

Voting with Social Influence: Using Arguments to Uncover Ground Truth

(Extended Abstract)

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ABSTRACT

In certain voting problems, a hidden ground truth is inferred by aggregating the opinions of an electorate. We propose a novel model of these underlying social interactions, and derive maximum likelihood estimators for the ground truth in these models, given the social network and votes. We also evaluate these new estimators, as well as existing ones, on a class of simulated social networks.

1. INTRODUCTION

This paper focuses on problems in the aggregation of opinions about an objective truth, a problem with applications to many problems in artificial intelligence, and multiagent systems in particular (e.g. recommender systems, multiagent resource allocation, aggregating noisy sensor data from a corporative swarm of agents). To date, most existing work in this domain has assumed that voters' individual impressions of the truth are generated *independently*. In practice, however, this may not be so. In recent work, Conitzer [3] considers the possibility that voters' impressions of the truth might be influenced by discussions with other members of the electorate. This additional factor can confound attempts to recover the truth. As a trivial example, consider the social network depicted in Figure 1.

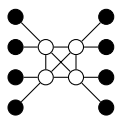


Figure 1: Example of voting on a social network.

if social interactions occur according to this graph, the members of clique could reinforce each others' opinions, while the isolated perimeter nodes perceive only opinions opposed to their own, and consequently might change their minds, and cast a vote for 'white' based on the local aggregation of information from their neighbors. Naively aggregating the opinions of the voters after social interactions of this kind

Each node represents a voter in this social network. A central 4-clique have the belief that the correct answer is 'white', while peripheral neighbors connected only to the members of the clique believe the correct answer is 'black'. The correct answer is black, and nodes are more likely (by a 2:1 ratio) to have observed the truth. However,

take place could, thus, result in an overwhelming majority for white, when in fact, the true answer is black. This suggests that aggregation should incorporate information about the network structure.

2. RELATED WORK

Social networks represent a model of how people communicate, interact and influence each other in their daily lives. Properly harnessed, a social network can be used to promote the spread of certain ideas, and curtail others. This idea motivated much interest in the area, with recent work including [1–6]. Our work examines social choice mechanisms in the presence of a social network. Our approach differs most markedly from that of opinion dynamics in that we are not examining long term behavior of the network; rather, we emulate the time critical aspects of real world problems, and conduct only a brief round of interactions before measuring opinions from the network.

The work most closely related to our own is Conitzer's recent papers proposing a maximum likelihood approach to voting over a social network [3]. In this model, given a set of voters V , the probability distribution of a voter's report is comprised from two independent parameters, $g(A_v|c)$ the probability of observing a vote A_v given that c was the correct answer, and $h(A_v, A_{N(v)})$ the probability of observing the joint profile of votes for A_v and its neighbors $A_{N(v)}$. Conitzer considers the problem of estimating the winner given the network structure and the votes cast after some (unspecified) social interactions have taken place on the network, but the initial model [2] was shown to reduce to a naive counting of votes, without incorporating network structure at all. The proposed alternative edge-based model, where opinions are distributed over conversations rather than voters, seems counter-intuitive. This motivates our work, which is an attempt to remedy the problems of the original, vertex-based model.

3. RIGHTEOUS ARGUMENT

Suppose that the joint probability of a voter's reported opinion and the opinions of its neighbors *did* depend on the outcome. What would the nature of this dependency be? We propose that, when a voter has the correct opinion, they should be more likely to convince their neighbors of that opinion than if they hold an incorrect opinion. We consider a model according to the one in [2] and make two assumptions to reflect the idea of a righteous argument. First, we assume that $h(A_v, A_{N(v)})$ is not indepen-

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dent of \hat{c} (the final outcome), but instead depends upon it. Second, we assume that $h(A_v, A_{N(v)}|\hat{c})$ can be partitioned into a product of independent pairwise interactions, according to some function h' that satisfies the ordering: $h'(A_v = c, A_u = c|\hat{c} = c) > h'(A_v = c, A_u = c'|\hat{c} = c) \sim h'(A_v = c', A_u = c|\hat{c} = c) > h'(A_v = c', A_u = c'|\hat{c} = c)$.

We assume that voters are assigned an initial judgment in favor of the correct answer (c) with probability $p > 0.5$, and opposed to the correct answer (c'), with probability $1-p$. We then suppose that voters interact with one another, and that their interactions are independent of one another. During these interactions, voters exchange information, and that the exchange of information is more likely than not to lead them closer to the truth. Let $h'(A_v, A_u|\hat{c} = c)$ be the probability that we observe vertices v and u , which are connected in the social network, cast votes A_v and A_u respectively, given that the correct answer is c .

Returning to the earlier vertex-based model [2], we can define a new Maximum Likelihood Estimator (MLE) for the observed vote profile in terms of h' : $\mathcal{L}(A_V|\hat{c} = c) \propto \prod_v g(A_v|\hat{c} = c) \left[\prod_{u \in N(v)} h'(A_v, A_u|\hat{c} = c) \right]$. To select a winner under this model, we simply find $\arg \max_{d \in C} \mathcal{L}(A_V|\hat{c} = d)$. Under the assumptions stated above, $g(A_v|c) = p$ when $A_v = c$ and $\tilde{p} = 1-p$ otherwise. Thus, the likelihood function reduces to $\mathcal{L}(A_V|\hat{c} = c) \propto p^i \tilde{p}^{|V|-i} \prod_{u \in N(v)} h'(A_v, A_u|\hat{c} = c)$ where i is the total number of reported votes for c in A_V . Notice that, if two adjacent nodes u and v cast differing votes, then the terms $h'(A_v = c, A_u = c'|\hat{c} = c)$ and $h'(A_v = c, A_u = c'|\hat{c} = c')$ will appear in the likelihood for both candidates, and so such discordant edges need not be considered. This leaves only the consensus edges where both parties agreed. Suppose that $h'(A_v = c, A_u = c|\hat{c} = c) = q$ for some $q > 0.3$, and that $h'(A_v = c', A_u = c'|\hat{c} = c) = \tilde{q}$ for some $\tilde{q} < 0.3$. Then we can write the final likelihood as $\mathcal{L}(A_V|\hat{c} = c) \propto p^i \tilde{p}^{|V|-i} q^x \tilde{q}^y$ where x is the number of connections between nodes with opinion c , and y is the the number of connections between nodes with a different opinion. Recall that p, q and \tilde{q} are actually unknown quantities. It follows that this model can produce a certain result only when one candidate has both a majority of votes and a majority of the consensus edges in the graph between vertices that voted for the winning candidate. Computing the most likely candidate is linear in the size of the network. There are natural extensions of this model to the directed and multi-candidate cases, but we omit them for space reasons.

4. EVALUATION

We evaluated the estimator on Erdős-Rényi and Barabási-Albert random graphs initialized by first assigning a fraction $\frac{n_c}{n}$ of the n voters the correct opinion. Then, a vertex is sampled with probability proportionate to $\frac{1}{n}$ if it has the correct opinion, and $\frac{1+N_c}{n}$ if it has the incorrect opinion, where N_c is the number of neighbors with the correct opinion. The sampled vertex has its opinion inverted, and then a new vertex is sampled with an updated probability distribution, until k vertices have been sampled in total. This simulates a process where a “correct” opinion is diffused into a network: once convinced of the correct opinion by good evidence, a voter is unlikely revert back.

The performance of our model depends on the relationship between p and q , as described above. It can be shown that a network that starts with $p = \frac{c}{n}$ and has no opinion dynamics

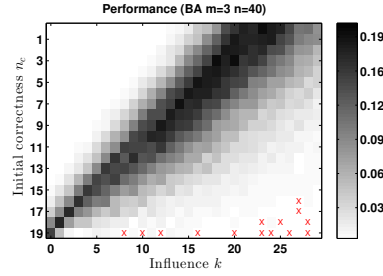


Figure 2: Simulation results. Positive (darker) values indicate advantage for the new model. Each cell is the average of 1,000 simulation runs on randomly generated networks.

will have $q \propto p$ in expectation, in which case the righteous argument model will perform exactly as well as the naive model. If we start with $p \sim 0.5$, and flip k , then $q > p$ in expectation, as long as k is not too large (since then $p \sim 1 \sim q$). This is in agreement with our results, shown in Figure 2.

5. CONCLUSION AND FUTURE WORK

In this paper, we proposed a new maximum likelihood approach to voting in the presence of a social network. Our new model started from the assumption of righteous argument: that it is easier to convince someone of the truth, than of a falsehood. We derived efficiently computable maximum likelihood estimators for the correct judgment under this assumption for the case of a binary election over an undirected graph and validated the new approach with simulations. Overall, we have demonstrated that there exists a novel and efficient model for incorporating information about the structure of social networks, which offers increased confidence in the outcomes it predicts.

The proposed model offers several interesting avenues for future work. The Righteous Argument model is a general model, that works over a large family of problems. However, additional performance advantages may be possible over a more restricted family of problems. For instance, a model that incorporates knowledge of the social dynamics used could consider possible starting positions that generated a particular configuration, in much the same way as Conitzer’s edge model.

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