Complexity of Manipulation, Bribery, and Campaign Management in Bucklin and Fallback Voting*

(Extended Abstract)

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ABSTRACT

A central theme in computational social choice is to study the extent to which voting systems computationally resist manipulative attacks seeking to influence the outcome of elections, such as manipulation (i.e., strategic voting) [1], control [7, 4], and bribery [5]. Bucklin and fallback voting are among the voting systems with the broadest resistance (i.e., NP-hardness) to control attacks. However, only little is known about their behavior regarding manipulation and bribery attacks. We comprehensively investigate the computational resistance of Bucklin and fallback voting for many of the common manipulation and bribery scenarios; we also complement our discussion by considering several campaign management problems for Bucklin and fallback.

Categories and Subject Descriptors

F.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity;
I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent Systems*

General Terms

Algorithms; Theory

Keywords

Computational social choice; manipulation; bribery; campaign management; Bucklin voting; fallback voting

1. BUCKLIN AND FALLBACK VOTING

An *election* is a pair (C,V), where $C = \{c_1,\dots,c_m\}$ is a set of m candidates and $V = (v_1,\dots,v_n)$ is a list of votes (or ballots) specifying the n voters' preferences over the candidates in C, where the representation of the preferences depends on the voting system used. We allow voters to be weighted, i.e., a nonnegative integer weight w_i is associated with each vote v_i. For example, a vote v_i of a voter with weight $w_i = 3$ is counted as if three voters with unit weight would have cast the same ballot. An unweighted election is the special case of a weighted election where each voter has unit weight. A voting system is a rule for how to determine the winner(s) of a given election. Both Bucklin and fallback voting use the notion of *(weighted) majority threshold in* V, which is defined by $maj(V) = \lfloor W/2 \rfloor + 1$, where $W = \sum_{i=1}^{n} w_i$ is the total weight of the votes in V. In *Bucklin voting (BV)*, votes are linear rankings of all candidates, denoted by, e.g., $c_2 > c_3 > c_1$, meaning that this voter (strictly) prefers c_2 to c_3 and c_3 to c_1. We call the top position in a vote *level* 1, the next position *level* 2, and so on. Starting with the top position and proceeding level by level through the votes in V, we determine the smallest level ℓ such that some candidate(s) occur(s) in at least $maj(V)$ votes up to this level and those occurring most often up to this level ℓ are the *Bucklin winners*. Fallback voting is a hybrid voting system combining Bucklin with approval voting. In *approval voting*, votes in an election (C,V) indicate for each candidate $c \in C$ whether c is approved by this voter or not. Every candidate with the highest approval score is an *approval winner*. In *fallback voting (FV)*, each voter first partitions the set of candidates into the approved ones and the disapproved ones and then provides a linear ranking of the approved candidates. For example, some voter might disapprove of c_1 and c_4, but approve of c_2 and c_3, preferring c_2 to c_3; this vote is denoted by $c_2 > c_3 \mid \{c_1, c_4\}$. To determine the winners, we first try to find the Bucklin winners when they exist. If so, all Bucklin winners are *fallback winners*. However, due to disapprovals there might be no Bucklin winner; in that case all approval winners are *fallback winners*.

2. RESULTS

Manipulation: Extending the work of Bartholdi et al. [1], Conitzer et al. [2] introduced the following decision problem to model manipulation by a coalition of weighted voters. For an election system $\mathscr{E}$, they define $\mathscr{E}$-CONSTRUCTIVE COALITIONAL WEIGHTED MANIPULATION ($\mathscr{E}$-CCWM) to be the problem where we are given a set C of candidates, a list V of nonmanipulative votes over C each having a nonnegative integer weight, where W_V is the list of these weights, a list W_S of the weights of k manipulators in S (whose votes over C are still unspecified) with $V \cap S = \emptyset$, and a designated candidate $c \in C$. We ask whether the votes in S can be set such that c is an $\mathscr{E}$ winner of $(C, V \cup S)$. The unweighted case $\mathscr{E}$-CCUM is the special case of $\mathscr{E}$-CCWM where all voters and manipulators have unit weight. By changing the question to "… such that c is *not* a winner in $(C, V \cup S)$?," we obtain the destructive variants, $\mathscr{E}$-DCWM and $\mathscr{E}$-DCUM. Table 1 gives an overview of our complexity results for manipulation in Bucklin voting ("BV") and fallback voting ("FV"). "P" ("NP-c.") means the problem is solv-

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able in polynomial time (is NP-complete). Note that all our results hold in both the nonunique-winner and the unique-winner model.

$\mathscr{E}$	$\mathscr{E}$-CCUM	$\mathscr{E}$-DCUM	$\mathscr{E}$-CCWM	$\mathscr{E}$-DCWM
BV	P	P	NP-c.	P
FV	P	P	P	P

Table 1: Results for manipulation

Bribery and Campaign Management: $\mathscr{E}$-CONSTRUCTIVE UNWEIGHTED BRIBERY ($\mathscr{E}$-CUB) denotes the standard bribery scenario proposed by Faliszewski et al. [5] where given an $\mathscr{E}$ election (C,V), a designated candidate p, and a nonnegative integer k, we ask whether p can be made a winner by changing the votes of at most k voters. This, again, can be extended by either considering voters with different weights ($\mathscr{E}$-CWB), or allowing that each voter has a different price for changing her vote ($\mathscr{E}$-CUB-\$), or both ($\mathscr{E}$-CWB-\$). By changing the question to ask whether p can be prevented from being a winner of the election by bribing some of the voters, we obtain the destructive variants of these bribery scenarios, which we denote by $\mathscr{E}$-DUB, $\mathscr{E}$-DWB, $\mathscr{E}$-DUB-\$, and $\mathscr{E}$-DWB-\$. The left-hand side of Table 2 shows our bribery results.

Problem	BV	FV	Problem	BV	FV
$\mathscr{E}$-CUB	NP-c.	NP-c.	$\mathscr{E}$-CUSB	NP-c.	NP-c.
$\mathscr{E}$-DUB	P	P	$\mathscr{E}$-DUSB	NP-c.	NP-c.
$\mathscr{E}$-CUB-\$	NP-c.	NP-c.	$\mathscr{E}$-CWSB	NP-c.	NP-c.
$\mathscr{E}$-DUB-\$	P	P	$\mathscr{E}$-DWSB	NP-c.	NP-c.
$\mathscr{E}$-CWB	NP-c.	NP-c.	FV-CUEB	–	P
$\mathscr{E}$-DWB	P	P	FV-DUEB	–	P
$\mathscr{E}$-CWB-\$	NP-c.	NP-c.	FV-CWEB	–	NP-c.
$\mathscr{E}$-DWB-\$	NP-c.	NP-c.	FV-DWEB	–	NP-c.

Table 2: Results for bribery and campaign management

Besides discussing attacks by bribery and manipulation, it is also quite natural to consider bribery scenarios through the lenses of running a political campaign. After all, in a successful campaign, the candidates spend their effort (measured in terms of time, financial cost of organizing promotional activities, and even in terms of the difficulty of convincing particular voters) to change the minds of the voters. Formally, this idea is very close to bribery; indeed, this view of campaign management was first presented in a paper whose focus was on the SWAP BRIBERY problem introduced by Elkind et al. [3]. This problem models a situation where a campaign manager, who is interested in the victory of a given candidate p, can organize meetings with specific voters (the unweighted variant of the problem) or with groups of like-minded voters (the weighted variant) and convince them to change their preference orders. However, the difficulty (or the cost) of changing the voters' preference orders depends both on the voter and on the extent of the change (e.g., it might be expensive to swap a voter's most preferred candidate with her least preferred one, but it might be very cheap to swap her two least preferred candidates). Elkind et al. [3] define so-called *swap-bribery price functions* that for each voter and for each pair of adjacent candidates give the cost of swapping these two candidates in the voter's preference order. In $\mathscr{E}$-CONSTRUCTIVE SWAP BRIBERY ($\mathscr{E}$-CUSB), where $\mathscr{E} \in \{\mathrm{BV},\mathrm{FV}\}$, we ask if there is a sequence of swaps of adjacent candidates that make a given candidate a winner. (Note that the swaps are performed in sequence; even if some candidates are not adjacent at first, they may become adjacent in the course of performing the swaps.) We define the weighted variant of the problem, $\mathscr{E}$-CWSB, and the destructive variants ($\mathscr{E}$-DUSB and $\mathscr{E}$-DWSB) in the standard way (as far as we can tell, the weighted variant of the problem has not been studied before). The definition of swap bribery is very natural for voting rules where each voter ranks all the candidates; for the case of fallback we need to extend the definitions. In our approach, we define swap bribery under fallback to allow the swaps within the approved parts of the votes only. Naturally, one could also define costs for including given disapproved candidates in the approved part as Elkind et al. [3] indeed did for SP-AV (a variant of approval voting). However, following Schlotter et al. [8], we believe that it is more informative to study the complexity of modifying the rankings within the approved parts and the complexity of modifying the sets of approved candidates separately.

The idea of extension bribery is to capture very noninvasive campaign actions, where we try to convince some voters to include the designated candidate at the end of the ranking of approved candidates, and the price for this action is given by the voters' *extension bribery price funtions*. In FV-CONSTRUCTIVE UNWEIGHTED EXTENSION BRIBERY (FV-CUEB), given a fallback election (C,V), a designated candidate p, the voters' extension bribery functions, and a nonnegative integer k, we ask if p can be made a winner by extending the approved parts of the voters' ballots without exceeding the budget k. Again, the weighted variant (FV-CWEB) is defined in the natural way and so are the destructive variants (FV-DUEB and FV-DWEB). The right-hand side of Table 2 summarizes our results for both swap and extension bribery.

Complementing the results regarding control, campaign management, and possible/necessary winner problems, we now have an almost complete picture of the (worst-case) complexity of Bucklin and fallback voting for all the standard election problems. Having this is particularly useful for Bucklin and fallback voting: For control problems they are among the hardest voting rules but for bribery and manipulation they often lay on the edge of (in)tractability. For detailed proofs of the presented results, see [6].

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