodology for using intervals to approximate solutions to infinite Bayesian games with distributional uncertainty. Our experiments show that intervals can be an effective approach for these more general Bayesian games; our algorithm is faster and results in higher quality solutions than previous methods.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: [Multiagent systems]

Keywords

game theory; algorithms; interval uncertainty; security games

1. INTRODUCTION

Security games [11, 22] are a general framework for modeling a wide variety of resource allocation decisions in adversarial security domains. These games are used to find optimal randomized strategies for a defender to deploy limited security resources to protect vulnerable targets from attacks. Recently, they have been used in a growing number of homeland security applications, including airport security [19, 20], scheduling for the Federal Air Marshals [23], and patrolling strategies for the United States Coast Guard [21]. Game theory is also used for applications in cybersecurity [3, 16].

An important concern with using game theory to model real-world security problems is that the models require very precise and accurate information about the capabilities and preferences of the players. In practice, models are constructed using information provided by subject matter experts knowledgeable about the resources available for protection, the security risks and vulnerabilities of different targets, and the motivations of possible attackers. Unfortunately, there is often a high degree of uncertainty associated with the information used to construct the models. For example, it is difficult to know exactly what value a terrorist might perceive for a successful attack against a given target, even through it may be clear that some targets are more attractive to attackers than others. This means that it may not be possible to give exact values for the payoffs in different attack scenarios in a security game.

There is a growing emphasis on developing models and algorithms for security games that are able to represent various kinds of uncertainty about the model, with the goal of generating robust security strategies that are not highly sensitive to modeling errors. The existing approaches are primarily based on Bayesian Stackelberg games that model uncertainty about payoffs, the observation capabilities of an attacker, and other factors [17, 18, 10, 12, 28]. All of these approaches suffer from problems with computational scalability and/or solution quality. Finite Bayesian Stackelberg games are NP hard to solve [6], and experimental results show that they are hard in practice as well. For infinite Bayesian Stackelberg games no exact algorithm exist, and none of the existing methods give bounds on solution quality [12].

The approach we take here is based on modeling uncertainty using intervals, rather than distributions. We take a worst-case optimization approach with respect to the interval uncertainty. In our model, the defender in a security game knows only that the attacker's payoffs are in some interval of possible values, and tries to maximize the worst case outcome for any possible realization of payoffs consistent with these intervals. Modeling uncertainty using intervals is common in robust optimization [5], and this idea has also been used to develop a notion of equilibrium in game theory based on robust optimization [1]. The most closely related model for security games is BRASS, which was introduced by Pita et al. for robustness against human decision-makers [18]. BRASS is a special case of our model.

The interval-based approach has advantages over Bayesian methods for modeling uncertainty. It is simpler for domain experts to understand and specify a model based on interval uncertainty because the model does not require eliciting detailed information about probability distributions. In many cases, an interval model is a more natural and effective way to represent the game. We show in this paper that interval models also have considerable computational advantages over Bayesian models. While Bayesian models are NP-hard, we introduce a polynomial-time approximation algorithm for interval representations that provides tight bounds on solution quality. For large security games, our algorithm may be the only computationally feasible approach for handling uncertainty.

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The following are the primary contributions of this paper: (1) we introduce an interval-based model of uncertainty for security games, (2) we present a very fast polynomial algorithm for solving interval security games, (3) we present a methodology for approximating security game with distributional uncertainty using intervals, which can be solved using our algorithm, and (4) we present experimental results showing the value of the interval method for increasing robustness, and showing that interval-based methods can also provide fast approximations with high solution quality even when distributional information is available.

2. RELATED WORK

One of the motivations for out work is applications of Stackelberg games to real-world security domains [22]. They have been used in fielded applications at the Los Angeles International Airport [19], the Federal Air Marshals Service [23], and the Coast Guard [21]. There is also work on using game theory for patrolling strategies for robots and unmanned vehicles [7, 2, 4] and applications of game theory in network security [3, 26, 16]. Much of this work is computational in nature, and progress on security games has been driven by algorithmic advances that can solve larger and more complex games [6, 17, 9].

Robustness and uncertainty have been recognized as important concerns in game theory by many authors. One of the most influential models, Bayesian games, was developed early in the history of game theory by John Harsanyi [8]. Worst-case approaches also have a history in game theory, including many approaches that build on the pessimistic notion of minmax strategies. However, many of these take a worst-case view with respect to opponent behaviors, rather than the specification of the underlying game model. One exception is robust equilibrium [1], which takes a worst-case approach inspired by the robust optimization literature. There are also several works that look at varying aspects of the problem of robustness in security games, most of which adopt a Bayesian framework for reasoning about uncertainty [17, 18, 10, 12, 28]. Finally, fuzzy set theory is related to our approach, and the literature has explored a variety of methods for decision-making based on intervals that could be explored as extensions of the model presented here in future work [27, 15].

3. SECURITY GAMES WITH INTERVALS

We first introduce the security game model [11], and then extend the model to include interval uncertainty about the attacker's payoffs. A security game has two players, a *defender*, Θ, and an *attacker*, Ψ. The defender is protects a set of *targets* $T = \{t_1, \dots, t_n\}$ (e.g., airport terminals or computer servers) against attacks using a limited number of resources, with the number of available resources denoted by m. We assume that all resources are identical and can be used to protect any target. The set of pure strategies for the attacker, denoted $\sigma_\Psi \in \Sigma_\Psi$, correspond to actions attacking exactly one target from T. Each pure strategy for the defender, denoted $\sigma_\Theta \in \Sigma_\Theta$, corresponds to a subset of targets from T with size less than or equal to m which the defender chooses to protect.

Following previous work on security games, we model the interaction as a Stackelberg game [25]. The defender first commits to a mixed strategy δ_Θ that is a probability distribution over the pure strategies from Σ_Θ. The attacker then observes this mixed strategy δ_Θ, and chooses a best response strategy from Σ_Ψ that gives the attacker the maximum possible payoff. As in the previous work on security games [22], we use Strong Stackelberg Equilbrium as the standard solution concept, so ties in the attacker's best responses are broken in favor of the defender.[1] In addition, we only need to consider pure strategies for the attacker, since there always exists a pure-strategy best response [17].

The payoffs for the game depend on which of the n targets is attacked, and whether or not the target is protected (covered) by the defender or not. Specifically, for an attack on target t, the defender receives a payoff $U^u_\Theta(t)$ if the target is uncovered, and $U^c_\Theta(t)$ if the target is covered. The payoffs for an attacker of type $\omega \in \Omega$ is $U^u_\Psi(t,\omega)$ for an attack on an uncovered target, and $U^c_\Psi(t,\omega)$ for an attack on a covered target. We say that an attack on a covered target is "unsuccessful" and an attack on an uncovered target is "successful." In a security game we also assume that $U^c_\Theta(t) \geq U^u_\Theta(t)$ and $U^u_\Psi(t) \geq U^c_\Psi(t)$ for all $t \in T$. In games with identical and unconstrained defender resources, we can use a compact representation for the defenders strategies [11]. We represent the defender strategies as *coverage vectors* which give the probability that there is a defender resource assigned to each target. These probabilities for each target t_i are denoted by c_i, with $\sum_{i=1}^{n} c_i = m$, and the full vector of probabilities is denoted by C. Once the coverage probabilities are determined, the full joint security policy can be extracted using a sampling algorithm similar to the comb sampling procedure described in Tsai et al. [24]

Extension to Interval Uncertainty. We now introduce the *Interval Security Games* (ISG) model, which extends the standard security game model so that the defender has uncertainty about the attacker's payoffs that is represented using intervals. We still assume that both the attacker and defender know their *own* payoffs with certainty. In addition, we do not need to model the attacker as having uncertainty about the defender's payoffs because the attacker is able to directly observer the strategy of the defender, and therefore does not require knowledge of the payoffs to predict the defender's strategy.

In the model, rather than having a single value representing the attacker's payoffs for the two cases of a successful and unsuccessful attack ($U^u_\Psi(t)$ and $U^c_\Psi(t)$), we have pairs of values that represent the maximum and minimum possible payoffs for a successful or unsuccessful attack on target t_i. We denote these values using the notation $U^{u,max}_\Psi(t)$ and $U^{u,min}_\Psi(t)$ for the uncovered case, and $U^{c,max}_\Psi(t)$ and $U^{c,min}_\Psi(t)$ for the covered case. The idea is that the defender knows only that the attacker's payoffs lie within a range of possible values, and not the precise value. The defender does not have information about the distribution of payoffs within these intervals, and therefore cannot compute an expected payoff. Therefore, we cannot apply the concept of Strong Stackelberg Equilbrium. Instead, we follow the literature on robust optimization and take a worst-case approach. The defender's goal in our framework is to select a coverage vector, C, that maximizes the defenders worst-case payoff over all of the possible ways that the attacker payoffs could be chosen from the defined intervals.

4. ANALYSIS OF ISG

In security games without intervals, we define the *attack set* to be the set of all targets that give the attacker the maximum expected payoff, given some coverage strategy C. For some classes of security games, finding the optimal coverage strategy can be reduced to finding a coverage strategy that induces the maximum attack set, while minimizing the attacker's expected payoff [11]. In our model

[1]The tie-breaking rule is not intuitive in adversarial games. While there are arguments to support it they rely on precise maximization by the attacker. We view this as one more reason to develop robust solutions like the ones described in this paper that do not rely heavily on tie-breaking rules.

we cannot directly apply the idea of the attack set, but we can generalize it as follows. We define the *potential attack set* for a coverage strategy C to be the set of all targets that *could* give the attacker the maximum expected value, for any realization of attacker payoffs consistent with the payoff intervals. For every target, the attacker has a range of expected payoffs:

$$v^{max}(t_i) = c_i \cdot U_\Psi^{c,max}(t_i) + (1 - c_i) \cdot U_\Psi^{u,max}(t_i) \tag{1}$$

$$v^{min}(t_i) = c_i \cdot U_\Psi^{c,min}(t_i) + (1 - c_i) \cdot U_\Psi^{u,min}(t_i). \tag{2}$$

Observe that for a given coverage vector C the attacker can guarantee a payoff of at least the maximum of the minimum values over all targets; let us denote this value by $R = \max_{t_i} v^{min}(t_i)$. Given the value of R we can identify the targets that could be attacked for some realization of the payoff values. Any target t_i with a maximum expected value $v^{max}(t_i) \geq R$ could be the best target for the attacker to attack. To see this, suppose that the the attacker's payoff for t_i is the maximum value in the interval, and the payoffs for all other targets is are the minimal values, so that the best possible value for attacking any target other than t_i is R. Therefore, the potential attack set, $\Lambda(C)$, is defined as:

$$\Lambda(C) = \{t_i : v^{max}(t_i) \geq R\} \tag{3}$$

The defender's expected payoff for each target is:

$$d_i = c_i \cdot U_\Theta^c(t_i) + (1 - c_i) \cdot U_\Theta^u(t_i). \tag{4}$$

The defender's objective is to select a strategy C to maximize the worst-case payoff over the targets in the potential attack set:

$$\max_C (\min_{t_i \in \Lambda(C)} d_i) \tag{5}$$

Note that this objective function specifies as worst-case approach to uncertainty about the attacker's payoffs, but still takes the expected payoffs with respect to the realizations of the coverage probabilities. This problem cannot be solved using linear programming because the set of targets $t_i \in \Lambda(C)$ depends on C. It can be expressed as a mixed-integer program (MIP) which is a slightly generalized version of the MIP used for BRASS [18]. We omit this MIP due to space constraints, but it can be found in [18].

The main idea of our approach is to transform the optimization problem specified in the equations above into a series of feasibility problems. Our first observation is that the defender's maximum possible expected payoff increases monotonically as the number of available resources m is increased. This follows as consequence of the fact that the defender's set of possible coverage strategies is strictly larger for larger m. Using this observation, we can frame the problem as a binary search in the space of defender expected payoffs. In each iteration, we test whether the defender payoff at the midpoint is feasible or not given the number of resources available. If it is not, the maximum payoff must be smaller than the midpoint. If it is feasible, the maximum payoff is greater than or equal to the midpoint. Using this strategy, we can approximate the maximum payoff to within a very small tolerance.

To implement this approach, we need to analyze the problem to determine whether a given defender payoff (denoted by D^*) is feasible given the resources available, m. Since we are interested in worst-case outcomes, this means that we need to guarantee that the defender will achieve *at least* D^* for any attacker payoffs in the known intervals. For D^* to be guaranteed by a coverage strategy C, one of the following two conditions must hold for every target:

1. The target is in the potential attack set $\Lambda(C)$, but the defender's expected payoff if the target is attacked is at least D^*.
2. The target is not in the potential attack set $\Lambda(C)$.

We now derive conditions that satisfy these conditions for each target using the minimum amount of resources (i.e., coverage probability). We can calculate the coverage required to satisfy condition 1 for each target (if it is in Λ) from the equation for the defender's payoff. The minimal coverage for target t_i is given by:

$$c_i^1 = \max(0, 1 - \frac{D^* - U_\Theta^u(t_i)}{U_\Theta^c(t_i) - U_\Theta^u(t_i)}). \tag{6}$$

The problem now reduces to finding the potential attack set that minimizes the overall coverage probability required to meet conditions 1 and 2 for all targets. A naïve approach would be to enumerate all of the possible attack sets and calculate the minimum coverage for each such set. For any given set, we can calculate the value of R directly, and then calculate the minimal coverage required for each target in Λ from Equation 6. For a given value of R we can also calculate the minimum coverage that would be required on each target t_i so that the target is *not* in Λ, which requires that the maximum possible expected payoff for the attacking t_i is less than R. We calculate the minimum coverage as follows, using the maximum attacker payoffs from the possible intervals:

$$c_i^2 = \max(0, 1 - \frac{R - U_\Psi^{u,max}(t_i)}{U_\Psi^{c,max}(t_i) - U_\Psi^{u,max}(t_i)}). \tag{7}$$

By summing the values of c_i^1 for targets in Λ and c_i^2 for the remaining targets, we get the minimum coverage required to guarantee D^* for this potential attack set. Unfortunately, the number of such sets is exponential in the number of targets, so enumerating them is inefficient. To avoid this problem we make another observation that allows us to efficiently explore the candidate solutions. For every set Λ there is a target, which we label $\hat{t}$, that has the maximum minimum expected payoff, R. This is the target that defines the value of R. Since there are only n targets, we can test each target as a candidate for $\hat{t}$ and construct a coverage vector that meets the necessary constraints using minimal resources. We present the details of this construction in the next section. If the solution is feasible for any one of the n targets that are candidates for $\hat{t}$, then the value of D^* is feasible. In the following section we describe an algorithm that uses this solution strategy to efficiently approximate the optimal coverage vector C for the defender.

5. ISG ALGORITHM

We now describe our algorithm for solving an ISG. The pseudocode is given in Algorithms 1 and 2. Algorithm 1 implements binary search in the space of possible defender payoffs. The feasibility check is presented in Algorithm 2. The goal is to construct a solution that will guarantee the defender D^* while using the minimum resources; if we can construct a solution that uses less than the available resources m we have found a feasible solution. The strategy is to divide the search into n possible cases, each of which corresponds to a different assumption about which target will have the maximum guaranteed payoff for the attacker, R. The algorithm iterates through each choice of t_i as a candidate for this special target $\hat{t}$. For each case the algorithm constructs a coverage vector using minimal coverage probability that guarantees the defender D^* based on the conditions 1 and 2 above.

First, for $\hat{t}$ the target is part of the potential attack set in this solution based on our assumption that it will have the maximum minimum expected payoff. Therefore, if this target is attacked it must give the defender an expected payoff of at least D^*, as calculated in Equation 6. We take the value $c_{\hat{t}}^1$ necessary to ensure D^* and use this to calculate the value of R. This value of R is as high as possible because we use the minimum coverage. We do not need to consider adding additional coverage to $\hat{t}$ to decrease the value of R because this could only increase the coverage needed for any other target. To see this, note that the values of c_i^1 are independent of R and the values of c_i^2 increase monotonically as R decreases.

Now that we have the value of R, we calculate the value of c_i^2 for every other target. We calculate the minimum coverage required to satisfy one of these two conditions by taking $\min(c_i^1, c_i^2)$ for each target. There is one final condition that must be met for each target for our initial assumption to hold: the target we are assuming is $\hat{t}$ must have the maximum minimum expected payoff. We enforce this by adding an additional constraint on the coverage probability assigned to each target so that the maximum attacker payoff for the target is less than the calculated value of R for $\hat{t}$:

$$c_i^3 = \max(0, 1 - \frac{R - U_\Psi^{u,min}(t_i)}{U_\Psi^{c,min}(t_i) - U_\Psi^{u,min}(t_i)}). \tag{8}$$

These values increase monotonically as R decreases. The overall minimum coverage for each target is given by $\max(c_i^3, \min(c_i^1, c_i^2))$. We sum these coverages over all targets and compare this with the available resources m to determine whether this selection of $\hat{t}$ yields a feasible solution. If this case does not yield a feasible solution, the algorithm continues testing the other targets as candidates for $\hat{t}$. As soon as a feasible solution is found, the subroutine terminates and the binary search continues.

The worst-case complexity of the algorithm is $O(n^2 \cdot \log(1/\epsilon))$ where ϵ is the error tolerance parameter for the binary search. Each feasibility check requires one iteration to test each target as $\hat{t}$, and each iteration does several constant-time operations on each target to determine the minimal coverage. Therefore, the feasibility check has complexity $O(n^2)$. Binary search requires $O(\log(1/\epsilon))$ iterations to converge within ϵ, giving the overall complexity of $O(n^2 \cdot \log(1/\epsilon))$.

Algorithm 1 ISG Solver

```
for all t_i ∈ T do
    c_i ← 0
end for
maxPayoff ← 0
minPayoff ← min_{t_i∈T} U^u_Θ(t_i)
while maxPayoff − minPayoff > ε do
    midPoint ← (maxPayoff − minPayoff)/2
    if feasibilityCheck(midPoint, m, C) then
        minPayoff ← midPoint
    else
        maxPayoff ← MidPoint
    end if
end while
return C
```

In addition to our own ISG algorithm we implemented a mixed-integer program (MIP) that computes an exact solution for interval security games. This MIP model is used as a benchmark in the experimental evaluation. We do not described this MIP here due to space constraints, but note that the formulation is a minor variation of the BRASS MIP formulation presented in Pita et al. [18].

Algorithm 2 feasibilityCheck

```
for all t_i ∈ T do
    c^1_i ← max(0, 1 − (midPoint − U^u_Θ(t_i)) / (U^c_Θ(t_i) − U^u_Θ(t_i)))
end for
for all t_i ∈ T do
    totalCov ← c^1_i
    c_i ← c^1_i
    if c_i > 1 then
        GOTO next t_i
    end if
    R ← (c^1_i · U^{c,min}_Ψ(t_i)) + ((1 − c^1_i) · U^{u,min}_Ψ(t_i)) − ε'
    for all t_j ∈ {T\t_i} do
        c^2_j ← max(0, 1 − (R − U^{u,max}_Ψ(t_j)) / (U^{c,max}_Ψ(t_j) − U^{u,max}_Ψ(t_j)))
        c^3_j ← max(0, 1 − (R − U^{u,min}_Ψ(t_j)) / (U^{c,min}_Ψ(t_j) − U^{u,min}_Ψ(t_j)))
        minCov ← max(c^3_i, min(c^1_i, c^2_i))
        if minCov < 0 || minCov > 1 then
            GOTO next t_i
        end if
        totalCov ← totalCov + minCov
        c_j ← minCov
    end for
    if totalCov ≤ m then
        return TRUE, C
    end if
end for
return FALSE
```

6. SECURITY GAMES WITH DISTRIBUTIONAL UNCERTAINTY

An alternative way to model uncertainty about payoffs is to use distributions instead of intervals to represent possible values. The distributional security games (DSG) model introduced by Kiekintveld et al. [12] uses this approach, and presents several approximation algorithms for computing solutions to DSG. The DSG model contains more information than our model because it has access to distributional information, however, this has two significant drawbacks: (1) The models are more problematic to accurately define, since they require the domain expert to specify a large number of payoff distributions. (2) It is computationally challenging to solve infinite Bayesian Stackelberg games that represent distributional information; no exact algorithms are known for this class of games, and even heuristic approximations are expensive.

For these reasons, it is often preferable to use an interval model to represent uncertainty instead of a distributional model. We also show in this paper that we can use our interval algorithm as an efficient way to approximate solutions even when distributional information is available. We begin by introducing the DSG model, and then show how we can transform a DSG into an interval game which we can solve with our efficient interval algorithm. In the experimental results we compare the approximation results using intervals with the best known approximation methods for DSG games.

A distributional security game extends the security game model (see Section 3) in a similar way to ISG. The difference is that the attacker's payoffs are represented by continuous probability density functions (e.g., uniform distributions or Gaussian distributions) instead of intervals. Formally, this becomes an infinite Bayesian Stackelberg game with an infinite number of attacker types, and the game unfolds as follows: (1) the defender commits to a mixed strategy (2) nature chooses a random attacker type $\omega \in \Omega$ with probability $Pb(\omega)$, (3) the attacker observes the defender's mixed strategy, and (4) the attacker responds to the mixed strategy with a best-response that provides the attacker (of type ω) with the highest *expected* payoff. We define the type distribution by replacing the payoffs values $U^c_\Psi(t,\omega)$, $U^u_\Psi(t,\omega)$ for each target $t \in T$ with two continuous probability density functions that represent the defender's *beliefs* about the attacker payoffs:

$$f^c_\Psi(t,r) = \int_{\omega\in\Omega} Pb(\omega)U^c_\Psi(t,\omega)d\omega \quad (9)$$

$$f^u_\Psi(t,r) = \int_{\omega\in\Omega} Pb(\omega)U^u_\Psi(t,\omega)d\omega \quad (10)$$

For example, the defender expects with probability $f^c_\Psi(t,r)$ that the attacker receives payoff r for attacking target t when it is covered. For some coverage vector C, let $X_t(C)$ be a random variable that describes the *expected* attacker payoffs for attacking target t, given C. We then define the probability that the attacker will choose target t for each target $t \in T$ as follows:

$$a_t(C) \quad = \quad Pb[X_t(C) > X_{t'}(C) \text{ for all } t' \in T \setminus t] \quad (11)$$

because the attacker acts rationally. Conceptually, this gives the probability that the attacker will choose to attack each target for a given coverage vector C and the probability distributions of the attacker's payoffs. Using these probabilities, we can calculate the expected payoff for the defender. The original paper presents a derivation of an analytic formula for these probabilities, but it cannot be solved directly. Instead, Monte Carlo simulation is used to estimate the attack probabilities. We generate one sample attacker type by sampling payoffs from each of the payoff distribution; these are the payoff values assigned to that type. Using those payoff values we can calculate the best-response for this attacker type against the coverage strategy C. We sample a large number of types to estimate the expected value of a coverage strategy for a DSG.

In our experiments, we benchmark again the Greedy Monte Carlo (GMC) algorithm introduced by Kiekintveld et al [12]. This was found to be the method that was fastest and had the highest quality solutions for instances of DSG, especially when scaling up to large games. The GMC method is based on sampling a large number of attacker types (thousands) using Monte-Carlo sampling. It uses a greedy heuristic to approximate the optimal defender coverage strategy against the sampled attacker types.

To apply our interval algorithm to distributional security games we translate the distribution for each payoff to an interval. There are many ways to do this but we use a simple method that centers the interval around the mean of the distribution and determines the size of the interval based on the standard deviation of the interval and a multiplier. The multiplier is a parameter of the algorithm, and allows us to have intervals that include a larger or smaller fraction of the possible payoff values in the distribution. The minimum value for the interval is calculated as $mean - (StdDev \cdot multiplier)$ and the maximum values is calculated as $mean + (StdDev \cdot multiplier)$. Figure 1 shows this visually.

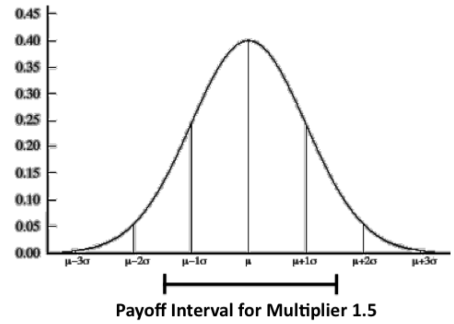


Figure 1: A payoff interval for a Gaussian distribution.

7. EXPERIMENTAL EVALUATION

We begin by evaluating the runtime and solution quality of the ISG solver on interval security games, and then present results on distributional security games.

7.1 Interval Game Experiments

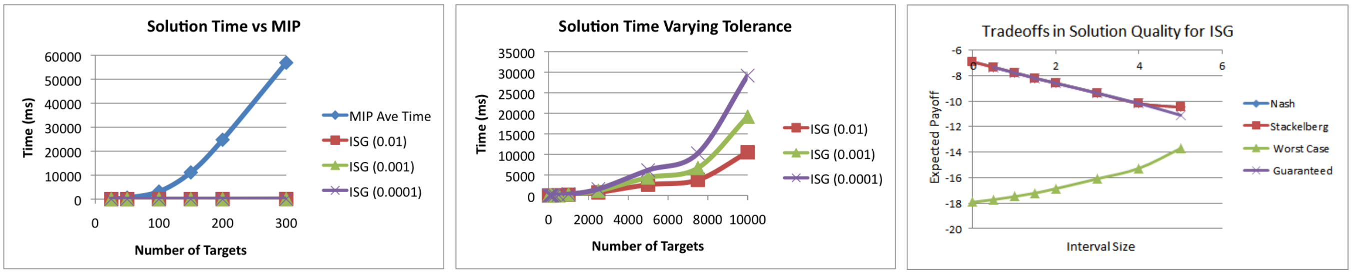
First, we tested the speed of the ISG solver against an exact MIP formulation based on BRASS. We tested the algorithms on 30 randomly generated sample games with defender payoffs for successful attacks drawn uniform random between 0 and -100, and the attacker payoffs for successful attacks drawn uniformly between 0 and 100. We modify the attacker payoffs to be intervals by using the first value drawn as the minimum value and setting the maximum value by adding a uniform random value between 0 and 20. The payoffs for unsuccessful attacks are 0 for both players. We fixed the number of resources at 20% of the number of targets.

In Figure 2(a) we present results for the MIP (solved using GLPK version 4.36) and ISG with three different tolerance settings. The tolerance settings control the accuracy of the binary search. All three ISG instances are much faster, even with an error tolerance of just 0.0001. Figure 2(b) shows results for the three ISG settings on much larger games. Here we see a modest increase in solution time with increasing accuracy. Even for 10000 targets and the highest accuracy setting, ISG solves the game in half the time required by the MIP to solve games with only 300 targets.

We also ran experiments to test the impact of interval uncertainty on solution quality under varying assumptions about the attacker strategy. For this experiment we used 200 randomly generated games with defender payoffs for successful attacks drawn from the range -20 to -10, and attacker payoffs between10 and 20. Payoffs for unsuccessful attacks were 0 for both players. The baseline case has no uncertainty about the attacker's payoffs. We solved these games using an existing linear-time solver for security games [14]. We then added increasing amounts of interval uncertainty to the attacker's payoffs, and solved the resulting games using the ISG solver.

The results are shown in Figure 2(c). On the x-axis is the size of the intervals for the attacker's payoffs. On the y-axis is the expected payoff for the defender. The four lines represent four different assumptions about the attacker. The Nash attacker always plays the optimal attacker strategy computed in the case with no uncertainty (in this case, the Stackelberg equilibrium strategy is the same as the Nash strategy [13]). The Stackelberg attacker is able to observe the exact coverage strategy and chooses a best-response, as in a Strong Stackelberg Equilibrium. The worst case attacker always chooses the worst possible target for the defender, without regard to the attacker's own payoffs. Finally, the guaranteed payoff is the payoff that the ISG method is able to guarantee against any rational attacker with payoffs that lie within the given intervals.

In the results there is a small decrease in the payoffs for the solutions to the interval games against the Nash and Stackelberg attackers. This is expected, and can be interpreted as the price of robustness to payoff uncertainty. The advantage of the ISG method is seen in the guaranteed and worst case payoffs. There is an in-



(a) Comparison of solution time for ISG and the MIP solved using CPLEX.

(b) The effect of varying the tolerance on ISG solution time.

(c) Impact of interval uncertainty on solution quality and robustness

Figure 2: Runtime and solution quality analysis for ISG.

creasing trend in the worst-case payoffs for ISG, with the strongest results for larger intervals. More importantly, the method is able to guarantee high payoffs for smaller amounts of possible variation in the attacker's payoffs, anywhere within the specified intervals.

7.2 Distributional Game Experiments

Our next set of experimental results evaluates the potential for ISG to be used as a fast approximation algorithm for distributional security games. We compare the performance of ISG using our methodology for transforming distributional security games into approximate versions based on intervals to the best existing methods for DSG, Greedy Monte Carlo (GMC) [12] and BRASS [18]. We run experiments on the same three classes of distributional games used by Kiekintveld et al., games with Uniform payoffs distributions games with Gaussian distributions with the same standard deviation for every payoff, and games with Gaussian distributions with varying standard deviations.

We generated 300 random instances of each class of games. The games were generated by first drawing random rewards and penalties for both players. All rewards were drawn from U[6; 8] and penalties were drawn from U[2; 4]. We then generated distributions of the correct type for the attacker's payoff, using the values from the first stage as the mean. In uniform games we vary the length of the uniform interval to increase or decrease uncertainty. For Gaussian games we vary the standard deviation, and all payoffs have the same amount of uncertainty. Gaussian variable games have a different standard deviation for each payoff distribution, which are drawn from U[0; 1] in our experiments.

All three of the algorithms we tested have parameters. For GMC the two main parameters that control the solution time and quality are the number of sample attacker types used in the calculation, and the size of the increment used in the greedy allocation of coverage probability. Solution quality improves with a larger number of types and a smaller increment, but solution time increases. We include both a "low" and "high" quality set of parameters for GMC; the specific settings are given in Table 1. The parameter for BRASS is ϵ, and reflects how far attackers may be from choosing the optimal target. The parameters for ISG are the multiplier used to generate the interval game from the distributional game, and the tolerance. The tolerance does not have a large effect on the solution quality (because we can always get very small error), so we fix this at 0.0001 in our experiments.

The BRASS epsilon parameter and the ISG multiplier can both have a significant effect on the solution quality, and there is no obvious way to set the value of these parameters. The best value can depend on the size of the game, the type of uncertainty, and the amount of uncertainty (e.g., the standard deviation of the distribu-

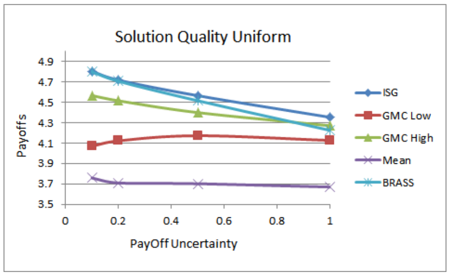
Table 1: Parameter settings for the algorithms.

Parameter	Values
ISG Multipliers	0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0
BRASS Epsilons	0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0
GMC Low	increment 0.05,1000 types
GMC High	increment 0.01,10000 types

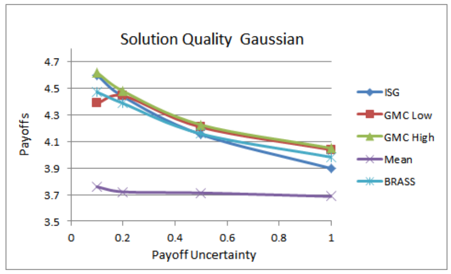
tion). We tested a variety of parameter settings to select the ones that gave the best results. The set of candidate values for each algorithm is shown in Table 1. Figure 3(g) shows an example of an experiment to find the best parameters games with Gaussian uncertainty for the ISG algorithm. The amount of uncertainty (i.e., standard deviation) varies along the x-axis, the value of the best parameter setting is given on the y-axis, and each line represents a class of games with a different number of targets. In general, the best multipliers are smaller for smaller games and games with more uncertainty. The need to do some experimentation to find a good parameter setting for ISG is not a significant practical concern. These settings could be used based on known values for similar games, and the algorithm is fast enough that testing a few different parameter settings on a specific game would not be prohibitive. In all of our experiments, we tested all of the values in the table for both BRASS and ISG, and selected the one that gave the best result for the specific setting.

The first three plots, 3(a), 3(b), and 3(c) compare the solution qualities achieved by the algorithms on games with 15 targets and 3 resources. For the Uniform and Gaussian games we vary the amount of payoff uncertainty on the x-axis by varying the standard deviations of the attacker's payoff distributions. For Gaussian Variable games we use only a single setting, since the amount of uncertainty varies on a per-payoff basis in these games. In all cases, the defender's expected payoff for the solution is plotted on the y-axis. This is evaluated after the algorithms return solutions by using a very large number of Monte-Carlo sample types (100000) to give a very accurate estimate of the expected payoff for the proposed coverage solution. We also include a final baseline called "Mean" that solves the game optimally by assuming that the mean of the distribution is the exact payoff value (in other words, it ignores the uncertainty in payoffs and solves it as a standard security game).

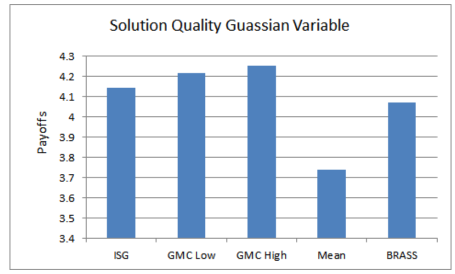
The mean baseline performs poorly in all cases. For uniform games, ISG has the highest solution quality, followed closely by BRASS. For Gaussian and Gaussian variable games ISG performs slightly worse than the GMC methods, particularly when there is a large amount of uncertainty, but the performance is still competitive. In the second set of plots, 3(d), 3(e), and 3(f), we fix the



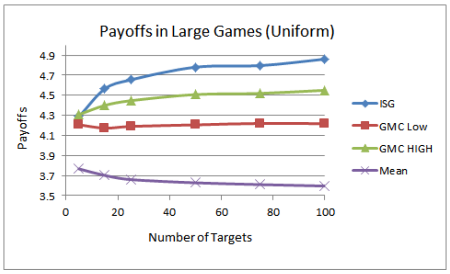
(a) Solution quality results for small games with uniform attacker payoff distributions.



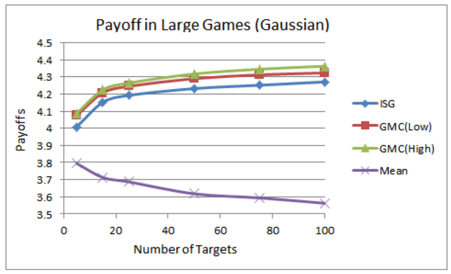
(b) Solution quality results for small games with Gaussian attacker payoff distributions.



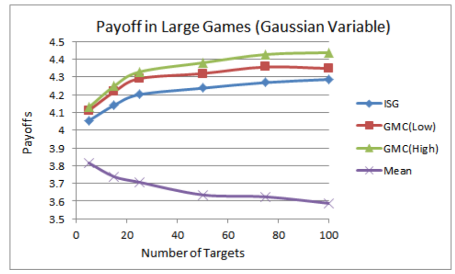
(c) Solution quality results for small games with Gaussian variable attacker payoff distributions.



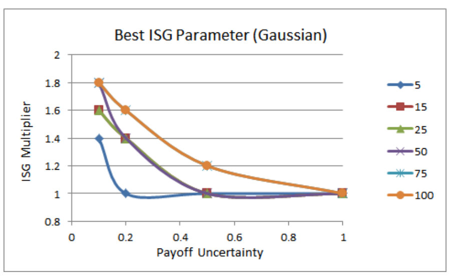
(d) Solution quality results scaling to larger games for uniform attacker payoff distributions.



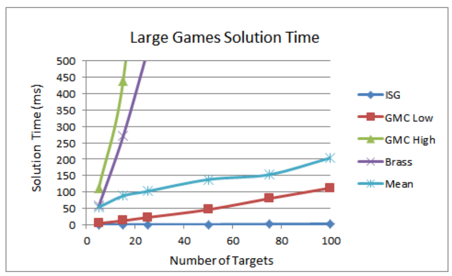
(e) Solution quality results scaling to larger games for Gaussian attacker payoff distributions.



(f) Solution quality results scaling to larger games for Gaussian variable attacker payoff distributions.



(g) Parameter optimization for ISG on games with Gaussian distributions.



(h) Comparison of solution times for solving large Gaussian games.

Figure 3: Solution quality and computation time comparisons.

amount of uncertainty for each the classes of games and vary the number of targets to assess performance on larger games. The standard deviation for uniform and Gaussian games is fixed at 0.5, and Gaussian variable games use the same distribution of standard deviations as before. BRASS is not included because it required too much memory to complete for the larger problems. The pattern of results is similar to the smaller games. In uniform games there is a greater separation, with ISG outperforming GMC. On Gaussian and Gaussian variable games, GMC has higher solution quality, but the overall difference between GMC and ISG is small.

The final result is presented in Figure 3(h). This plot compares the runtimes for computing solutions on the large Gaussian games (results for the other classes of games are very similar. Here we see that the solution times for both BRASS and GMC high rapidly increase as the size of the game increases. The solution times for GMC low and mean grow more reasonably. However, ISG is by far the fastest algorithm. It is fast enough to scale well beyond 100 targets, as seen in the previous set of results. Overall, ISG offers very fast solutions and either superior or competitive solution quality for approximating distributional games, depending on the type of uncertainty. It is a particularly good choice for accounting for uncertainty in very large games or situations where very fast performance is needed; in these cases it may be the only feasible method from a computational perspective that can account for uncertainty.

8. CONCLUSION

Security games have important real-world applications, but one of the critical questions is how to account for uncertainty and error in building the analyzing the game models. If the solutions are not robust to the kinds of errors and uncertainties that arise in modeling real problems, then they will not be useful in many situations. However, many of the standard approaches for handling uncertainty, such as Bayesian games, are very challenging from both a model elicitation standpoint and a computational standpoint.

We have introduced a new model of security games with uncertainty that is based on using intervals to represent possible payoffs, and takes a worst-case approach to uncertainty. This approach is motivated in part by the literature on robust optimization and more recently work on robust game theory concepts. We show that modeling uncertainty using intervals has distinct computational advantages. We present a highly efficient polynomial algorithm for approximating solutions to interval security games within very small (negligible) error bounds. Our experiment results show that this algorithm is much faster than equivalent MIP formulations.

In addition, we show that intervals can be used to approximate infinite Bayesian games with distributional uncertainty. We develop a methodology for modeling the infinite games using games with intervals, which can then be solved using our fast algorithm for ISG. In our experiments, the solutions found using this interval-based approach are surprisingly good–in all cases they are competitive with the best known methods for directly approximating the solutions to the Bayesian games, and in some cases the quality is even better. In addition, the speed of the solutions is much faster, and we can scale to extremely large games using this approach. This provides a computationally feasible way to account for uncertainty even in very challenging cases of security games. Our success in applying interval-based models in this case also suggests interesting directions for future work in applying these principles to manage uncertainty in more general classes of games.

8.1 Acknowledgments

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9. REFERENCES

[1] M. Aghassi and D. Bertsimas. Robust game theory. *Mathematical Programming*, 107:231–273, 2006. 10.1007/s10107-005-0686-0.

[2] N. Agmon, S. Kraus, G. A. Kaminka, and V. Sadov. Adversarial uncertainty in multi-robot patrol. In *IJCAI-09*, 2009.

[3] T. Alpcan and T. Basar. A game theoretic approach to decision and analysis in network intrusion detection. In *Proc. of the 42nd IEEE Conference on Decision and Control*, pages 2595–2600, 2003.

[4] N. Basiloco, N. Gatti, and F. Amigoni. Leader-follower strategies for robotic patrolling in environments with arbitrary topologies. In *AAMAS*, 2009.

[5] A. Ben-Tal and A. Nemirovski. Robust optimization – methodology and applications. *Mathematical Programming*, 92:453–480, 2002. 10.1007/s101070100286.

[6] V. Conitzer and T. Sandholm. Computing the optimal strategy to commit to. In *ACM EC-06*, pages 82–90, 2006.

[7] N. Gatti. Game theoretical insights in strategic patrolling: Model and algorithm in normal-form. In *ECAI-08*, pages 403–407, 2008.

[8] J. C. Harsanyi. Games with incomplete information played by Bayesian players (parts i–iii). *Management Science*, 14, 1967–8.

[9] M. Jain, E. Kardes, C. Kiekintveld, M. Tambe, and F. Ordonez. Security games with arbitrary schedules: A branch and price approach. In *AAAI-10*, 2010.

[10] M. Jain, M. Tambe, and C. Kiekintveld. Quality-bounded solutions for finite bayesian stackelberg games: Scaling up. In *AAMAS-11*, 2011.

[11] C. Kiekintveld, M. Jain, J. Tsai, J. Pita, F. Ordonez, and M. Tambe. Computing optimal randomized resource allocations for massive security games. In *AAMAS-09*, 2009.

[12] C. Kiekintveld, J. Marecki, and M. Tambe. Approximation methods for infinite Bayesian Stackelberg games: Modeling distributional payoff uncertainty. In *AAMAS-11*, 2011.

[13] D. Korzhyk, Z. Yin, C. Kiekintveld, V. Conitzer, , and M. Tambe. Stackelberg vs. Nash in security games: An extended investigation of interchangeability, equivalence, and uniqueness. *Journal of AI Research (JAIR) (to appear)*, 2011.

[14] O. Lerma, V. Kreinovich, and C. Kiekintveld. Linear-time resource allocation in security games with identical fully protective resources. In *Proceedings of the AAAI Workshop on Applied Adversarial Reasoning and Risk Modeling (AARM)*, 2011.

[15] W. Ma, X. Luo, and W. Xiong. A model for decision making with missing, imprecise, and uncertain evaluation of multiple criteria. *International Journal of Intelligent Systems*, 20(1), 2013.

[16] K. C. Nguyen and T. A. T. Basar. Security games with incomplete information. In *Proc. of IEEE Intl. Conf. on Communications (ICC 2009)*, 2009.

[17] P. Paruchuri, J. P. Pearce, J. Marecki, M. Tambe, F. Ordonez, and S. Kraus. Playing games with security: An efficient exact algorithm for Bayesian Stackelberg games. In *AAMAS-08*, pages 895–902, 2008.

[18] J. Pita, M. Jain, F. Ordóñez, M. Tambe, S. Kraus, and R. Magori-Cohen. Effective solutions for real-world Stackelberg games: When agents must deal with human uncertainties. In *AAMAS-09*, 2009.

[19] J. Pita, M. Jain, C. Western, C. Portway, M. Tambe, F. Ordonez, S. Kraus, and P. Parachuri. Depoloyed ARMOR protection: The application of a game-theoretic model for security at the Los Angeles International Airport. In *AAMAS-08 (Industry Track)*, 2008.

[20] J. Pita, M. Tambe, C. Kiekintveld, S. Cullen, and E. Steigerwald. GUARDS - game theoretic security allocation on a national scale. In *AAMAS-11 (Industry Track)*, 2011.

[21] E. Shieh, B. An, R. Yang, M. Tambe, C. Baldwin, J. Direnzo, G. Meyer, C. W. Baldwin, B. J. Maule, and G. R. Meyer. PROTECT : A Deployed Game Theoretic System to Protect the Ports of the United States. *AAMAS*, 2012.

[22] M. Tambe. *Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned*. Cambridge University Press, 2011.

[23] J. Tsai, S. Rathi, C. Kiekintveld, F. Ordóñez, and M. Tambe. IRIS - A tools for strategic security allocation in transportation networks. In *AAMAS-09 (Industry Track)*, 2009.

[24] J. Tsai, Z. Yin, J. young Kwak, D. Kempe, C. Kiekintveld, and M. Tambe. Urban security: Game-theoretic resource allocation in networked physical domains. In *National Conference on Artificial Intelligence (AAAI)*, 2010.

[25] H. von Stackelberg. *Marktform und Gleichgewicht*. Springer, Vienna, 1934.

[26] K. wei Lye and J. M. Wing. Game strategies in network security. *International Journal of Information Security*, 4(1–2):71–86, 2005.

[27] W. Xiong, X. Luo, and W. Ma. Games with ambiguous payoffs and played by ambiguity and regret minimising players. In *Australasian Conference on Artificial Intelligence*, pages 409–420, 2012.

[28] Z. Yin and M. Tambe. A unified method for handling discrete and continuous uncertainty in Bayesian Stackelberg games. In *International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, 2012.