Single Item Auctions with Discrete Action Spaces

(Extended Abstract)

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1. INTRODUCTION

An implicit assumption in truthful mechanism design is that revelation of one's true type is always feasible. Indeed, this is not a problem in standard mechanism design setups, where it is up to the designer to determine the action spaces. However, this assumption fails to hold in many practical scenarios, where there are natural, exogenous constraints on the set of possible actions. For example, in combinatorial auctions [5] where bidders have combinatorial preferences on bundles of items, truthful revelation of such a preference requires a bidder to communicate a value on each subset of items, results in an exponential blow up in communication complexity. A practical combinatorial auction often imposes constraints on the number of package bids a bidder can place. The above observations motivate an active line of research that concerns the *expressiveness* of mechanisms [1, 2, 3, 4].

We consider a practical single-item auction design setting with restricted expressiveness. In particular, the action space of all bidders is restricted to a set of discrete *bid levels*, while the values are in continuous spaces. With this interface, truth revelation is not feasible and the revelation principle fails to hold (We will address this point later).

Tailored for this setting, we put forward an auction, coined the *extended second price auction* (ESP). Our auction resembles the second price auction when there are multiple winners tied at the highest bid; however, when there is a unique winner, our auction charges a bit more than the second price auction. We show that, our auction satisfies the following desirable properties:

- Anonymity: if we replace the bids of any two bidders, their winning probability and payments (if win) will also be replaced.
- Truthfulness: each bidder finds in her best interests to bid the highest bid level below her true type;
- Not dominated in welfare: there exists no other auction that can dominate our auction in welfare for every possible type profile;
- Not dominated in revenue: there exists no other ex-post individually rational (IR) truthful auction that can dominate our auction in revenue for every possible type profile;

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In our setting, truthfulness is different from standard definition because of the restriction on expressiveness. However, similar to the other settings, in a truthful mechanism, it is "straightforward" for bidders to decide their bids and their bids truthfully (partially in our setting) reveal their values. An equivalent understanding of our truthfulness is as follows. The bid levels separate the value space into many intervals, such that two consecutive intervals are separated by a bid level. Let *value intervals* denote these intervals. A mechanism is truthful if each bidder finds in her best interests to report the value intervals she belongs to.

2. THE SETTINGS

We consider a setting that is built upon the standard independent private value (IPV) setting. In an IPV setting, there is a seller who has an indivisible item for sale. Her valuation of the item is zero. There is a set M of m bidders that are interested in the item. The degree of interest is expressed by a number called valuation. Each bidder i knows her own valuation v_i. Other bidders, denoted by $-i$, treat v_i as a random variable that is distributed according to some distribution function $F_i(v_i)$ that is positive everywhere on $[0, v_{max}]$. v_i is independent of v_j for any $i \neq j$. An (direct) auction then solicits bids from the bidders, allocates the item to the bidders according to its allocation rule and charges payments according to its payment rule.

Distinct from the standard IPV setting, which places no restrictions on the set of bids to report, our setting only allows bidders to select bids from a given list of *bid levels* $\mathcal{B} = \{l_1, l_2, \ldots, l_n\}$ such that $v_{max} > l_1 > \ldots, l_n = 0$. $b_i \in \mathcal{B}$ denotes agent i's bid. Distinct from the standard IPV setting, in our setting the seller does not have any knowledge about valuation distribution. Therefore, our setting effectively excludes cases such as full-information setting where the seller knows the exact value of each bidder, as well as Myerson's standard setting of optimal Bayesian auction design. The utility of bidder i is $u_i = v_i - p_i$.

3. EXTENDED SECOND PRICE AUCTION

In cases where there are several highest bids tied at some bid level. The ESP is exactly the same as SP. However, when the highest bid is different from the second highest, the winner needs to pay a bit more than what they pay in SP, i.e., the second highest bid. The winner's payment is independent from their own bid, which ensures the truthfulness of the auction. The detailed description of ESP is as follows.

Mechanism 1 The ESP auction

Input: Bid levels $\mathcal{B}$ and a bidding profile $(b_1, b_2, \ldots, b_n)$.
Output: Price profile $P = \{p_1, p_2, \ldots, p_n\}$ where p_i is the price for bidder i; the winner w ($w \in \{1, 2, \ldots, n\}$).

Let b^* denote the highest bid, b' denote the second highest bid.
if $b^* = b'$ **then**
  Uniformly randomly pick one bidder who bids b^* as the winner w.
  $p_w = b^*$. For $i \neq w$, $p_i = 0$.
else
  w is the bidder who bids b^*.
  Let $n(b)$ be the number of bidders who bid b. b'' denotes the lowest bid level above b'. $p_w = \frac{n(b')b''+b'}{n(b')+1}$; For $i \neq w$, $p_i = 0$.
end if

Remark:
(b', b'', b^*) can also be (l_k, l_{k-1}, l_i). The $\frac{n(b')b''+b'}{n(b')+1}$ is in fact an indifference point. If we charge more than this amount, the winner has incentive to underbid; otherwise, some bidder may have incentive to overbid.

It can be proved that ESP can obtain great properties as follows.

Theorem 1. *The ESP auction is truthful.*

Proof. Our proof will discuss two cases. First, we show that agents cannot benefit from overbidding. Then, we show that they cannot benefit from underbidding either.

We consider an arbitrary bidder i, whose type is v_i and the highest bid level under v_i is b_i^*. In the following discussion, we compare to the truthful case where i bids b_i^* and show that overbidding or underbidding cannot increase i's utility.

- First, we argue that i cannot benefit from reporting higher than b_i^*.
  - If i is the only possible winner[1], reporting a higher bid will not reduce her payment.
  - If someone else bid b' higher than v_i, i needs to bid at least b' to be possible to get the item, and pay at least b'. As $b' > v_i$, i will lose money.
  - If i is tied with $k (k \geq 1)$ bidders at the top bid, i gets $(v_i - b_i^*)/(k+1)$ in expectation. If i reports higher, i will be the winner, but charged $(b_i'k + b_i^*)/(k+1)$, where b_i' is the level closest above b_i^*. It results in a lower utility since
  $$v_i - (b_i'k + b_i^*)/(k+1) < (v_i - b_i^*)/(k+1).$$
- Then we argue that i cannot benefit from reporting lower than b_i^*.
  - If i is not the only possible winner, underbid disqualifies i as a winner. Since obviously i gets non-negative utility whenever she is a winner, underbid will not increase her utility in this case.
  - Now, we consider the case where i is the only possible winner. If i reports less but higher than the second highest bid. i's utility remains the same. If i reports less than the second highest bid, i will no longer get the item. If i bid the same as the second highest bid, i's utility will become $(v_i - b_{sec})/(k+1)$, which is less than the original utility $v_i - (b'_{sec}k + b_{sec})/(k+1)$. Here, b_{sec} is the second highest bid, k is the number of bidders who bid b_{sec}. b'_{sec} is the bid level closest above b_{sec}

To sum up, we have shown that overbid or underbid does not increase the utility of an agent. □

[1] i bids strictly higher than the second highest bidder.

Theorem 2. *The ESP auction is not dominated on social welfare.*

Theorem 3. *The ESP auction is not dominated in revenue among all truthful, ex-post IR auctions.*

Theorem 4. *Given the distributions of bidders' types are i.i.d. and all the bidders truthfully report, the revenue of ESP can be at most $\frac{1}{e}MG$ more than SP, where MG is the difference between the highest and the lowest bid levels.*

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