Computing Quantal Response Equilibrium for Sponsored Search Auctions

(Extended Abstract)

Jiang Rong
The Key Laboratory of Intelligent Information Processing, ICT, CAS
University of Chinese Academy of Sciences
Beijing 100190, China
rongjiang13@mails.ucas.ac.cn

Tao Qin
Microsoft Research
Beijing 100080, China
taoqin@microsoft.com

Bo An
School of Computer Engineering
Nanyang Technological University
Singapore 639798
boan@ntu.edu.sg

ABSTRACT

Sponsored search auctions (SSAs) have attracted much research attention in recent years and different equilibrium concepts have been studied to understand advertisers' behaviors. However, the assumption that bidders are perfectly rational in these studies is unrealistic in the real world. In this work, we investigate the quantal response equilibrium (QRE) for SSAs. QRE is powerful in characterizing the bounded rationality in the sense that it only assumes that an advertiser chooses a better strategy with a larger probability instead of choosing the best strategy deterministically. We propose a homotopy-based method to compute the QRE of SSAs. We further show that there are many nice properties of the SSAs compared with general normal formal games, which can be used to improve the computational performance. Our experimental results indicate that our algorithm outperforms the basic traversal method.

Categories and Subject Descriptors

J.4 [**Computer Applications**]: Social and Behavioral Sciences—*Economics*

Keywords

Quantal Response Equilibrium; Sponsored Search Auctions; Bounded Rationality; Homotopy Principle

1. INTRODUCTION

Sponsored search has become a major monetization means for commercial search engines. Most of the time, there are many more advertisers bidding for the query than the number of available ad slots. Hence, the search engines need an auction mechanism to sell the ad slots. The most popular mechanism used by commercial search engines is the Generalized Second Price (GSP) auction which has attracted much research attention recently . Among those studies, equilibrium analysis is a hot topic to understand advertisers' behaviors [2, 10]. A critical limitation of existing studies on equilibrium analysis is that they assume the full rationality of advertisers. In practice, an advertiser may be incapable to estimate the bid strategies of his competitors and to take the "best-response" action on that basis. In this paper, we, for the first time, introduce the *quantal response equilibrium* [6] into SSAs considering that it can deal with limited rationality situations and has presented very good performance in general normal form games. QRE is a mixed strategy equilibrium in which strategies with higher utilities are more likely to be chosen than those with lower utilities, but the best is not chosen with certainty due to the limited rationality of participants.

In our work, an efficient algorithm is proposed to compute the QRE for SSAs. We show that this problem is equivalent to finding a solution of a continuous non-linear function. Basic Newton-type algorithms are usually locally convergent and work well only when we could provide a good starting point which is difficult to find in SSAs. To address this problem, we introduce the *homotopy* principle [1], which has been successfully used for equilibrium computation [9, 3]. Advantages of homotopy-based methods include their numerical stability and potential to be globally convergent. We noticed that Gambit [5] used the similar method to compute the QRE for normal form games with homogeneous player precisions and logit quantal response [6, 9], which are different from the settings in SSAs. We further show that there are many nice properties of the SSAs which can be used to refine the computational procedure.

2. GSP MECHANISM FOR SSAS

There are N bidders competing for K ad slots ($N > K$). Let v_i denote the private value of bidder i, which expresses the maximum per-click price he is willing to pay. b_i represents the bid submitted by i to participate in the auction. θ_{ik} is the click-through-rate (CTR) of i's ad when placed at slot k, which is usually assumed to be the product of the ad CTR α_i and the slot CTR β_k. In the GSP mechanism, bidders are ranked in the descending order of their ranking scores ($\alpha_i b_i$). Suppose advertisers are labeled such that $\alpha_i b_i \geq \alpha_{i+1} b_{i+1}$, then the utility of bidder i is $u_i = (v_i - \frac{b_{i+1}\alpha_{i+1}}{\alpha_i})\alpha_i\beta_i$, $i = 1, 2, \ldots, K$.

Appears in: *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2015), Bordini, Elkind, Weiss, Yolum (eds.), May 4–8, 2015, Istanbul, Turkey.*

3. COMPUTING QRE FOR SSAS

Bidder i's expected utility is represented as $\overline{u}_i(\sigma_{-i})$, where σ_{-i} is other bidders' mixed strategy profile[1]. In the QRE model, any bidder's mixed strategy π_i is a *quantal response* to $\overline{u}_i(\sigma_{-i})$ given his *precision parameter* λ_i. A profile σ is a QRE if it satisfies the following set of equations:

$$\sigma_{ij} = \pi_{ij}(\sigma_{-i}|\lambda_i) \quad (1)$$

for any bidder i and any pure strategy j. Computing a QRE of SSAs is equivalent to finding a zero point of the nonlinear functions

$$F_{ij}(\sigma) = \pi_{ij}(\sigma_{-i}|\lambda_i) - \sigma_{ij}. \quad (2)$$

If a good initial point which is close to a zero point of F is available, we can directly apply Newton-style iteration methods. However, we have little information about such a good initial point. As pointed by Allgower and Georg [1], Newton-style iteration methods often fail because poor start points are very likely to be chosen. Hence we turn to the homotopy method.

The basic idea of the homotopy is composed by two steps: given a problem we want to solve, first, define a problem $G(\sigma)$ with a unique easily-computed solution and then build a continuous transformation H with $H(\sigma, 0) = G(\sigma)$ and $H(\sigma, 1) = F(\sigma)$; second, begin with the solution of $G(\sigma)$ and trace solutions of the associated problems $H(\sigma, t)$, $t \in [0, 1]$, until finally finding the solution of $F(\sigma)$. The method to trace the solutions is called predictor-corrector (PC) [1], which begins with $\mu_1 = (\sigma_0, 0)$, the solution of $H(\sigma, 0) = G(\sigma)$, and then numerically generates a sequence of points $\mu_i = (\sigma, t)_i, i = 2, 3, \ldots$ satisfying $\|H(\mu_i)\| \leq \varepsilon$ for some $\varepsilon > 0$ by the Euler predictor and Gauss-Newton corrector.

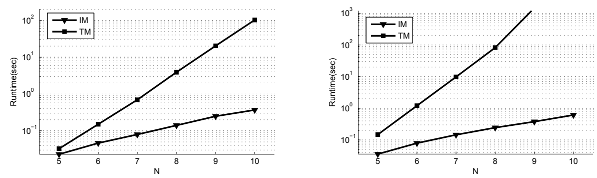
4. EFFICIENT COMPUTATION FOR SSAS

We need to compute the Jacobian matrix H' of H at each predictor and corrector step when tracing the solutions with PC method. Since the dimension of H could be large, the efficiency of calculating H' will significantly affect the speed of the homotopy method. We discuss how to efficiently calculate H' by leveraging the properties of SSAs.

The elements in H' consist of the expected utilities of bidders. The traditional traversal method (TM for short) for computing the expected utility in normal form games is exponential ($O(M^N)$). Fortunately, the expected utilities in SSAs with the GSP mechanism have many special properties that could be utilized to reduce the computational complexity. That is, a bidder's utility only depends on how many bidders' ranking scores are greater and equal to him and on the the maximal ranking score below him, but not on who they are or exactly what their bids are. These properties indicate that SSAs have considerable context-specific independence structure and can be represented compactly by an Action Graph Game with Function Nodes (AGGFN) [4, 8]. Thus we can reduce the complexity for computing a bidder's expected utility to $O(KN^3M)$. Besides, we find some methods to further avoid redundant calculation when computing the elements of H'. We call our improved method the IM.

5. EVALUATION

We make a comparison between the TM and the IM for computing H' with games of different sizes ($K = \lfloor N/2 \rfloor$). we see from the figure that both of the two methods are efficient and the improvement of IM is not apparent with small games. However, the speed of TM slows down dramatically with N increasing especially when M is large, e.g., by more than 10000 times when $M = 8$. Moreover, there is also a sharp degradation of TM's performance as M grows with large N's, but the efficiency of IM is not affected obviously. These observations further confirm the improvement of IM.



(a) M=5 (b) M=8

6. ACKNOWLEDGEMENTS

This work is supported by NSFC grant No. 61202212 and Singapore MOE AcRF Tier 1 grant MOE RG33/13.

REFERENCES

[1] E. L. Allgower and K. Georg. *Numerical continuation methods*, volume 13. Springer-Verlag Berlin, 1990.

[2] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. Technical report, National Bureau of Economic Research, 2005.

[3] P. W. Goldberg, C. H. Papadimitriou, and R. Savani. The complexity of the homotopy method, equilibrium selection, and lemke-howson solutions. *ACM Transactions on Economics and Computation*, 1(2):9, 2013.

[4] A. X. Jiang and K. Leyton-Brown. A polynomial-time algorithm for action-graph games. In *Proceedings of the 21st AAAI Conference on Artificial Intelligence*, pages 679–684. AAAI Press, 2006.

[5] R. D. McKelvey, A. M. McLennan, and T. L. Turocy. Gambit: Software tools for game theory, version 14.1.0. http://www.gambit-project.org. 2014.

[6] R. D. McKelvey and T. R. Palfrey. Quantal response equilibria for normal form games. *Games and economic behavior*, 10(1):6–38, 1995.

[7] J. Rong, T. Qin, and B. An. Quantal response equilibrium for sponsored search auctions: Computation and inference. In *Proceedings of The 10th workshop on Ad Auctions, in conjunction with the 15th ACM Conference on Electronic Commerce*, 2014.

[8] D. R. M. Thompson and K. Leyton-Brown. Computational analysis of perfect-information position auctions. In *Proceedings of the 10th ACM conference on Electronic commerce*, pages 51–60. ACM, 2009.

[9] T. L. Turocy. A dynamic homotopy interpretation of the logistic quantal response equilibrium correspondence. *Games and Economic Behavior*, 51(2):243–263, 2005.

[10] H. R. Varian. Position auctions. *International Journal of Industrial Organization*, 25(6):1163–1178, 2007.

[1]More details are included in the workshop paper [7]