Competing Intermediary Auctions

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ABSTRACT

Bidders' selection of an auction is important in the study of competing auctioneers. However, in a variety of settings, such as in the spot pricing of online display advertisements, bidders, represented by autonomous agents, are not allowed to bid directly but only via specialized intermediaries. Motivated by the emergence of these markets, we analyze a scenario where a single indivisible good is auctioned at a central seller via two intermediary auctions. We study, for the first time, the selection problem faced by the buyers who have to decide in which intermediary auction to bid after observing the set reserve prices. We find that, when the reserve prices of the intermediaries are sufficiently different, the unique pure-strategy equilibrium is for all buyers to select the low-reserve intermediary. When this is not the case, the equilibrium strategy depends on the valuation of the buyers and consists of three intervals. In this unique equilibrium, buyers in the low-valuation interval always choose the low-reserve intermediary. Buyers in the middle interval follow a strictly mixed strategy. Finally, the strategy of the buyers in the high-valuation interval is for all of them to go to either the high-reserve intermediary, or the low-reserve one, but not both.

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1. INTRODUCTION

A rich theory of auctions has been developed over the last decades for the setting of a monopolist who is auctioning a good to a number of buyers. However, much less is known for the case of multiple auctioneers that compete for buyers' attention. A handful of researchers have dealt with this problem, usually assuming an infinite population of agents [7, 10] or duopoly competition [4, 9], mainly due to tractability issues. Our work takes the latter approach, but, in contrast to the previous literature, focuses on the effects of competition between intermediary auctions. These are contingent auctions that are held among buyers before the actual selling of the good at a central auction.

Our motivation stems from the online advertising exchange marketplace [8], whereby impressions are sold using real time auctions, following the sponsored search paradigm. Nowadays, autonomous agents are used to bid on behalf of their owning advertisers in these auctions that take place billions of times per day, a fact that has attracted the attention of the multi-agents society. The introduction of these markets has resulted in increased complexity for the advertisers, who are usually not allowed or are not able to bid directly in the central exchange, but can only participate via certified intermediaries, such as ad networks or demand side platforms. The latter typically run their own auction before forwarding their bid on behalf of the advertiser onto the exchange.

Against this background, in this paper we apply game theoretic concepts to the problem of how a buyer should choose an intermediary, and discuss how this decision affects the policies (in particular, the reserve prices) of the intermediaries. This is the first work that considers the problem of competing intermediaries where buyers are *non-captive*, i.e. can choose an intermediary. More specifically, we study an abstracted setting where buyers are willing to buy an indivisible good from a central auctioneer by placing a bid at one of two intermediaries, each running a local auction among their delegated buyers. These intermediaries then compete at the center for the good. Our formulation is general enough to encompass other markets such as real estate, treasury bills or procurement auctions where intermediaries are prominent. Buyers then face the following problem: after determining their valuations for the good and observing the reserve prices of the intermediaries, they must simultaneously select one of them to bid in, i.e. bidders *"single-home"*. This is a natural assumption for a qualitative analysis on the effect of intermediary competition, as advertisers cannot typically fully multi-home with hundreds of intermediaries. This is due to the high costs for managing each campaign and technology integration given the real time nature of the auctions. In addition to the impact on their own utility, buyers' strategy also has important implications for the design of the auctions, both at the level of the intermediaries, and at the exchange. Note that our setting departs from standard models of two-sided markets where emphasis is given on network effects [11]. In contrast, intermediaries

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here are one-sided platforms representing the demand side of the market.

Our contributions are as follows. We identify resulting equilibria of the intermediary selection subgame when buyers have probabilistic information about their competitors' valuations. We find that the presence of intermediaries fundamentally changes the incentives of both the buyers and the auctioneers compared to the setting with independent auctions. Interestingly, even in this simple scenario, the buyers' resulting strategy is fairly complicated, illustrating the complexity of the problem, and thus partially explaining simpler strategies that are likely to be encountered in practice. However, our equilibrium analysis serves as a useful benchmark against these strategies, and provides insights regarding the auction design problem faced by the intermediaries and the exchange. More specifically, we show that, whenever the reserve prices are sufficiently high or different, there is a unique pure-strategy equilibrium where all buyers select the low-reserve intermediary. When this is not the case, there is a unique equilibrium strategy that depends on the valuation of the buyers and consists of three intervals, whose limits we call *cut-off points*. In this equilibrium, buyers in the interval with low valuations always go to the low-reserve intermediary; buyers with valuations in the rightmost interval either select the high-reserve or the low-reserve intermediary (but not both); finally, the equilibrium strategy for buyers with valuations in the middle interval is strictly mixed. This is in contrast to previous works on competing Vickrey auctions, where it was shown that buyers' selection admits simple cut-off strategies involving uniform randomization among the eligible auctions [2, 4, 7, 9, 10].

The paper is structured as follows. In Section 2, we provide an overview of related work on competing auctions. Then, Section 3 provides a formal description of our model. We define the intermediary selection problem and detail our results in Section 4. Finally, Section 5 concludes.

2. RELATED WORK

Our model extends that of Feldman et al. [5], who study the reserve price setting problem faced by the center and multiple intermediaries implementing Vickrey auctions with reserve prices. The authors show that intermediaries will use stochastic reserve prices over an interval, whereas the center maximizes its revenue by setting a reserve price that depends on the number of intermediaries. However, crucially, they consider symmetric intermediaries (equal number of buyers with identical type distributions) and *captive* buyers (i.e. they cannot move between the intermediaries). By contrast, we assume that the buyers can choose their intermediary, and so the distribution at an intermediary is endogenously defined by the buyers' selection strategies.

Competition between auctions that do not involve intermediaries in a non-captive setting has first been studied by McAfee [7] and Peters and Severinov [10]. They identify a symmetric equilibrium where buyers follow strategies involving cut-off points, so that buyers with valuations between two consecutive cut-off points equally randomize between eligible auctions. Moreover, they show that the reserve prices equal sellers' production cost in the limit. However, their results require an infinite number of sellers, and break down in the case of oligopolies which naturally arise in our setting. Given this, Burguet and Sákovics [4] studied the competition between two sellers implementing Vickrey auctions with reserve prices. The authors identify a unique Bayes-Nash equilibrium for the selection problem faced by the buyers, involving a single cut-off point, w, so that buyers with valuations less than w always prefer the low-reserve auction, whereas buyers with higher types randomize equally between the two auctions. Hernando-Veciana [2] extended these results to more than two auctions, assuming a finite set of reserve prices for the auctioneers. Based on this work, Gerding et al. [6] studied the duopoly competition between auctions in presence of a mediating institution and identified a pure-strategy equilibrium for the sellers with asymmetric production costs. Furthermore, competing auctions in the context of sponsored search were studied by Ashlagi et al. [3] who consider a setting with two auctions that differ in their click through rates. One of their findings is that, when sellers offer VCG auctions with reserve prices, there is a unique equilibrium for the buyers' selection subgame, uniquely defined by two cut-off points, so that buyers with valuations in the interval defined by the cut-off points follow a strictly mixed strategy, whereas buyers with valuations outside of this interval follow pure strategies. However, none of the previous works considers the problem of competing intermediaries in a non-captive setting, whose presence fundamentally changes the nature of the problem. More specifically, the auctions are no longer independent, as they compete additionally as bidders at the central auction. This provides incentives to increase their reserve prices, and hence their contingent payments, that in turn decreases demand from buyers.

3. MODEL

We consider a setting with a unique indivisible good (corresponding to an impression) for sale by a single auctioneer, c, called the *center*. There is also a population of $n \in \mathbb{N}, n \geq 3$, ex ante symmetric, purely profit maximizing (risk-neutral) buyers that compete for this good, but are allowed to participate only via two qualified *intermediaries*, s_1, s_2. The center and the intermediaries are also profit maximizing but have no value for the good itself. The preferences of buyers and auctioneers are described by von Neumann and Morgenstern utility functions.

We study a standard independent private values model, where each buyer i has a valuation, υ_i, $i \in \{1, ..., n\}$, for the good to be traded over a compact support $V = [0, 1]$. It is assumed that valuations, υ_i, $i \in \{1, ..., n\}$, for the good to be traded are i.i.d. drawn from a commonly known distribution F with a continuous, positive density f and support $V = [0, 1]$. F, f and the total number of buyers, n, are assumed to be common knowledge. The expected utility for a buyer i with valuation υ_i is $\Pi_\ell(\upsilon_i) = \alpha_\ell(\upsilon_i)(\upsilon_i - \rho_\ell)$, where $\alpha_\ell : V \mapsto [0, 1]$ is the probability of obtaining the good in intermediary s_ℓ's local auction ($\ell = 1, 2$), and $\rho_\ell \in [0, 1]$ the price to be paid to the intermediary.

In our setting, each intermediary, s_ℓ, $\ell = 1, 2$, runs a Vickrey auction with a reserve price, r_ℓ, $\ell = 1, 2$, respectively[1]. Without loss of generality, for the remainder of this paper we will assume that $r_1 \leq r_2$. The center runs a Vickrey auction without a reserve price[2] and a fair tie-breaking rule,

[1]This is the mechanism implemented in large, automated ad networks, such as Google AdWords.

[2]This is done for simplification reasons. We could also assume that the center sets a reserve price, but this has no

and each intermediary is allowed to submit a single bid[3]. Buyers must select their (single) intermediary after learning their valuations based on the announced reserve prices and available information. We note that the selection problem is independent of the actual bidding. Thus, after selecting their desired intermediary, it is a weakly dominant strategy to submit their true valuations.

Each intermediary, s_ℓ, $\ell = 1, 2$, runs a *contingent* auction among its set of buyers, denoted K_ℓ, to determine the winning bid, $w_\ell = \max_{i \in K_\ell}\{v_i\}$, the price to be paid from the winning buyer conditional on its winning at the central auction, $\rho_\ell = \max\{v_{\arg\max\{v_i \in K_\ell \setminus \{w_\ell\}\}}, r_\ell\}$, as well as the bidding amount to be submitted to the center. Given that the auction at the center is dominant-strategy incentive compatible, it is a weakly dominant strategy for the intermediaries to bid their contingent payments, ρ_ℓ, $\ell = 1, 2$. Hence, the expected profit of an intermediary s_ℓ is $\Pi_{s_\ell}(\rho_\ell) = \alpha_c(\rho_\ell)(\rho_\ell - \rho_m)$, $m \neq \ell$, where $\alpha_c : V \mapsto [0, 1]$ is the probability of obtaining the good at the center. In more detail, the game proceeds as follows:

1. Buyers learn their valuations for the good.
2. Intermediaries announce their reserve prices, r_1, r_2.
3. Buyers select their preferred intermediary, s_ℓ, $\ell = 1, 2$, and submit a bid to their selected intermediary.
4. Intermediaries run their auctions for the good among their local buyers and submit their bids, ρ_ℓ, $\ell = 1, 2$, to the center.
5. The center runs its auction given the bids submitted by the intermediaries, transfers the good to the winning intermediary and receives payment.
6. The winning intermediary transfers the good to its winning buyer and receives payment.

Given this formulation, we will now study the intermediary selection problem faced by the buyers. The following examples illustrate some of the implications stemming from the introduction of the intermediaries, namely a pressure for intermediaries to increase the reserve prices and the inapplicability of the revenue equivalence theorem. Suppose that there are three buyers, $1, 2, 3$ with valuations $v_1 = 1$, $v_2 = 0.9$ and $v_3 = 0.8$. First, suppose that the reserve prices are $r_1 = 0.85$ and $r_2 = 0.89$ (i.e. buyer 3 cannot participate). Then, if buyer 1 selects s_1 and buyer 2 selects s_2, the intermediaries submit their reserve prices at the center and s_2 wins, giving the good to buyer 2 for a price of r_2 and s_2 obtains a profit of $r_2 - r_1$. In this case, it is an equilibrium for buyer 1 to select the high-reserve intermediary, s_2, that, although offering him a higher price, guarantees that he will always win the good at the center. It is clear here that intermediaries have incentives to increase their reserve prices. In another example, suppose that $r_1 = 0.7$ and $r_2 = 0.75$. Buyer 3 can never obtain the good, so we can assume that he randomizes equally between the intermediaries. Then, if both buyers 1 and 2 select the same intermediary, buyer 1 always wins for a utility of $v_1 - v_2 = 0.1$ (as the selected intermediary submits 0.9 at the center, whereas the opponent intermediary will only submit its reserve price half of the time, when buyer 3 is there, and zero otherwise). However, if they select different intermediaries, then buyer 1's expected utility is $\frac{1}{2}(v_1 - v_3) = 0.1$ and buyer 2's expected utility is $\frac{1}{2}(v_2 - v_3) = 0.05$ (each of them wins whenever he is in the same intermediary auction as buyer 3). In this scenario, there exist two (weak) pure strategy Nash equilibria, where buyers 1 and 2 select different intermediaries. This means, that there can be situations where the good is not allocated to the buyer with the highest valuation.

effect on the buyers' choice of intermediary.

[3]Variations of the Vickrey auction with reserve price are used in all major ad exchanges.

4. BUYER EQUILIBRIUM STRATEGIES

Having illustrated our model and examples, we begin our analysis. Since we are in a probabilistic environment, our equilibrium concept is *symmetric Bayes-Nash*. We denote by $\theta : V \mapsto [0, 1]$ the selection strategy of the buyers, which is a mapping from a buyer's valuation to the probability of selecting the *low-reserve* intermediary, s_1. Thus, $1 - \theta(v)$ is the probability that the same buyer selects intermediary s_2.

As we will show, there is a unique pure-strategy Nash equilibrium (PSNE) where all buyers select the low-reserve intermediary when the reserve prices are sufficiently different. If this is not the case, the intermediary selection problem admits a unique mixed-strategy Nash equilibrium (MSNE) involving three intervals defined by two cut-off points. In this equilibrium, buyers with valuations below the low cut-off point always select the low-reserve intermediary. Also, buyers with valuations between these cut-off points randomize between the intermediaries according to a function whose form will be different for different distributions of valuations assumed. Finally, buyers above the high cut-off point will all select either the low-reserve intermediary, or the high-reserve one, but never both. Due to space limitations, parts of some proofs are moved to an online appendix [1]. In what follows, we start by providing a closed form expression for the utility from each intermediary that the buyers expect.

The expected utilities for a buyer with valuation v from selecting the low- and high-reserve intermediary, $\Pi_1(v), \Pi_2(v)$ respectively, when $r_1 < r_2$, can be written as:

$$\Pi_1(v) = \begin{cases} 0 & \text{if } v \in [0, r_1) \\ \int_{r_1}^{v} F_1^{(n-1)}(y)dy & \text{if } v \in [r_1, r_2) \\ (v - r_1)F_1^{(n-1)}(r_1) + \int_{r_1}^{r_2} (v - y) f_1^{(n-1)}(y)dy + & \\ + \int_{r_2}^{v} (v - y)\theta(y) f_1^{(n-1)}(y)dy + & \\ + \int_{r_2}^{v} \int_{y_2}^{1} (v - y_2)(1 - \theta(y_1))\theta(y_2) f_{1,2}^{(n-1)}(y_1, y_2)dy_1 dy_2 & \\ & \text{if } v \in [r_2, 1] \end{cases} \tag{1}$$

$$\Pi_2(v) = \begin{cases} 0 & \text{if } v \in [0, r_2) \\ (v - r_2)F_1^{(n-1)}(r_2) + \int_{r_2}^{v} (v - y)(1 - \theta(y)) f_1^{(n-1)}(y)dy + & \\ +(v - r_2) \int_{0}^{r_2} \int_{r_2}^{1} \theta(y_1) f_{1,2}^{(n-1)}(y_1, y_2)dy_1 dy_2 + & \\ + \int_{r_2}^{v} \int_{y_2}^{1} (v - y_2)\theta(y_1)(1 - \theta(y_2)) f_{1,2}^{(n-1)}(y_1, y_2)dy_1 dy_2 & \\ & \text{if } v \in [r_2, 1] \end{cases} \tag{2}$$

where $F_1^{(n-1)}(y) = F^{n-1}(y)$, $f_1^{(n-1)}(y) = (n-1)F^{n-2}(y)f(y)$ are the cumulative distribution and density functions of the first-order statistic, and $f_{1,2}^{(n-1)}(y_1, y_2) = (n-1)(n-2)f(y_1) f(y_2)F^{n-3}(y_2)$ is the joint density of the first- and second-order statistics among $n - 1$ bids.

The first term in (1) represents a buyer's expected utility from the low-reserve intermediary, s_1, when his valuation is in $[r_2, 1]$ and all opponent bids are less than or equal to r_1. The buyer also expects positive utility from s_1 when the expected highest opponent bid over the population of buyers is higher than r_1, lower than his valuation, and is submitted in the same auction (second and third terms in (1)), as this bid will always win at the center. Finally, he expects positive utility from s_1 when the expected second highest opponent bid over the population of buyers is higher than r_2, lower than his valuation, and is submitted in the same auction, and, at the same time, the expected highest opponent bid is submitted in the high-reserve intermediary (fourth term in (1)). This is because the local second highest bids compete at the center, and hence his local second highest bid (which will be the third highest global bid) is guaranteed to win against the local second highest bid in the other auction (which will be at most the fourth highest global bid or r_2).

Similarly, a buyer with valuation in $[r_2, 1]$ expects positive utility from the high-reserve intermediary, s_2, when all opponent bids are less than or equal to r_2 (first term in (2)), or when the expected highest opponent bid over the population of buyers is higher than r_2, lower than his valuation, and is submitted in the same auction (second term in (2)). He also expects positive utility from s_2 when the expected second highest opponent bid over the population of buyers is higher than r_2, lower than his valuation, and is submitted in the same auction, and, at the same time, the expected highest opponent bid is submitted in the low-reserve intermediary auction (fourth term in (2)). Finally, the third term in (2) corresponds to the case where the expected highest opponent bid is higher than r_2 and submitted in s_1, and, at the same time, the expected second highest opponent bid is less than r_2. Then, the buyer's expected payment is r_2, as the forwarded bid by s_1 (which will be at most the third highest global bid or r_1) will always be less than r_2.

We begin our analysis with the special case of equal reserves and then provide results for the more general case where r_1 is strictly less than r_2.

4.1 Special Case: Equal Reserve Prices

When $r_1 = r_2 = r$, assuming a fair tie-breaking rule by the center, the expected utilities for a buyer with valuation υ from the low- and high-reserve intermediary simplify to:

$$\Pi_1^{eq}(\upsilon) = \begin{cases} 0 & \text{if } \upsilon \in [0, r) \\ (\upsilon - r)F_1^{(n-1)}(r) + \int_r^\upsilon (\upsilon - y)\theta(y)f_1^{(n-1)}(y)dy + \\ +\frac{1}{2}(\upsilon - r)\int_0^r \int_r^1 (1-\theta(y_1))f_{1,2}^{(n-1)}(y_1, y_2)dy_1dy_2 + \\ +\int_r^\upsilon \int_{y_2}^1 (\upsilon - y_2)(1-\theta(y_1))\theta(y_2)f_{1,2}^{(n-1)}(y_1, y_2)dy_1dy_2 \\ & \text{if } \upsilon \in [r, 1] \end{cases} \quad (3)$$

$$\Pi_2^{eq}(\upsilon) = \begin{cases} 0 & \text{if } \upsilon \in [0, r) \\ (\upsilon - r)F_1^{(n-1)}(r) + \int_r^\upsilon (\upsilon - y)(1-\theta(y))f_1^{(n-1)}(y)dy + \\ +\frac{1}{2}(\upsilon - r)\int_0^r \int_r^1 \theta(y_1)f_{1,2}^{(n-1)}(y_1, y_2)dy_1dy_2 + \\ +\int_r^\upsilon \int_{y_2}^1 (\upsilon - y_2)\theta(y_1)(1-\theta(y_2))f_{1,2}^{(n-1)}(y_1, y_2)dy_1dy_2 \\ & \text{if } \upsilon \in [r, 1] \end{cases} \quad (4)$$

The third term in both equations above represents the expected utility of a buyer when the highest opponent bid is submitted in the other intermediary, whereas all remaining bids are less than the reserve price set by the intermediaries. In this case, both intermediaries will submit r at the center, where a fair tie breaking rule yields the same probability of winning the auction. The following theorem illustrates the equilibrium intermediary selection strategy of the buyers in this scenario.

THEOREM 1. *Whenever $r_1 = r_2 = r$, it is a mixed-strategy Nash equilibrium for the buyers in the intermediary selection problem to equally randomize between the intermediaries. Moreover, there exists a pure-strategy Nash equilibrium where all buyers select the low-reserve intermediary auction if the reserve prices are such that $F(r) = 0$ or $F(r) = 1$.*

PROOF. It is easy to see that the only mixed equilibrium strategy $\theta(\upsilon) \in (0, 1)$ equals $\frac{1}{2}$ for all $\upsilon \in [r, 1]$, due to the symmetry of (3) and (4). For the PSNE, suppose without loss of generality that all (other) buyers select intermediary s_1, i.e. $\theta(\upsilon) = 1$ for all $\upsilon \in [r, 1]$. Then, the utility difference that a buyer with valuation $\upsilon \in [r, 1]$ expects will be:

$$\Pi_1(\upsilon) - \Pi_2(\upsilon) = \int_r^\upsilon F_1^{(n-1)}(y)dy - (\upsilon - r)[\frac{n-1}{2}F^{n-2}(r) - \frac{n-3}{2}F^{n-1}(r)] \quad (5)$$

The second partial derivative of this function w.r.t. υ is:

$$\frac{\partial^2}{\partial \upsilon^2}(\Pi_1(\upsilon) - \Pi_2(\upsilon)) = (n-1)F^{n-2}(\upsilon)f(\upsilon) \geq 0 \quad (6)$$

This means that the function is convex, so its global minimum at a valuation we denote υ_c will satisfy the FOC:

$$F^{n-1}(\upsilon_c) = \frac{n-1}{2}F^{n-2}(r) - \frac{n-3}{2}F^{n-1}(r) \quad (7)$$

For the existence of a PSNE, we would like that $\Pi_1(\upsilon_c) - \Pi_2(\upsilon_c) \geq 0$. Using (7), this means that $\int_r^{\upsilon_c} F^{n-1}(y)dy \geq F^{n-1}(\upsilon_c)(\upsilon_c - r)$. However, from the first mean value theorem for integration, $\int_r^{\upsilon_c} F^{n-1}(y)dy = F^{n-1}(\omega)(\upsilon_c - r)$, where $r < \omega < \upsilon_c$. So, we would have that $F^{n-1}(\omega)(\upsilon_c - r) \geq F^{n-1}(\upsilon_c)(\upsilon_c - r)$, which can only happen for $\upsilon_c = r$, since $f > 0 \Rightarrow F(\omega) < F(\upsilon_c)$. Using this last fact in (7) yields:

$$F^{n-1}(r) = \frac{n-1}{2}F^{n-2}(r) - \frac{n-3}{2}F^{n-1}(r) \Rightarrow F(r) = 0 \text{ or } F(r) = 1 \quad (8)$$

□

4.2 Pure Nash Equilibrium Strategy

Having analyzed the equal reserve prices scenario, for the remainder of this report, we will assume that $r_1 < r_2$. In this general case, as the next theorem shows, buyers have an incentive to select s_1 when the reserve prices are sufficiently different.

THEOREM 2. *There exists a pure-strategy Nash equilibrium in the buyer intermediary selection problem where all buyers select the low-reserve auction if the reserve price of the low-reserve intermediary, r_1, is lower or equal than a critical value $r_c \in [r_1, r_2)$, satisfying*

$$\int_{r_c}^{\upsilon_c} F_1^{(n-1)}(y)dy = F_1^{(n-1)}(\upsilon_c)(\upsilon_c - r_2) \quad (9)$$

where υ_c is such that

$$F_1^{(n-1)}(\upsilon_c) = F_1^{(n-1)}(r_2) + (n-1)(1 - F(r_2))F_1^{(n-2)}(r_2) \quad (10)$$

PROOF. Consider the case that all (other) buyers select the low-reserve auction, i.e. $\theta(\upsilon) = 1$ for all $\upsilon \in [r_1, 1]$. Then, using (1) and (2), we can write the utility difference that a buyer with valuation $\upsilon \in [r_2, 1]$ expects as:

$$\Pi_1(\upsilon) - \Pi_2(\upsilon) = \int_{r_1}^\upsilon F_1^{(n-1)}(y)dy - \quad (11)$$
$$- [F_1^{(n-1)}(r_2) + (n-1)(1 - F(r_2))F_1^{(n-2)}(r_2)](\upsilon - r_2)$$

The second partial derivative of this function w.r.t. v is:

$$\frac{\partial^2}{\partial v^2}(\Pi_1(v) - \Pi_2(v)) = f_1^{(n-1)}(v) \geq 0 \tag{12}$$

This means that the function is convex, so its global minimum at a valuation we denote v_c will satisfy the FOC:

$$F_1^{(n-1)}(v_c) = F_1^{(n-1)}(r_2) + (n-1)(1 - F(r_2))F_1^{(n-2)}(r_2) \tag{13}$$

For this to be a PSNE, we want $\Pi_1(v_c) - \Pi_2(v_c) \geq 0$. The equality $\Pi_1(v_c) - \Pi_2(v_c) = 0$ gives us an upper bound for r_1, which we call the critical reserve price, r_c. As can be seen from (13), v_c is only dependent on r_2. Additionally, $\Pi_1 - \Pi_2$ is a decreasing function of r_1 (see (11)). This means that, for a given r_2, setting $r_1 = r_c + \epsilon$, where $\epsilon > 0$ is a strictly positive quantity, $\Pi_1(v_c) - \Pi_2(v_c) < 0$. This bound is strict as $F(\cdot)$ is strictly increasing. On the other hand, setting $r_1 = r_c - \epsilon$ gives a positive utility difference, i.e. $\Pi_1(v_c) - \Pi_2(v_c) > 0$. So, a PSNE exists for any $r_1 \leq r_c$. □

In what follows, we prove, through a number of steps, that the equilibria of Theorem 2 are the only PSNE. First, we show that it is not a PSNE for the buyers to always select the high-reserve intermediary.

Proposition 1. *There is no pure-strategy Nash equilibrium where all buyers always select the high-reserve intermediary.*

Proof. Similarly, when all other buyers select the high-reserve intermediary, the difference in expected utilities for a buyer with valuation $v \in [r_2, 1]$ will be:

$$\Pi_2(v) - \Pi_1(v) = \int_{r_2}^{v} F_1^{(n-1)}(y)dy - \int_{r_1}^{r_2} F_1^{(n-1)}(y)dy - (v - r_2)F_1^{(n-1)}(r_2) \tag{14}$$

By taking the first and second partial derivatives w.r.t. v, we get:

$$\frac{\partial}{\partial v}(\Pi_2(v) - \Pi_1(v)) = F_1^{(n-1)}(v) - F_1^{(n-1)}(r_2) \tag{15}$$

$$\frac{\partial^2}{\partial v^2}(\Pi_2(v) - \Pi_1(v)) = f_1^{(n-1)}(v) \geq 0 \tag{16}$$

This means that the function is convex, so there is a global minimum at v_c where $F_1^{(n-1)}(v_c) = F_1^{(n-1)}(r_2)$. But $\Pi_2(v_c) - \Pi_1(v_c) < 0$, so this can never be a symmetric PSNE. □

The only remaining case for the existence of PSNE is to consider a selection strategy consisting of a number of intervals whereby the strategy remains constant but changes between two successive intervals. Suppose that there are $k \geq 1$ points, $w_i, i = 1, ..., k$, in V, which we call *cut-off points*, so that $\theta(v) = \theta_1$ for $v \in [r_2, w_1)$, $\theta(v) = \theta_2$ for $v \in [w_1, w_2)$ and so on, where $\theta_i \in \{0, 1\}$ for $i = 1, ..., k+1$ and $\theta_i \neq \theta_{i+1}$. Moreover, it has to hold that $\Pi_1(w_i) = \Pi_2(w_i), \forall i = 1, ..., k$, i.e. a buyer with a valuation equal to a cut-off point has to be indifferent between choosing either intermediary. Proposition 2 shows that no PSNE with such cut-off points exists.

Proposition 2. *Let $w_1 < w_2 < ... < w_k \in (r_2, 1]$, $k \in \mathbb{N}^*$ denote cut-off points and let $\theta : V \mapsto [0, 1]$ be a strategy profile where $\theta(v) = \theta_1$ if $r_2 \leq v < w_1$, $\theta(v) = \theta_2$ if $w_1 \leq v < w_2$ and so on, $\theta(v) = \theta_{k+1}$ if $w_k \leq v \leq 1$, for a buyer with valuation $v \in V$, where $\theta_i \in \{0, 1\}$ for $i = 1, ..., k+1$, $\theta_i \neq \theta_{i+1}$ for all $i = 1, ..., k$. Then, $\theta(\cdot)$ is not a pure Nash equilibrium strategy profile.*

Proof Sketch. Given that $\Pi_1(r_2) > \Pi_2(r_2) = 0$, it should be that $\theta(r_2) = 1$ and hence $\theta_1 = 1 \ \forall v \in [r_2, w_1)$ by continuity of Π_1, Π_2. This means that $\theta_{\lambda+1} = 1$ for even λ and $\theta_{\lambda+1} = 0$ for odd λ. It can also be shown that the selection strategy, $\theta_{\lambda+1} \in \{0, 1\}$, controls the convexity of $\Pi_1 - \Pi_2 = D_\lambda$ in $[w_\lambda, w_{\lambda+1})$, by taking its second partial derivative with respect to v. More specifically, D_λ is convex when $\theta_{\lambda+1} = 1$, and concave otherwise. We consider two cases, $k = 1$ and $k \geq 2$.

When $k \geq 2$, for the existence of a PSNE, we would have a non-negative decreasing convex $\Pi_1 - \Pi_2$ at $[r_2, w_1)$ ($\theta_1 = 1$) followed by a non-positive concave $\Pi_1 - \Pi_2$ at $[w_1, w_2)$ ($\theta_2 = 0$), which first decreases and then increases up to zero, followed by a non-negative convex $\Pi_1 - \Pi_2$ at $[w_2, w_3)$ ($\theta_3 = 1$) and so on. But then there should be discontinuities at the optima of the corresponding intervals, which is in contrast with the well-defined first derivative of $\Pi_1 - \Pi_2$. Hence, there cannot be a PSNE with $k \geq 2$ cut-off points.

When $k = 1$, w is a saddle point. So, from FOC:

$$\begin{aligned} &F_1^{(n-1)}(w) + (n-1)(1 - F(w))F_1^{(n-2)}(w) = \\ &F_1^{(n-1)}(r_2) + (n-1)(1 - F(r_2))F_1^{(n-2)}(r_2) \end{aligned} \tag{17}$$

However, the function $x^{n-1} + (n-1)(1-x)x^{n-2}$ is strictly increasing for $0 < x < 1$, which means that $F(w) = F(r_2)$. So, it should be that $w = r_2$. Nevertheless, $\Pi_1(r_2) - \Pi_2(r_2) > 0$, whereas we require that $\Pi_1(w) - \Pi_2(w) = 0$. Hence it cannot be a PSNE. The full proof can be found in [1]. □

Given this last result, we derive the following corollary.

Corollary 1. *The equilibrium of Theorem 2 is the unique pure-strategy Nash equilibrium of the buyer intermediary selection problem.*

This can be directly derived by the nonexistence results of propositions 1 and 2 that cover all possible cases, combined with the existence result of Theorem 2.

4.3 Mixed Nash Equilibrium Strategy

Given this last result, when the reserve prices do not satisfy the conditions of Theorem 2, buyers should follow a mixed strategy $\theta_m(v) \in (0, 1)$ at an appropriate interval in equilibrium. The following lemma provides the conditions that $\theta_m(\cdot)$ should satisfy, no matter what the form of the pure strategies (if any) before and after randomizing.

Lemma 1. *Let $\theta : V \mapsto [0, 1]$ be a mixed Nash equilibrium strategy profile involving an interval $[w, a] \subseteq (r_2, 1]$, where $\theta(v) = \theta_m(v) \in (0, 1)$ for a buyer with valuation $v \in [w, a]$ and $\theta(v) = \theta^*(v) \in \{0, 1\}$ for $v \in (a, 1]$. Then $\theta_m(\cdot)$ satisfies the condition*

$$\begin{aligned} &[2F(v) + (n-2)(1 - F(v))]\theta_m(v) = \\ &= (n-2)\left[\int_v^a \theta_m(y)f(y)dy + \int_a^1 \theta^*(y)dy\right] + F(v) \end{aligned} \tag{18}$$

Proof. Suppose that buyers follow a pure strategy $\theta(v) = \theta_p(v)$ for all $v \in [r_2, w)$, then follow a mixed strategy $\theta(v) = \theta_m(v) \in (0, 1)$ for all $v \in [w, a]$ and then follow again a pure strategy $\theta(v) = \theta^*(v)$ for all $v \in (a, 1]$, i.e. the selection strategy involves an interval $[w, a]$ where buyers randomize between the two intermediary auctions. Then, for the existence of a MSNE, $\Pi_1(v) - \Pi_2(v)$ as well as all of its higher order derivatives should be zero for all $v \in [w, a]$. Under this assumption, the second order derivative of the utility difference for a buyer with valuation υ in $[w, a]$ can be written as (see [1] for the derivation):

$$\frac{\partial^2(\Pi_1(\upsilon) - \Pi_2(\upsilon))}{\partial \upsilon^2} = (n-1)F^{n-3}(\upsilon)f(\upsilon)\Big\{[2F(\upsilon) + (n-2)(1-F(\upsilon))]$$
$$\theta_m(\upsilon) - F(\upsilon) - (n-2)[\int_{\upsilon}^{a} \theta_m(y)f(y)dy + \int_{a}^{1} \theta^*(y)dy]\Big\} \quad (19)$$

where setting $\frac{\partial^2(\Pi_1(\upsilon)-\Pi_2(\upsilon))}{\partial \upsilon^2} = 0$ gives the condition of (18). $\square$

We will now show that there can only be a single cut-off point, $w \in (r_2, 1]$, before and at most a single cut-off point, $a \in (w, 1]$, after randomizing, where w, a are such that $\Pi_1(w) = \Pi_2(w)$, $\Pi_1(a) = \Pi_2(a)$, and $\theta(r_2 \leq \upsilon < w) = 1$, $\theta(w \leq \upsilon \leq a) = \theta_m(\upsilon) \in (0, 1)$, $\theta(a < \upsilon \leq 1) \in \{0, 1\}$.

Lemma 2. *The mixed equilibrium intermediary selection strategy of a buyer can only involve a single cut-off point $w \in (r_2, 1]$ before randomizing and at most a single cut-off point, $a \in (w, 1]$ after randomizing.*

Proof Sketch. As has been mentioned at the proof of Proposition 2, there will be at least a single cut-off point $w \in (r_2, 1]$ so that buyers with valuations in $[r_2, w)$ always select the low-reserve intermediary, as their utility from the low-reserve intermediary is positive, whereas their expected utility from the high-reserve intermediary is arbitrarily close to zero. Suppose there are $\sigma' \geq 2$ such cut-off points $w_1 < w_2 < ... < w_{\sigma'} \in (r_2, 1]$, so that $\theta(\upsilon) = \theta_{p_1} = 1$ for $\upsilon \in [r_2, w_1)$, $\theta(\upsilon) = \theta_{p_2} = 0$ if $\upsilon \in [w_1, w_2)$, and so on, $\theta(\upsilon) = \theta_{p_{\sigma'}} \in \{0, 1\}$ if $\upsilon \in [w_{\sigma'-1}, w_{\sigma'})$. Similarly, we can show that $\theta_{p_{\lambda+1}}$, controls the convexity of $D_\lambda = \Pi_1(\upsilon) - \Pi_2(\upsilon)$ for valuations $\upsilon \in [w_\lambda, w_{\lambda+1})$, $\lambda = 0, ..., \sigma' - 1$ (where we denote $w_0 = r_2$). This means that when $\theta_{p_{\lambda+1}} = 1$, the corresponding utility difference is convex, whereas when $\theta_{p_{\lambda+1}} = 0$, it is concave. Hence, in equilibrium, the utility difference for $\upsilon \in [r_2, w'_\sigma)$ will comprise pairs of non-negative convex intervals and non-positive concave intervals, which cannot happen unless there are discontinuities at the local optima, a fact which is not supported by the first order derivative, so there can only be a single cut-off point w before randomizing.

Similarly, if there are $k' \geq 2$ cut-off points, $a_1 < a_2 < ... < a_{k'} \in (w, 1]$, so that $\theta(\upsilon) = \theta^*_\lambda \in \{0, 1\}$ for valuations $\upsilon \in [a_\lambda, a_{\lambda+1})$, $\Pi_1(a_\lambda) = \Pi_2(a_\lambda)$ and $\theta^*_\lambda \neq \theta^*_{\lambda+1}$ for all $\lambda = 1, ..., k'$, we would then have a series of non-negative convex utility difference intervals and non-positive concave alternating intervals if $\theta^*_1 = 1$, or the opposite when $\theta^*_1 = 0$. This means that there should be discontinuities at the local optima of the corresponding intervals, which is in contrast with the well defined first order derivative of $\Pi_1 - \Pi_2$, and hence there can only be at most a single cut-off point a in $(w, 1]$. The full proof can be found in [1]. $\square$

We have shown that there has to be a single cut-off point, w, before randomizing, and at most a single cut-off point, a, after randomizing. We posit that the equilibrium strategy should, in general, involve exactly one a. Equation (18) is a Volterra integral equation of the second kind. However, solving it requires, in general, knowledge of the distribution function. Hence, the form of $\theta_m(\cdot)$ will depend on our assumptions about the distribution of valuations, parameterized by a. Nevertheless, when $a = 1$, (18) has a solution $\theta_m(\upsilon) = \frac{1}{2}$ for all $\upsilon \geq w$. This strategy is identical to the one proposed by Burguet and Sákovics for two independent auctions. Substituting the proposed $\theta_m(\cdot)$, $\Pi_1(\upsilon) - \Pi_2(\upsilon)$ will have the following form:

$$\Pi_1(\upsilon) - \Pi_2(\upsilon) = \int_{r_1}^{w} F^{n-1}(y)dy + \frac{n-1}{2}(1-F(w))\int_{r_2}^{w} F^{n-2}(y)dy +$$
$$+ r_2[(n-1)F^{n-2}(r_2) - (n-2)F^{n-1}(r_2)] + w[\frac{n-3}{2}F^{n-1}(w) -$$
$$- \frac{n-1}{2}F^{n-2}(w)] + \upsilon[(n-2)F^{n-1}(r_2) - (n-1)F^{n-2}(r_2) -$$
$$- \frac{n-3}{2}F^{n-1}(w) + \frac{n-1}{2}F^{n-2}(w)] \quad (20)$$

For this to be a NE for all $\upsilon \geq w$, w must set both the first-order and zero-order coefficients of this polynomial to zero. Nevertheless, this can only be true for at most a single pair of reserve prices: in the zero-order coefficient, w is uniquely defined by both r_1, r_2, whereas in the first-order it only depends on r_2. So, given that the system of equations is underdefined, w cannot be the solution of both equations for all valid pairs of r_1, r_2. This means that there should be exactly one more cut-off point, $a < 1$.

Given this last result, we have shown that the mixed-equilibrium selection strategy of the buyers will involve three intervals defined by two cut-off points: buyers with valuations in the first interval always select the low-reserve intermediary, buyers with valuations in the middle interval will randomize between the intermediaries with a probability that is given by the solution to (18), and buyers whose valuations lie in the third interval will also follow a pure strategy. We formalize this finding in the following theorem where we additionally prove uniqueness of the resulting equilibrium and also give the conditions for w and a.

Theorem 3. *Let $\theta : V \mapsto [0, 1]$ be a strategy profile where $\theta(\upsilon) = 1$ if $\upsilon \in [r_2, w)$, $\theta(\upsilon) = \theta_m(\upsilon)$ if $\upsilon \in [w, a]$, and $\theta(\upsilon) = \theta^* \in \{0, 1\}$ if $\upsilon \in (a, 1]$, for a buyer with valuation $\upsilon \in [r_2, 1]$, where $\theta_m(\cdot)$ satisfies the condition*

$$[2F(\upsilon) + (n-2)(1-F(\upsilon))]\theta_m(\upsilon) =$$
$$= (n-2)[\int_{\upsilon}^{a} \theta_m(y)f(y)dy + \theta^*(1-F(a))] + F(\upsilon) \quad (21)$$

and w, a are given by:

$$F^{n-2}(w)\int_{r_1}^{w} F^{n-1}(y)dy - F^{n-1}(w)\int_{r_2}^{w} F^{n-2}(y)dy = [(w-r_2)F^{n-2}(w)$$
$$- \int_{r_2}^{w} F^{n-2}(y)dy]F^{n-2}(r_2)[F(r_2) + (n-1)(1-F(r_2))] \quad (22)$$

$$F^{n-2}(w)\{F(w) + (n-1)[1 - F(w) - \int_{w}^{a} \theta_m(y)f(y)dy -$$
$$- \theta^*(1-F(a))]\} = F^{n-2}(r_2)[F(r_2) + (n-1)(1-F(r_2))] \quad (23)$$

Then, $\theta(\cdot)$ is a unique mixed Nash equilibrium strategy profile.

Proof Sketch. (21) can be directly derived from Lemmas 1 and 2, where we have used the fact that there can only be one w and one a. Given that this makes the utility difference a linear function of the valuation, for the existence of a MSNE, both the utility difference and its first order derivative should be zero for all $\upsilon \in [w, a]$. We will apply these conditions at $\upsilon = w$ to get the solutions for w and a. More specifically, FOC at w yields:

$$F^{n-2}(w)[F(w)+\theta^*(n-1)(1-F(a))] = F^{n-2}(w)[2F(w)+(n-1)$$
$$(1-F(w)) - \int_w^a \theta_m(y)f(y)dy)] - F^{n-2}(r_2)[F(r_2)+(n-1)(1-F(r_2))] \tag{24}$$

Then, setting $\Pi_1(w) - \Pi_2(w) = 0$ yields:

$$\int_{r_1}^{w} F^{n-1}(y)dy + (n-1)\int_{r_2}^{w} F^{n-2}(y)dy[1-F(w) - \int_w^a \theta_m(y)f(y)dy-$$
$$-\theta^*(1-F(a))] = (w-r_2)F^{n-2}(r_2)[F(r_2)+(n-1)(1-F(r_2))] \tag{25}$$

The system of (24) and (25) provides the conditions that w and a should jointly satisfy. Eliminating a from these equations, we obtain the following equation for w:

$$F^{n-2}(w)\int_{r_1}^{w} F^{n-1}(y)dy - F^{n-1}(w)\int_{r_2}^{w} F^{n-2}(y)dy = [(w-r_2)F^{n-2}(w)-$$
$$-\int_{r_2}^{w} F^{n-2}(y)dy]F^{n-2}(r_2)[F(r_2)+(n-1)(1-F(r_2))] \tag{26}$$

and then find a by substituting the w found in any of (24) or (25).

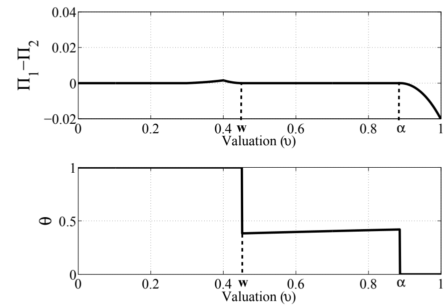
We have reasoned about the existence of these cut-off points (Theorem 2 and equation (20) respectively). Uniqueness of w is proved by taking the first order derivatives of the LHS and RHS in (22) w.r.t. w, where it is easy to see that the former is always lower than the latter, given that $\int_{r_1}^{w} F^{n-1}(y)dy - (w-r_2)F^{n-2}(r_2)[F(r_2)+(n-1)(1-F(r_2))] < 0$, as can be directly derived from (25). Similarly, taking the first order derivatives of the LHS and the RHS in (23) w.r.t. a, we can see that the former is strictly higher (lower) than the latter for $\theta^* = 1$ ($\theta^* = 0$ respectively). Finally, the existence and uniqueness of the solution to (21) is guaranteed as this integral equation can be transformed to a first order linear differential equation with continuous coefficients (Existence and Uniqueness Theorem). The full proof is provided in [1]. $\square$

To gain intuition, two examples of the equilibrium selection strategy when $\theta^* = 0$ and $\theta^* = 1$ for different pairs of reserve prices, a uniform distribution $U(0,1)$ and 5 buyers are given in Figure 1. In this case, the mixed equilibrium selection strategy, $\theta_m(\cdot)$, will have the form:

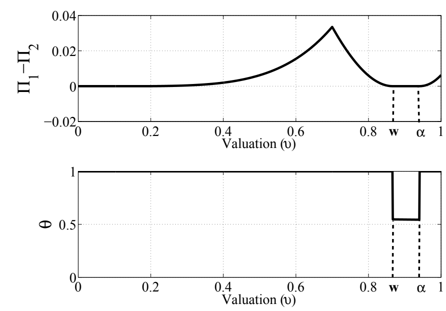
$$\theta_m(v) = \begin{cases} \frac{1}{2} - \frac{(1-2\theta^*)(1-a)}{2\exp(-a)}\exp(-v) & \text{if n=4} \\ \frac{1}{2} + (n-2)\frac{(1-2\theta^*)(1-a)}{2[(n-4)a-(n-2)]^{\frac{n-2}{n-4}}}[(n-4)v-(n-2)]^{\frac{2}{n-4}} & \\ & \text{otherwise} \end{cases} \tag{27}$$

and w, a can be directly derived from (22) and (23).

Figure 2 illustrates two examples showing existence of all NE with $r_1 < r_2$ when valuations are i.i.d. drawn from a uniform distribution $U(0,1)$ when 5 (top) or 10 (bottom) buyers are present. As can be seen, there is a region of PSNEs where all buyers select s_1 (right), followed by a region of MSNEs where buyers with valuations $v \in (a,1]$ always select s_1 but buyers with valuations in $[w,a]$ follow mixed strategies (center). Finally, there is a region where buyers with high valuations ($v \in (a,1]$) always select the high-reserve intermediary, s_2 (left). This means that buyers will only prefer the low-reserve intermediary when the reserve prices are high or when they are sufficiently different, so intermediaries have incentives to increase their reserve prices up to a point, in contrast with the classical setting without intermediaries. Moreover, as can be seen, the region where buyers (always or after mixing) select the low-reserve intermediary shrinks as the number of buyers increases, making the pressure for high reserve prices more apparent.



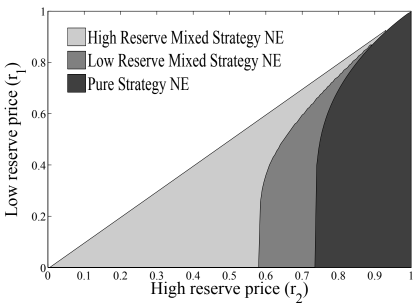
(a) r_1=0.3 and r_2=0.4.



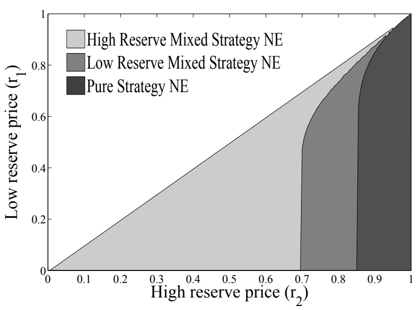
(b) r_1=0.2 and r_2=0.7.

Figure 1: Two examples of the equilibrium strategy, θ, (bottom in each) and the corresponding utility difference, $\Pi_1 - \Pi_2$, (top in each) for the buyers' intermediary selection problem when there are 5 buyers whose valuations are i.i.d. drawn from $U(0,1)$. In the top subfigure, buyers with high valuations select the high-reserve intermediary ($\theta^* = 0$), whereas in the bottom subfigure the opposite happens ($\theta^* = 1$).

As we have shown, the problem of competition between intermediaries is fundamentally different than the classical setting with independent auctions. The form of the equilibrium strategy is significantly more complex and involves two cut-off points, and additionally buyers' randomization between the auctions will be different for different distributions on the valuations assumed. In contrast, for two independent auctions, the equilibrium strategy comprises a single cut-off point and buyers randomize uniformly between the auctions. Moreover, we can see a pressure for the reserve prices to increase, whereas reserve prices have been shown to converge to zero for the case of independent auctions [4]. This is due to the fact that intermediaries participate in the central auction as buyers, thus facing a trade-off between profit maximization and probability of winning at the center. Furthermore, the intermediaries submit to the center their second highest local bid. This triggers different behaviour for the buyers: in order to win the good, the buyer with the highest valuation must now select the same intermediary as the buyer with the second or third highest valuation.



(a) 5 buyers.



(b) 10 buyers.

Figure 2: Two examples of Nash equilibria for the buyers' intermediary selection problem with 5 and 10 buyers respectively whose valuations are i.i.d. drawn from $U(0,1)$ when $r_1 < r_2$. There are three distinct regions for the reserve prices: (i) PSNE: buyers always select the low-reserve intermediary (right), (ii) MSNE: buyers with valuations in $[r_2, w)$ select the low-reserve intermediary, buyers with valuations in $[w, a]$ randomize between the intermediaries, and buyers with valuations in $(a, 1]$ either select the low-reserve (center) or the high-reserve intermediary (left).

5. CONCLUSIONS AND FUTURE WORK

In this paper we have studied, for the first time, the selection problem of buyers in auctions with intermediaries in a Bayesian setting. As we have shown, the buyers' strategy is fairly complex even in the simplest scenario with two intermediaries. More specifically, this problem admits a unique pure symmetric Bayes-Nash equilibrium where all buyers prefer the low-reserve intermediary when the difference between the reserve prices is large enough or when both reserve prices are sufficiently high. If this is not the case, there is a unique mixed-strategy Nash equilibrium involving two cut-off points. In this equilibrium, buyers with valuations lower than the first cut-off point select the low-reserve intermediary, buyers with valuations that are higher than the second cut-off point either select the high-reserve or the low-reserve intermediary (when the reserve prices are low or moderately high respectively), whereas buyers with valuations lying inside this interval follow a strictly mixed strategy.

Our study is a first step towards understanding competition between auctions involving intermediaries and provides useful insights in the design of both bidding strategies and auctions in the display advertising marketplace. Nevertheless, there are many remaining challenges to be addressed. First, we intend to investigate the problem of optimal auction design for the intermediaries and the center. This is an important task in oligopolies, given that a seller's selection of mechanism affects the number and distribution of visiting buyer types, as well as the response of competing sellers, leading to an infinite regress. Primary simulation results for the uniform distribution case show that reserve prices are driven high, as expected, and that intermediaries are unlikely to follow pure reserve price strategies in equilibrium. This is in accordance with the results of Feldman et al. [5]. Second, the complexity of the intermediary selection problem for the buyers and our results for this simple duopoly competition show the need for computational techniques, such as fictitious play or iterative best response, as well as for simplification of the model so as to circumvent issues of tractability in more general settings. Finally, an important area for future research is the study of information asymmetries between buyers (i.e. different intermediaries provide different user information that affects buyers' valuations).

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