Power and Welfare in Noncooperative Bargaining for Coalition Structure Formation

(Extended Abstract)

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ABSTRACT

In this paper, we investigate a noncooperative sequential bargaining game for allowing a group of agents agents to partition themselves into non-overlapping coalitions. We focus on the issue of how a player's position on the bargaining agenda affects his power. We also analyse the relationship between the distribution of the power of individual players, the level of democracy, and the welfare efficiency of the game.

Categories and Subject Descriptors

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Keywords

Bargaining, Coalition structures, Externalities

1. INTRODUCTION

In this paper, we focus on the problem of how a group of agents can partition themselves into a coalition structure through a process of noncooperative bargaining. We assume externalities and non-transferable utilities. We introduce *coalition structure games* (CSGs) that encompass many important classes of coalitional games including hedonic games and NTU (non-transferable utility) games in partition function form. Using a CSG as the underlying game, we investigate a noncooperative bargaining protocol.

The protocol (built on [3, 4, 2]) runs in a series of rounds; the agents take turns to propose an offer, i.e., a coalition structure. An agreement must be reached by a deadline. For this protocol, we provide a quantitative analysis of power and efficiency. We show how a players position on the bargaining agenda influences his bargaining power, and how the distribution of the players' powers influences system efficiency. Our work is an extension of [1].

2. THE MODEL

We begin by defining CSGs and the bargaining game. A coalition structure game is a tuple $\mathbb{G} = \langle N, \succ_1, \ldots, \succ_n \rangle$ where $N = \{1, \ldots, n\}$ is the set of players and $\succ_i \subseteq \Pi(N) \times \Pi(N)$ is a preference relation for each player $i \in N$. Player i's preference for a coalition structure is given by a *rank* ρ_i.

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For a coalition structure game $\mathbb{G}$, we explore the following noncooperative bargaining game (BG) for forming a coalition structure. This is a finite-horizon game in which the players take turns in proposing offers where an offer is a coalition structure. The sequence in which the players are called to make offers is called the *bargaining agenda*. The bargaining agenda ρ is a permutation of the first n integers. The game can run for at most n time periods. Bargaining starts in the first time period. To begin, all the players in N are in the set IN. The set OUT is initially empty. At $t = 1$, mover 1 offers a coalition structure $\pi \in \Pi(IN)$. After an offer is proposed, the game will end with probability δ. With probability $(1-\delta)$ it will continue to the next round when mover 2 will propose an offer and so on. When a player's offer is rejected, he is moved from IN to OUT.

A BG is a 4-tuple $G = \langle N, \succ_1, \ldots, \succ_n, \delta, \rho \rangle$, but, for notational convenience, we will denote it $G(n, \delta, \rho)$, $G(n, \rho)$, $G(n)$, $G(\rho)$, or just G when the other parameters are clear from context. P is the set of all possible preferences combinations for the n players. All other parameters remaining the same, we can obtain different BGs by varying $\succ_1, \ldots, \succ_n$. There are $|P| = ((Bell(n))!)^n$ possible bargaining games. $\mathcal{G}$ is the set of these $|P|$ games.

3. EQUILIBRIUM ANALYSIS

For a BG $G = \langle N, \succ_1, \ldots, \succ_n, \delta, \rho \rangle$, we provide subgame perfect equilibrium strategies and analyse i) how a player's power is related to his position on the agenda and ii) how the distribution of power affects the efficiency of the equilibrium. To this end, we measure a player's power by considering his ability to secure a preferable equilibrium structure:

Definition 1. *Player ρ_i's ($i \in N$) power index over the set of games in $\mathcal{G}$ is defined as:*

$$\mathbb{P}_{\rho_i}(\mathcal{G}) = 1 - \big((\mathbb{E}(\rho_i) - 1)/(Bell(n) - 1)\big);$$

where $\mathbb{E}(\rho_i)$ denotes ρ_i's average expected rank in the equilibrium for the games in $\mathcal{G}$ and defined as follows:

$$\mathbb{E}(\rho_i) = \frac{1}{(Bell(n)!)^n} \sum_{G \in \mathcal{G}} er_{\rho_i}(\pi_1^*(G)).$$

Definition 2. *A welfare maximizing coalition structure is one that minimizes the sum of ranks of the players and is denoted π_{SW}.*

A coalition structure bargained using G may not the be the same as the socially optimal structure π_{SW}. In order to measure how far a bargained structure is from π_{SW}, we define *efficiency ratio*.

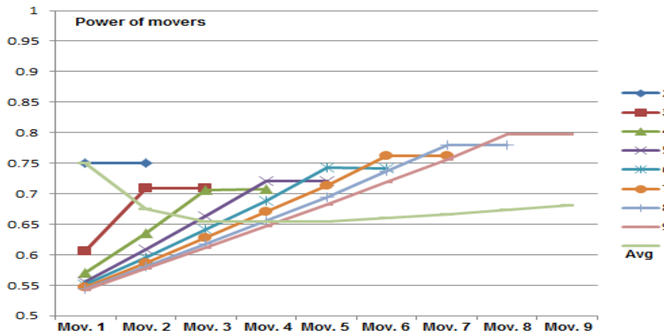

Figure 1: The power in games of different size ($\delta = 0$).

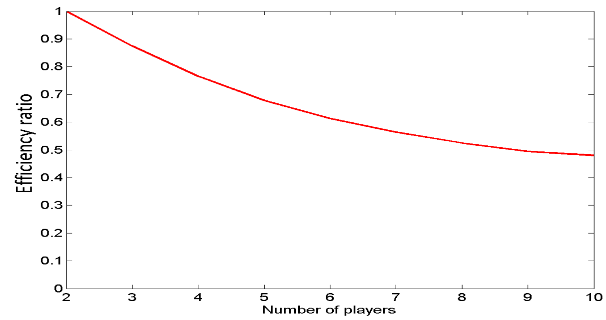

Figure 2: Average efficiency ratios for games with $\delta = 0$.

DEFINITION 3. *For a give n and δ, the efficiency ratio, $\mathcal{E}(G)$ for a game G is the ratio of the sum of the players' ranks for the globally optimal coalition structure for G and the sum of ranks for the bargained structure, i.e., we have:*

$$\mathcal{E}(\mathcal{G}) = \left(\sum_{i=1}^{n} r_i(\pi_{SW})\right) / \left(\sum_{i=1}^{n} er_i(\pi_1^*(G))\right)$$

Since the sum of ranks for a bargained structure can never be lower than that for the globally optimal structure, we have $\mathcal{E}(G) \leq 1$ for any n and δ. Below, we analyse the power and efficiency of our game starting with the description of the simulation setup.

3.1 Simulation Results

The model was implemented and the power and efficiency were measured for a range of games with different n. The results are shown in Figures 1 and 2 for democratic games (i.e., $\delta = 0$).

Power: We can distinguish two types of power: the power to *propose* and the power to *reject*. From the definition of the BG, observe that, for $n \geq 2$, mover 1 has only the power to propose, while mover n has only the power to reject. The following holds:

PROPOSITION 1. *For $n = 2$, ρ_1's power to propose is equal to ρ_2's power to reject.*

Sketch of proof: Consider the number of preference orderings in which both movers can exercise their power. We have only two coalition structures $\pi_1 = \{\{1, 2\}\}$ and $\pi_2 = \{\{1\}, \{2\}\}$ and four possible preference orderings: $(1; 2)_1 = (\pi_1 \succ \pi_2; \pi_1 \succ \pi_2)$, $(1; 2)_2 = (\pi_2 \succ \pi_1; \pi_2 \succ \pi_1)$, $(1; 2)_3 = (\pi_1 \succ \pi_2; \pi_2 \succ \pi_1)$, and $(1; 2)_4 = (\pi_2 \succ \pi_1; \pi_1 \succ \pi_2)$. We have $\mathcal{G} = \{G(N, (1; 2)_1), G(N, (1; 2)_2), G(N, (1; 2)_3), G(N, (1; 2)_4)\}$. There is no conflict for the first two games and for each of these two games, the outcome is a structure that is most preferred by both ρ_1 and ρ_2. For $G(N, (1; 2)_3)$, ρ_2 has the power to reject ρ_1's offer in order to bring about his most preferred structure $\{\{1\}, \{2\}\}$. Thus, the outcome of this game will be $\{\{1\}, \{2\}\}$ which is ranked 1 by ρ_2 and 2 by ρ_1. Here, ρ_1 is powerless. The opposite happens for $G(N, (1; 2)_4)$. Now, ρ_1 will offer his most preferred structure $\{\{1\}, \{2\}\}$ and ρ_2 cannot gain anything by rejecting ρ_1's offer. Thus, ρ_2 has the power to propose. The outcome of this game will be $\{\{1, 2\}\}$ which is ranked 1 by ρ_1 and 2 by ρ_2. We therefore have $\mathbb{E}(\rho_1) = \mathbb{E}(\rho_2) = 5/4$ and $\mathbb{P}_{\rho_1}(\mathcal{G}) = \mathbb{P}_{\rho_2}(\mathcal{G}) = 3/4$.

PROPOSITION 2. *For $n \geq 2$, the last two players ρ_{n-1} and ρ_n have equal power, i.e., $\mathbb{P}_{\rho_{n-1}}(\mathcal{G}) = \mathbb{P}_{\rho_n}(\mathcal{G})$.*

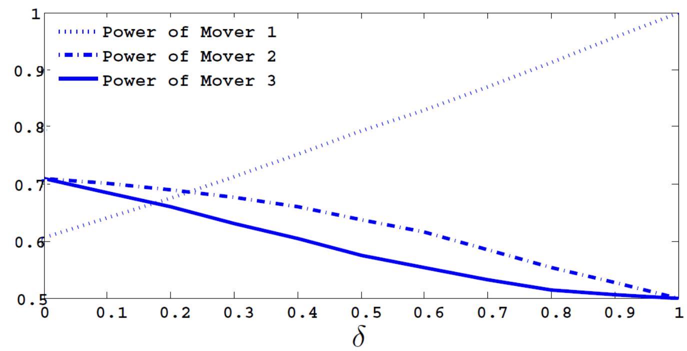

Figure 3: Power and efficiency in G for $n = 3$.

The fact that the ρ_{n-1} and ρ_n have equal power is convenient when computing power in bigger games. Figure 1 presents the results for $n = 2, 3, \ldots, 9$, from which we observe the following:

(a) For all $n \geq 2$, the power of movers n and $n - 1$ is identical (Proposition 2).

(b) For all $n \geq 3$, the power of movers $1 \ldots, n - 2$ increases monotonically, i.e., $\forall_{3 \leq i \leq n-2} \mathbb{P}_{\rho_i}(G) < \mathbb{P}_{\rho_{i+1}}(G)$.

(c) For all $n \geq 2$, the power of the first mover decreases with n, i.e., $\forall_{n \geq 2} \mathbb{P}_{\rho_1}(G(n)) < \mathbb{P}_{\rho_1}(G(n + 1))$.

Efficiency: Figure 2 shows the average efficiency ratios for games of $n = 2, \ldots, 9$ players. We observe that the efficiency ratio is decreasing with growing n. The reasons can be sought in the distribution of power in the game. As visible in Figure 1, with growing n, the discrepancies between the power of agents increase. This means that more and more often powerful agents are able to secure favourable outcomes at the expense of powerless agents—a conflict which results in the overall efficiency loss. Figure 3 shows that power and efficiency for three player games.

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