Signal Structure and Strategic Information Acquisition: Deliberative Auctions with Interdependent Values

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ABSTRACT

The ability to gather information can affect outcomes in auctions and other games of incomplete information. We investigate situations where agents have a choice about which information, or *signals*, to observe, and are informed about the signal choices of others. Our models cover common-value games where agents decide whether to coordinate on observed information, and games mixing private- and common-value components. We find that the dependence structure among available signals can produce qualitatively distinct behaviors in equilibrium, including some cases where strategic agents implicitly collude to acquire less than maximally informative signal combinations.

Categories and Subject Descriptors

J.4 [**Social and Behavioral Sciences**]: Economics

General Terms

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Keywords

Deliberative Auctions; Information Acquisition

1. INTRODUCTION

In planning any non-routine purchase, it is natural to undertake research on the value of the good. When purchasing a used car, research reveals hard information, such as the mileage and the year, as well as signals of the car's quality such as interior condition, the sound of the engine, and rate of acceleration. Learning these attributes will help resolve uncertainty about the value of the car. If information gathering is costly, one can assess the *(expected) value of information* (VoI) to decide which pieces of information to learn.

This information will affect the value of the car to the purchaser and also to others, and if that value can influence the outcome—as in a competitive bidding situation—these effects must also be taken into account. Therefore, in *games of incomplete information*, information gathering is a strategic action, and VoI alone does not provide a reliable guide to information gathering.

Prior work [11] showed that the structure of probabilistic dependence among signals can be related to qualitative properties of game solutions. In a similar spirit, we examine auction games where the agents choose information-gathering actions prior to a bidding stage. For concreteness, we assume two-player second-price sealed-bid (SPSB) auctions, where agents bid simultaneously, the higher bidder wins the good, and pays the lower bidder's price.

We explore two classes of auction games. In the first, we consider a common-value auction where the agents face symmetric information-gathering options, and the key issue is whether they coordinate their information gathering. We call this the *coordination scenario*. Whereas intuition suggests that agents would be better off selecting signals about distinct attributes, we find that under some circumstances they choose to coordinate on the same information. The second, which we call the *private versus common scenario*, features an interdependent-value SPSB auction where agents must decide whether to acquire private information about their value, or common information about both agents' value. This scenario highlights the tradeoff between choosing to observe the attribute that has the higher VoI versus choosing to observe the attribute that generates a large strategic advantage.

2. PRIOR LITERATURE

In *deliberative* auction models, agents can acquire information about valuations prior to bidding. Deliberation covers any actions that modify the agent's beliefs, including sensing the world, purchasing data, or computing. Thompson and Leyton-Brown [8] investigate deliberation strategies for several auction games where agents have *independent private values* (IPV). The authors also provide a useful taxonomy classifying literature on mechanisms with deliberation. This paper presents a novel combination of attributes in terms of that taxonomy.

Thompson and Leyton-Brown [9] showed in an IPV setting that the only dominant-strategy mechanism is a sequential posted price (SPP) auction, in which bidders are sequentially given take-it-or-leave-it prices until the good is sold. Celis et al. [2] validated that SPP auctions have dominant strategies in many deliberative settings, and provided an efficient mechanism to get within a small factor of the optimal revenue. Larson and Sandholm [4] describe *strategic deliberation*, in which agents can gather information about the preferences of other agents. They show that it is impossible

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to create a strategy-dependent mechanism in which: (1) the mechanism does not deliberate for the agents, (2) agents do not strategically deliberate in equilibrium, and (3) in equilibrium one agent cannot believe another's true preferences are impossible.

Several works do not address deliberation per se, but offer results on information asymmetry in common-value auctions that are relevant for extensions to endogenous information-gathering. Abraham et al. [1] develop a new equilibrium concept that helps overcome the problem of equilibria multiplicity, particularly those that arise in asymmetric second price auctions. They use this equilibrium concept to study common-value auctions with information asymmetry and find surprising results relevant to ad auctions. Syrgkanis et al. [7] study information asymmetry in a two-player common-value hybrid first/second-price auction. The authors get around the equilibrium selection problem of second-price auctions by examining limiting behavior as the auction approaches second price. The result is significant freeloading by uninformed agents.

In light of this prior work, our new contributions to deliberative auctions are to accommodate interdependent values, and to consider the implications on information-gathering strategies of alternative dependence relations among signals available to the agents.

3. COORDINATION SCENARIO

In the first scenario we investigate, two agents choose between two signals they could observe, each providing information relevant to a distinct attribute of the underlying state. Given the signals provide equivalent quality of information, the question is whether the agents would (or should) decide to observe signals corresponding to the *same*, or *different* state attributes. That is, will the agents *coordinate*, or *anti-coordinate* in information gathering?

As an example, consider the auction of extraction rights for some resources (say oil and gas) on a specified plot of land. The value to energy companies of these rights depends on the unknown amounts of extractable resources, and the agents may have a variety of means to gather information about this. Suppose—to make the example stark—that a company has sufficient time to research only one of the resource types. Given that it can assess the reserves of oil or gas, but not both, which should it choose? Should competing companies choose to investigate the same resource type, or different types?

3.1 Model Description

We explore an abstract instance of the coordination question in a *common-value* auction setting [3], where agents have the same fundamental value for the good up for bid, but uncertain information about this value. The good's value, v, depends on an underlying state ω, which is defined by two components, or *attributes*, $\omega = \langle \omega_1, \omega_2 \rangle$. Each attribute is associated with signals potentially observed by the respective agents. We represent the signal/value dependence structure with a graphical model, shown in Figure 1.

For simplicity we assume that the state attributes and signals are binary random variables. The attributes may take on values *Good* (G) or *Bad* (B), and the signals may take on values *High* (H) or *Low* (L). All signals have an *accuracy* of $a \in (1/2, 1)$, which is defined as the probability of getting a *High* signal from a *Good* attribute or a *Low* signal from a

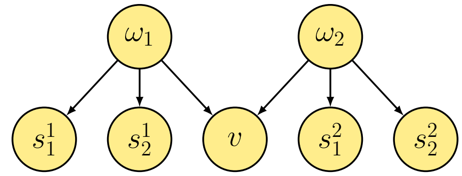

Figure 1: Graphical model of the coordination scenario information structure. The common valuation is determined by two independent state attributes (ω_1, ω_2). Variable s_i^j is agent i's *potential* signal from attribute j.

Bad attribute. When both agents choose a signal from the same attribute, they observe conditionally independent signal values. Without loss of generality, we scale the good's value to $[0, 1]$. We assume that bid prices are commensurately scaled, so that overall utility to the winning bidder equals value minus payment. The value of the good is zero if neither state attribute is *Good*, one if both are *Good*, and $g \in (0, 1)$ if only one is *Good*. The parameter g captures the degree of substitutability between *Good* realizations. $g = 1$, the attributes are perfect substitutes, since valuation is maximal as long as one attribute is *Good*. Conversely, if $g = 0$, both attributes must be *Good* for the good to have any value, hence the attributes are perfect complements.

Formally, we describe the value model as follows:

$$
\begin{aligned}
j &\in \{1, 2\} \\
\omega_j &\in \{G, B\} \\
\Pr[\omega_j = G] &= 1/2 \\
v(\omega) &= \begin{cases} 0 & \text{if } \sum_j \mathbb{I}\{\omega_j = G\} = 0 \\ g & \text{if } \sum_j \mathbb{I}\{\omega_j = G\} = 1 \\ 1 & \text{if } \sum_j \mathbb{I}\{\omega_j = G\} = 2 \end{cases}
\end{aligned}
$$

where $\mathbb{I}$ is the indicator function. For $i, j \in \{1, 2\}$, the signal model is given by

$$
\begin{aligned}
s_i^j &\in \{H, L\} \\
\Pr[s_i^j = H \mid \omega_j = G] &= \Pr[s_i^j = L \mid \omega_j = B] = a,
\end{aligned}
$$

where s_i^j is agent i's signal from attribute j.

In our games (Figure 2), each agent first chooses at most one signal to observe. Then the agents observe their own signals (generated independently from the specified distribution) and find out what signal their opponent chose (but not its realization). Finally the agents play an SPSB auction to allocate the good and determine payment. An agent's utility is zero if it loses the auction, and the good's value minus the payment if it wins.

To simplify analysis throughout this paper, we invoke the following.

Assumption 1. *Agents play only weakly undominated strategies.*

This assumption rules out implausible or uninteresting equilibria, such as when one agent bids nothing (essentially opting out of participating), and the other agent bids some very high amount to get the good for free.

Assumption 2. *Agents play only equilibrium bids in each bidding subgame.*

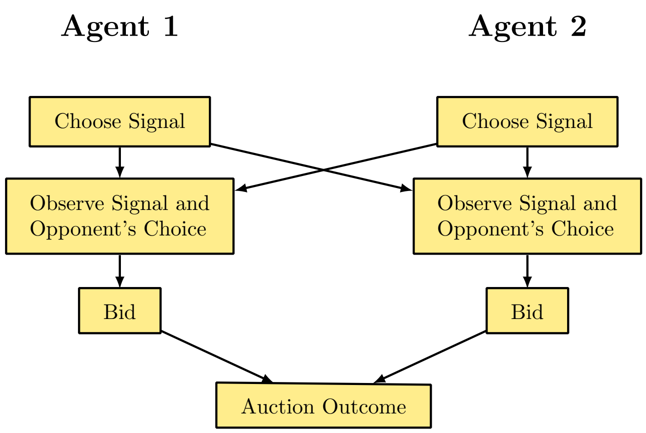

Figure 2: Two-stage game: simultaneous information gathering and observation, followed by SPSB auction.

This assumption forces a focus on subgame perfect equilibria of the full game.

3.2 Equilibria

Our prior intuition was that agents would choose *different* signals in this scenario. Observing different signals reveals the most information about the valuation upon combining both signals. Further, by observing different signals, the agents maximize the probability of observing different signal values, which in symmetric bidding equilibrium is the only way to earn positive utility from an SPSB auction.

Our analysis starts by deriving subgame bidding equilibria for every combination of information-gathering actions. With symmetry, there are four distinct bidding subgames, where: both agents choose not to observe a signal, one agent chooses to observe a signal, two agents observe the same signal, and two agents observe different signals. We then fold back the game tree by employing the expected utilities of subgame equilibria to construct a normal-form representation of the information-gathering stage[1]

Due to the nature of a SPSB mechanism, any bid above the maximum expected value for any possible signal realization is weakly dominated by a bid of the maximum expected value, because every situation where only the higher bid would win will always result in a loss. Any bid below the minimum expected value for any signal realization is weakly dominated by the minimum expected value by similar reasoning. Given the fact that each agent only observes a single binary signal and Assumption 1, each agent's bid must fall in the range from its expected value given its signal and a *Low* opponent signal, to its expected value given its signal and a *High* opponent signal. If neither agent observes a signal, this collapses to each agent biding their prior expected value as the only weakly undominated strategy. In this setting, each agent will bid identically and make no profit.

If only one agent observes a signal, then the weakly dominant strategy for the observing agent is to bid its expected value conditioned on its observed signal. However, because the observing agent acquires some information, there is a range of weakly undominated bids for the unobserving agent. Assumption 1 restricts the unobserving agent's bid to the range from $\mathbb{E}[v \mid s_{-i} = L]$ to $\mathbb{E}[v \mid s_{-i} = H]$. Whenever the unobserving agent wins, it will pay its expected value and never make a profit. Therefore every weakly undominated bid also constitutes an equilibrium bid. If the unobserving agent bids its expected value given its opponent saw a *High* signal, then the observing agent will either lose or win and pay the good's value, and therefore make no profit.

If both agents observe the same signal, than the assumption of weakly undominated bidding strategies restricts an agent's *Low* signal bid to be between $\mathbb{E}[v \mid s_i^j = L, s_{-i}^j = L]$ and $\mathbb{E}[v \mid s_i^j = L, s_{-i}^j = H]$, and restricts its *High* signal bid to be between $\mathbb{E}[v \mid s_i^j = H, s_{-i}^j = L]$ and $\mathbb{E}[v \mid s_i^j = H, s_{-i}^j = H]$. The only equilibrium in these ranges corresponds to the famous result of [6] that a symmetric equilibrium for SPSB with interdependent values is for each agent to bid its expected value, conditional on the most favorable opponent signal being equal to its own.[2][3] To see that it is the only equilibrium in weakly undominated strategies, suppose that one agent deviates to a point in the interior of one or both ranges. The range bounds and symmetry of signals ensure that its bid given a *High* observation is at least that of its bid given *Low*. The agent's opponent now can gain advantage by slightly underbidding it in the *Low* case, and/or slightly overbidding it in the *High* case. The agent's bid is clearly worse than matching the opponent, and so not in equilibrium. Indeed, the only stable point lies at the extremes of these ranges, as concluded above. Since agents bid identically and equal to their expected value if they observe the same signal value, the only time an agent makes profit is when it observes a *High* signal and its opponent observes a *Low* signal.

If the agents observe different attributes, then the possible bids are restricted to the same ranges, and due to equal signal accuracy the situation is analogous to that above. Thus the unique solution bids are the expected valuation conditioned on the opponent observing the same signal value. An agent similarly only profits when it observes a *High* signal and its opponent observes a *Low* signal.

Let u_{xy}, $x, y \in \{\varnothing, 1, 2\}$, denote the expected utility for an agent that chooses to observe attribute x when its opponent chooses attribute y. The equilibrium bidding strategies produce the expected utilities and corresponding information-stage normal-form game presented in Table 1[4] The bound on N is due to the range of equilibrium bids for the unobserving agent when only one agent observes a signal.

Proposition 1. *The pure-strategy Nash equilibrium (PSNE) where neither agent acquires information is always an equilibrium in the coordination scenario.*

Proof. Suppose that in the case when only one agent observes, the unobserving agent bids its expected value conditioned on its opponent getting a *High* signal. This is a possible equilibrium behavior for that subgame, and the

[1] Full symbolic derivations are included in Appendix A.1 at `http://hdl.handle.net/2027.42/102737`

[2] See also discussions of this setting by Menezes and Monteiro [5, Theorem 5], Krishna [3, Section 6.2], and Wellman [10, Section 3.3.3.1].

[3] We do not assume symmetry as a constraint; for this game all non-symmetric equilibria are eliminated by Assumption 1.

[4] Full derivations of equilibrium utilities are included in Appendix A.2 at `http://hdl.handle.net/2027.42/102737`

Table 1: Coordination scenario information normal-form game.

		Agent 2 ∅	Agent 2 1	Agent 2 2
Agent 1	∅	$(0,0)$	$(0,N)$	$(0,N)$
Agent 1	1	$(N,0)$	(S,S)	(D,D)
Agent 1	2	$(N,0)$	(D,D)	(S,S)

$$u_{\varnothing\varnothing}, u_{\varnothing 1}, u_{\varnothing 2} = 0$$
$$N \equiv u_{1\varnothing}, u_{2\varnothing} \le \frac{1}{4}(2a-1)$$
$$S \equiv u_{11}, u_{22} = \frac{1}{4}(2a-1)\frac{a(1-a)}{a^2+(1-a)^2}$$
$$D \equiv u_{12}, u_{21} = \frac{1}{4}(2a-1)[(1-a)(1-g)+ag]$$

equilibrium profit for the observing agent is zero in such situations, or in other words, every N in Table 1 takes value zero. Inspection of the game matrix thus reveals there is no beneficial deviation from the strategy profile where neither agent acquires information. □

Since payoffs are identically zero in this equilibrium, we classify it as degenerate and focus on information-gathering equilibria.

Proposition 2. *If* $a < \frac{\sqrt{1-g}(\sqrt{1-g}-\sqrt{g})}{1-2g}$, *then coordination on the same attribute is the only information-gathering behavior supported in PSNE. If the inequality is reversed, then specialization on distinct attributes (anti-coordination) is the only PSNE information-gathering behavior.*

Proof. From Table 1, if an agent does not observe a signal, it obtains zero utility. Since S and D are strictly positive, deviating to observing when the opponent observes information is beneficial. From the definitions of S and D, if $a < \frac{\sqrt{1-g}(\sqrt{1-g}-\sqrt{g})}{1-2g}$ then $S > D$, and therefore deviation from choosing different signals to choosing the same signal is beneficial. If the inequality is reversed, then $D > S$ and deviating to both agents choosing different signals is beneficial. □

Note that the inequality in Proposition 2 can hold only if $g < 1/2$, that is, if the attributes are complements. Figure 3 depicts the information-gathering pure-strategy equilibrium regions characterized by Proposition 2. The possibility of same-signal equilibrium contradicts our initial intuition about the problem. When the attributes are sufficiently complementary, and the signals are noisy, the agents choose to observe the same attribute. With low g and a, getting information about just one of the attributes provides only weak information about value. Under these conditions, agents may gain by implicitly colluding to get limited joint information. With a significant chance (better at low accuracy) that one agent observes a true positive and the other a false negative, the same-signal profile provides reasonable expected utility. Thus, while both signals have the same value of information, the relative strategic value between choosing the same or different signals changes according to game parameters.

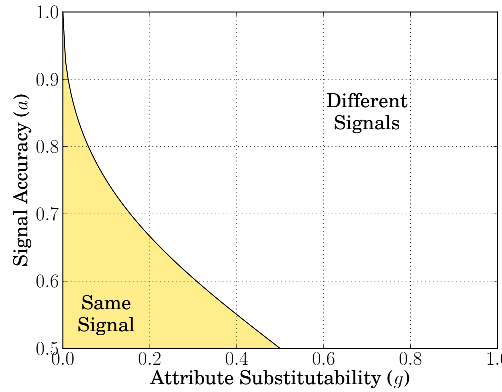

Figure 3: In the coordination scenario, agents in equilibrium observe the same signal when signal accuracy is low and the attributes are complementary. When accuracy is high or the signals are substitutable, agents observe different signals. Choosing not to acquire information is an equilibrium for all game parameters.

4. PRIVATE VS. COMMON SCENARIO

The second scenario features both private and common factors bearing on valuation. Bidding agents decide between two signals to observe: one that reveals information about an attribute affecting only their own valuation, and one that reveals information about an attribute affecting both agents' valuations. In what follows, we derive conditions under which the agents will gather private information and when they will choose to coordinate on the common information.

As an example, again consider an auction of extraction rights for some resources (say oil), except that in this scenario, the energy companies have specialized abilities to deploy their individual drilling technologies that may help them extract the oil more cheaply. The value to the energy companies of these rights depends on the unknown amount of oil available, and their special efficiency in extracting it. Each agent may have several possible actions they could take to gather information about either aspect of the total value. Suppose—to illustrate our point—that a company has time to research either the amount of oil available, or the efficiency of their specialized drilling technologies. Should competing companies investigate the common amount of oil, or their private drilling capabilities?

4.1 Model Description

The private versus common scenario shares several elements with the coordination scenario. The good's value depends on an underlying state ω, which in this case is defined by three attributes: $\omega = \langle \omega_0, \omega_1, \omega_2 \rangle$. The common attribute ω_0 affects both agents' valuations, whereas private attributes ω_1 and ω_2 affect only their respective agent's valuation. An agent cannot observe a signal from its opponent's private attribute. We refer to the signal about an agent's private attribute as its private signal, and similarly for the common attribute. This signal/value dependence is expressed by the graphical model of Figure 4.

As with the coordination scenario, we assume that all of the state attributes are binary random variables. The at-

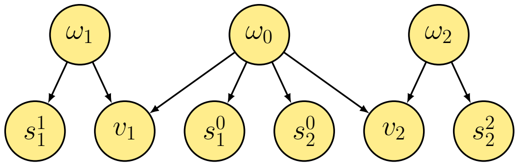

Figure 4: Graphical model of the private versus common scenario information structure. An agent's value (v_i) is a function of its private state attribute (ω_i) and the common state attribute (ω_0). s_i^j is agent i's potential signal from attribute j.

tributes are either *Good* (G) or *Bad* (B), and the signals are *High* (H) or *Low* (L). We scale the good's value to the range $[0, 1]$. An agent's value is zero if neither relevant state attribute is *Good*, one if both are *Good*, $c \in (0, 1)$ if only the common attribute is *Good*, and $p \in (0, 1)$ if only the private attribute is *Good*. All signals have an *accuracy* of $a \in (1/2, 1)$, which is defined as the probability of getting a *High* signal from a *Good* attribute or a *Low* signal from a *Bad* attribute. Section 4.5 discusses a relaxation where the common and private signals have different accuracies.

In what follows, we characterize $c - p \in (-1, 1)$ as the *degree of commonality* of the auction. As $c - p$ approaches 1, the private attribute vanishes in importance, resulting in an essentially common-value auction. As $c - p$ approaches -1, the situation approaches IPV. Furthermore, if the degree of commonality is negative then the private signal has a higher VoI, and if it is positive then the common signal has a higher VoI. See Section 4.4 for further details about Value of Information.

Formally we describe the value model as

$$
\begin{aligned}
i &\in \{1, 2\} \\
j &\in \{0, 1, 2\} \\
\omega_j &\in \{G, B\} \\
\Pr[\omega_j = G] &= 1/2 \\
v_i(\omega_0, \omega_i) &= \begin{cases} 0 & \text{if } \omega_0 = B, \omega_i = B \\ p & \text{if } \omega_0 = B, \omega_i = G \\ c & \text{if } \omega_0 = G, \omega_i = B \\ 1 & \text{if } \omega_0 = G, \omega_i = G \end{cases}
\end{aligned}
$$

For the corresponding $j \in \{0, i\}$, the signal model is given by

$$
\begin{aligned}
s_i^j &\in \{H, L\} \\
\Pr[s_i^j = H \mid \omega_j = G] &= \Pr[s_i^j = L \mid \omega_j = B] = a.
\end{aligned}
$$

The game proceeds as in the coordination scenario (Figure 2). Each agent chooses which signal to observe, if any. Then the agents observe their own signals (generated independently from the corresponding distribution) and their opponent's choice (but not its realization). The agents then play an SPSB auction.

4.2 Equilibria

As in the coordination scenario, we derive bidding equilibria and the corresponding expected utilities for every combination of information-gathering actions, then fold back the game tree by employing these expected utilities to construct a normal-form representation of the information-gathering game. There are six distinct bidding subgames. For brevity, we refer to an information-gathering strategy profile by the two symbols that describe the chosen signals (e.g., in profile PP both agents choose their private signal)[5]

The expected value forms of the equilibrium bidding strategies from the coordination scenario also apply to subgames of the private versus common scenario where agents do not observe their private signal. When one of the attributes is marginalized out of the coordination scenario and both private attributes are marginalized out of the private versus common scenario, the only difference is the value function, which only differs by scaling.

If neither agent observes the common signal, then an opponent's bid is independent of its valuation, and an agent's weakly dominant strategy is to bid its expected value conditioned on its signal.

If one agent observes the common signal and the other its private signal, the common-signal agent's value is independent of the private-signal agent's bid, and therefore its only weakly undominated strategy is for it to bid its expected value conditioned on its signal. Assumption 1 limits the private-signal agents *Low* signal bid to be in the range from $\mathbb{E}[v \mid s_P^P = L, s_C^C = L]$ to $\mathbb{E}[v \mid s_P^P = L, s_C^C = H]$, and limits its *High* signal bid to be in the range from $\mathbb{E}[v \mid s_P^P = H, s_C^C = L]$ to $\mathbb{E}[v \mid s_P^P = H, s_C^C = H]$. Note that not all of these are valid equilibrium bids. Since the signals are always informative ($a > 1/2$), when the private-signal agent observes a low signal, its expected value is always less than the common-signal agent's value, and the private-signal agent should always underbid on a *Low* signal. Similarly, the private-signal agent should always overbid on a *High* signal. The common-signal agent will win and make a profit whenever the private-signal agent observes a *Low* signal, and conversely the private-signal agent will win and make a profit every time it observes a *High* signal.

Let u_{xy}, $x, y \in \{\varnothing, P, C\}$, denote the expected utility for an agent that chooses to observe attribute x when its opponent chooses attribute y. The aforementioned combinations of equilibrium bidding strategies yield the expected utilities and resulting information stage normal-form game in Table 2[6] The bounds on D are a combination of restricting bidding to weakly undominated strategies (Assumption 1) and the bounds necessary to ensure an equilibrium bid.

Proposition 3. *Both agents choosing the common signal is the only pure-strategy equilibrium if $c - p > 1 - 4a(1 - a)$.*

Proof. From Table 2, $c - p > 1 - 4a(1 - a)$ implies $C > P$ and $D > P$. $C > P$ implies the strategy profile where both agents choose the common signal has no beneficial deviations. Since $D > P$, every other strategy profile has at least one beneficial deviation. □

Proposition 4. *Strategy profiles where one agent chooses the common signal and the other chooses its private signal are the only pure-strategy equilibria when $c - p > 0$ and $c - p < 1 - 4a(1 - a)$.*

[5]Full derivations are included in Appendix B.1 at `http://hdl.handle.net/2027.42/102737`

[6]Full derivations of the expected subgame utilities are included in Appendix B.2 at `http://hdl.handle.net/2027.42/102737`

Table 2: Private versus common scenario information normal-form game.

		Agent 2		
		$\varnothing$	P	C
Agent 1	$\varnothing$	$(0,0)$	(P,P)	$(0,N)$
	P	(P,P)	(P,P)	(P,D)
	C	$(N,0)$	(D,P)	(C,C)

$$u_{\varnothing\varnothing}, u_{\varnothing C} = 0$$

$$N \equiv u_{C\varnothing} \leq \frac{1}{4}(2a-1)(c-p+1)$$

$$C \equiv u_{CC} = \frac{1}{4}(2a-1)\frac{a(1-a)}{a^2+(1-a)^2}(c-p+1)$$

$$D \equiv u_{CP} \leq \frac{1}{8}(2a-1)^2(c+p-1) + \frac{1}{4}(2a-1)$$

$$u_{CP} \geq \frac{1}{8}(1+c+p) - \frac{1}{2}(a^2c + (1-a)^2p + a(1-a))$$

$$u_{CP} > \frac{1}{8}(2a-1)(c-p+1)$$

$$P \equiv u_{PP}, u_{PC}, u_{P\varnothing}, u_{\varnothing P} = \frac{1}{8}(2a-1)(p-c+1)$$

Proof. From Table 2, $c-p < 1-4a(1-a)$ implies $P > C$; thus the private-signal agent has no beneficial deviations from the PC profile. $c-p > 0$ implies $D > P$ and therefore the common-signal agent has no beneficial deviations from the PC profile. Table 2 rules out all other pure-strategy equilibria. □

Proposition 5. *Strategy profiles where at least one agent chooses its private signal are the only pure-strategy equilibria when* $c-p < 0$.

Proof. There are three information-gathering profiles where at least one agent observes its private signal: PC, PP, and P∅. Suppose that in the PC subgame the private-signal agent bids sufficiently low for its *Low* bid, such that the expected utility of the common-signal agent is always greater than the expected utility of the private-signal agent; in other words $D \geq P$. From Table 2, this is an equilibrium bid if $c-p < 0$. Further $c-p < 0$ implies $C < P$. Inspection reveals that there are no beneficial deviations given this strategy.

Now suppose in every PC subgame the private-signal agent bids sufficiently high for its *Low* bid, such that the expected utility of the common-signal agent is always less than the expected utility of the private-signal agent. Table 2 again ensures that this is a possible equilibrium of the subgame. Given this strategy, there are no beneficial deviations from the PP profile.

Suppose the above is true, and that the unobserving agent in the C∅ profile always bids its expected value conditioned on its opponent getting a *High* signal. The subgame profit for the common-signal agent is zero in this case; in other words every N in Table 2 takes value zero. Inspection reveals that there are no beneficial deviations from the P∅ profile.

By inspection, every other information-gathering profile has a beneficial deviation. □

Figure 5 depicts the pure-strategy equilibria regions characterized by Propositions 3–5. The existence of the CC equilibrium logically resembles the *same* equilibria in the coordination scenario. When the signals are noisy, it is favorable for the agents to implicitly collude on an uninformative signal (in terms of which agent has a higher valuation), because there is a high chance that the opponent will significantly underbid the true value, resulting in large expected utility. The implicit collusion results in the only allocatively inefficient equilibrium in this game, details follow in Section 4.3. The boundary between the CC and PC equilibria in Figure 5 highlights the tension between the informativeness of a signal and its strategic value in context. When the degree of commonality is positive, the common signal has higher VoI, but as the accuracy increases the strategic value of the common signal decreases to the point where CC is no longer an equilibrium.

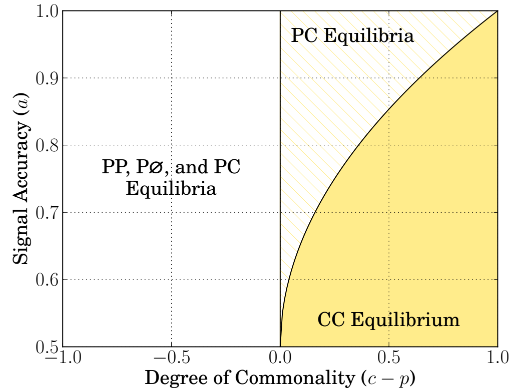

Figure 5: Private versus common equilibria as a function of game parameters. Higher degree of commonality means the common attribute has greater relative influence on valuation. As $c-p \to 1$, the game approaches common value.

4.3 Efficiency

Allocative efficiency measures how close the social welfare of a mechanism outcome approximates the maximum possible. In auctions the social welfare is traditionally the total of all players' utility including the seller, which—since the payments incurred and received cancel out—is simply the expected value of the good to the winner. The allocative efficiency of an auction is the ratio between the expected value to the winner in equilibrium and the expected maximal valuation among auction participants. We assess expectations conditional on knowledge of all the signals, whether or not they are observed by players in the game.

Observing the common signal reveals no information about which agent values the good more. Therefore, when both agents either observe the common signal or nothing, the good is randomly assigned, and the winner's resulting value is simply the expected value of the good. Consider the situation when an agent observes its private signal. If it gets a *High* signal, then the other private signal at best ties, so regardless of the other signals allocating the good to this agent maximizes expected social welfare. Similarly, if the agent gets a *Low* signal, allocating to the other agent is the social optimum. In all equilibrium settings where at least one agent observes its private signal, the good is indeed allocated in this way. Therefore, profiles with no private signal

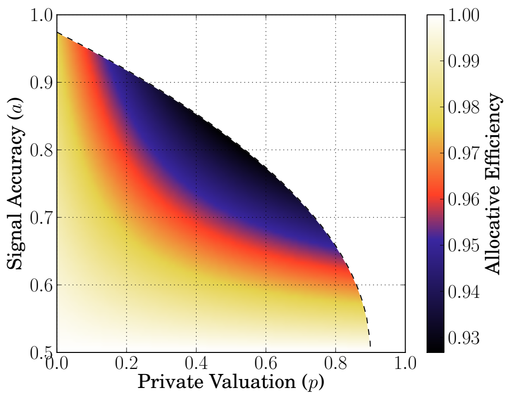

Figure 6: Private versus common scenario allocative efficiency of the CC equilibrium when $c = 0.9$. The CC equilibrium does not exist in the white area above the dashed curve.

observation are maximally inefficient (assigning the good at random), and profiles where at least one agent observes its private signal are perfectly efficient (always assigning the good to an agent with greatest expected value).

The maximum social welfare is therefore equal to the expected value of the winner in the subgame where one agent observes its private signal, and the other agent observes nothing[7]

$$\text{CC Efficiency} = \frac{2c + 2(p+1)}{(3-2a)c + (1+2a)(p+1)}$$

The degree of commonality along with accuracy do not uniquely determine CC equilibrium efficiency, therefore we plot in Figure 6 a slice of the CC equilibrium efficiency for $c = 0.9$.

4.4 Value of Information

In many domains, assessing the value of alternative information sources is an efficient way to make decisions about what signals to acquire. In strategic settings the VoI of available signals is not trivial to calculate, as it generally depends on the behavior of other agents [8].

Let us define the *single-agent* (or *non-strategic*) *VoI* of a signal in this scenario as the expected increase in profit after acquiring the information when bidding against an agent that has no information and bids some fixed value. For example, if the opponent's bid is the prior expected value of the good, the single-agent VoI would be $1/2(\mathbb{E}[v \mid H] - \mathbb{E}[v])$. Half the time the agent would observe a high signal and make the difference between posterior and prior profit; half the time it would observe the low signal and lose the auction. Not acquiring the information yields zero profit, because the agent's bid would equal the opponent's bid.

Proposition 6. *If the* degree of commonality *is positive, then for any value of the opponent's bid, the single-agent VoI for the common signal is greater than or equal to that of the private signal. If the degree of commonality is negative, then the single-agent VoI for the common signal is less than or equal to that of the private signal.*

Proof. A positive degree of commonality implies $\mathbb{E}[v \mid s^C = H] > \mathbb{E}[v \mid s^P = H]$ and $\mathbb{E}[v \mid s^C = L] < \mathbb{E}[v \mid s^P = L]$. The opponent's bid ($b$) can be broken into five distinct regions.

$\mathbb{E}[v \mid s^C = H] \leq b$**:** The agent can never make any profit.

$\mathbb{E}[v \mid s^C = H] > b \geq \mathbb{E}[v \mid s^P = H]$**:** A *High* common signal will provide enough information to make a profit, but the private signal will never provide enough information to make winning the good worthwhile.

$\mathbb{E}[v \mid s^P = H] > b \geq \mathbb{E}[v \mid s^P = L]$**:** Observing the common signal over the private signal will result in more profit from a *High* signal, and none from a *Low* signal.

$\mathbb{E}[v \mid s^P = L] > b \geq \mathbb{E}[v \mid s^C = L]$**:** The common signal will result in more profit from seeing a *High* signal, then the sum of profits that private signal can make. The common signal bidder avoids buying a low quality good.

$\mathbb{E}[v \mid s^C = L] > b <$**:** Both signals result in the same profit because every auction is won.

The result for $c - p < 0$ follows the exact same form. □

4.5 Symmetric Accuracy Relaxation

The value model in the private versus common scenario dictates that the private and common signals have the same accuracy. We now relax that constraint and provide distinct accuracies for the private (a_P) and common (a_C) signals. As an example, consider the auction for extraction rights example from Section 4, where each agent has some private technology. Figuring out how much oil is on the land may be a very difficult task and have a very low accuracy, while investigating a company's existing drilling technologies may be much more accurate.

Notice that when we change the signal model to allow for different accuracies, the only subgame utility expressions from Table 2 that change are for the subgames where one agent observes the common signal and the other observes its private signal (u_{CP}, u_{PC}). In every other subgame, the accuracy term in equilibrium utilities can be substituted with the corresponding accuracy of the observed signal in that subgame. When solving for equilibrium bidding in this new subgame we get the following utilities:[8]

$$\begin{aligned} D \equiv u_{CP} &\leq \frac{1}{8}(p+c+1) - \frac{1}{2}[(1-a_P)(1-a_C) \\ &\quad + p(1-a_P)a_C + ca_P(1-a_C)] \\ u_{CP} &\geq \frac{1}{8}(p+c+1) - \frac{1}{2}[a_C(1-a_P) \\ &\quad + ca_Pa_C + p(1-a_P)(1-a_C)] \\ u_{CP} &> \frac{1}{8}(2a_C-1)(c-p+1), \\ P \equiv u_{PC} &= \frac{1}{8}(2a_P-1)(p-c+1). \end{aligned}$$

Note that changing the signal accuracies does not change the private-signal agent's utility (P).

Proposition 7. *Both agents choosing the common signal is an equilibrium if*

$$a_P < (2a_C - 1)\frac{a_C(1-a_C)}{a_C^2 + (1-a_C)^2}\frac{1+(c-p)}{1-(c-p)} + \frac{1}{2}.$$

[7]The full derivation is included in Appendix B.3 at `http://hdl.handle.net/2027.42/102737`

[8]The modified symbolic bidding strategies are included in Appendix B.4 at `http://hdl.handle.net/2027.42/102737`

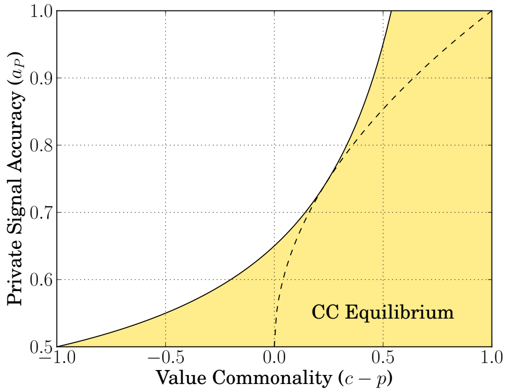

Figure 7: CC equilibrium region with utility-maximizing common signal accuracy (a_C^*). The dashed curve is the PC/CC equilibrium boundary from Figure 5.

Proof. From Table 2 and the new definition of C and P for asymmetric signal accuracies, the above equation implies that $C > P$ and therefore deviation from the CC equilibrium is not beneficial. □

Notice that if the common signal is too accurate or too noisy, the utility drops to zero, but there is an optimal common signal accuracy that maximizes utility of the CC profile. The common signal accuracy that maximizes the CC equilibrium profit is

$$a_C^* = \frac{1}{2}\sqrt{\sqrt{5} - 2} + \frac{1}{2} \approx 0.74$$

Figure 7 shows the range of parameters where the CC equilibrium exists when common signal accuracy is defined by a_C^*.

Whereas the informativeness of a signal always rises with accuracy, we see that beyond a_C^*, the strategic value of the common signal actually decreases. If the agent had exclusive access to a signal, it would naturally want it to be as accurate as possible, but since the signal is mutually accessible, more accuracy can be undesirable.

5. CONCLUSION

Because information-gathering decisions set the stage for subsequent strategic interactions, these choices must also be considered from a strategic perspective. Through analysis of two novel deliberative auction scenarios, we illustrate how the dependence structure of signals and complementarity of valuations interact with signal accuracy in the determination of equilibrium information-gathering strategies. In the coordination scenario, observing different attributes is more jointly informative and the preferred choice when signals are accurate or attributes are substitutes. With complementary attributes and noisy signals, however, we find a somewhat surprising equilibrium where agents implicitly collude to get less information by observing the same attribute. In the private versus common scenario, which attribute is more informative about value depends on the degree of commonality of the valuation function. Even when value is predominantly common, however, as signals become more accurate, the equilibrium strategy is for the agents to observe signals about their distinct private attributes. When the symmetric accuracy constraint is relaxed, we find that beyond a certain point, agents would prefer not to have available a more accurate signal of common value. This provides perhaps our most striking demonstration of the disparity between informativeness of a signal (as captured by standard VoI concepts) and its strategic value in the context of a game of incomplete information.

Many situations present choices among possibly overlapping, private- and common-relevant signals. The simple models analyzed here help us to think about the factors that induce coordination or specialization of information-gathering. Since the patterns they embody are ubiquitous, they can serve as building blocks to understand more complex scenarios.

One interesting extension would consider signals observable at a cost, as opposed to a strict limit on number of signals to observe. This extension could include the option to gain accuracy by obtaining multiple signals corresponding to the same attribute. Another interesting extension would investigate the implications of how much agents know about which signal their opponents chose.

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