Analysis of Condition for the Cooperation Achievement on Arbitrary Networks

(Extended Abstract)

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ABSTRACT

We analyze relations between evolution of cooperation on networks and network structures. Most researchers have used simple networks, such as scale-free networks and random networks. We performed decision tree analysis on various networks. The decision tree is constructed from cooperation degree and five network feature. Here, these five network features are selected from a lot of network features. We estimate their cooperation degree by SVR. As a result, estimation is successful because estimation values are correlated with measurement values strongly in the correlation coefficient of 0.8. Therefore, it is said that cooperation degree can be explained by five network features. The decision tree is constructed with cooperation degree and five network features, and then analyzed. We found that the conditions for achieving cooperation are having few hub nodes, being not very disassortative, and having a short average shortest-path length.

Keywords

prisoner's dilemma; complex network; network model; network feature

1. INTRODUCTION

Our goal is to understand the relation between network structure and cooperation. Most researchers approached this problem under limited conditions, it cannot be said that general knowledge about it has been obtained so far. For example, Rong et al. [3] studied cooperation focusing on assortativity. They generated scale-free networks with assortativity $r = 0.0 - 0.3$ by the Xulvi-Sokolov algorithm, and observed the relation between assortativity and cooperation. However there is a correlation between assortativity and average shortest-path length of the networks, and it is unclear which features affect cooperation degree. From the above reasons, it cannot be analyzed sufficiently without a compound analysis of the structure feature.

In this paper, we make clear which network features affect evolution of cooperation by the analysis of a decision tree. We simulate SPD games on various networks, and construct a decision tree. However, there are not enough guarantees to explain evolution of cooperation by adopted explanatory variables. Therefore, we estimate the achievement degree of the cooperation (cooperation degree) from explanatory variables. If it is possible to estimate at a high level, it shows that cooperation degree is explained sufficiently by these features.

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2. NETWORK FEATURES THAT CAN EXPLAIN COOPERATION

In this paper, 5 network features adopted as explanatory variables to exlain cooperation degree. However there are not enough guarantees to explain their cooperation degree. Then, we estimate cooperation degree from network structures. If cooperation degree can be estimated, it can be said that the adopted network features are sufficient to explain cooperation. In this paper, we employ support vector regression (SVR).

2.1 Estimation setting

Analyzed features are the following: average shortest-path length L, cluster coefficient C, assortativity r, α of Beta distribution, β of Beta distribution. We construct decision trees using five explanatory variables.

Cooperation degree P_c is defined as an objective variable of the decision tree. In this paper, Spatial Prisoner's dilemma is adopted [3]. Payoff matrix is defined as follows:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1.0 & 0.0 \\ 1.2 & \epsilon \end{pmatrix}, \tag{1}$$

where ϵ is a minimum value.

2.2 DataSet

In this paper, 4,000 networks are generated as the network dataset. Table 1 shows statistics of the network dataset. It is said values of each feature are various values. Therefore, the dataset includes networks with various network structures.

2.3 Estimation Result

We try to estimate cooperation degree P_c from explanatory variables by SVR. A gauss kernel is used for the kernel function of SVR. We perform 8-division cross validation, and the parameters are optimized by simulated annealing [2].

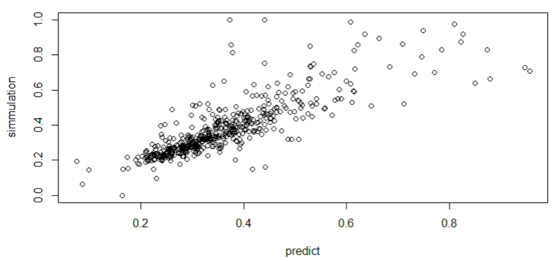

Figure 1: Estimation result

Figure 1 shows the estimation values calculated by SVR and measurement values calculated by the SPD game simulation. It is said that estimation succeeded because estimation values correlated with measurement values strongly in the correlation coefficient of 0.824. Therefore, cooperation degree can be estimated from the adopted network features uniquely; that is, cooperation degree can be explained with adopted explanatory variables.

3. ANALYSIS OF CONDITION WHERE COOPERATION BECOMES DOMINANT

3.1 Result of Analysis

In this section, we analyze how and which features affect cooperation degree. A decision tree is constructed for analyzing them. The cross validation error of the decision tree is 0.325.

There are three conditions in which the cooperator becomes the majority, as shown in Table 2. We consider the condition of highest cooperation degree. It is suggested that the lower average shortest-path length L leads to high cooperation degree. However, if average shortest-path length L is high, the cooperation degree can be high, as shown in the second highest condition. In the second highest condition, the condition in which α of degree distribution is low is added instead of the condition in which average shortest-path length L is low. The networks with high α have a lot of low degree nodes. Therefore, nodes with small degree are required for achievement of cooperation.

Next, not so low assortativity $r \geq -0.261$ leads to cooperation achievement. It is common to the second- and third-highest condition. Therefore, it can be said that disassortative networks cannot keep cooperators. In addition, in the third-highest condition, very high assortativity r leads to low cooperation degree. It is suggested that very disassortative networks and very assortative networks cannot keep cooperators.

Finally, high β leads to high cooperation degree. It is common to three conditions. The degree distribution with high β has a long tail. It is suggested that cooperation degree is affected by a long tail.

Table 1: Statistics of network dataset.

	Range	Average	Standard diviation	Max	Min
L	$1 \leq L$	3.49	1.07	6.08	2.23
C	$0 \leq C \leq 1$	0.291	0.201	0.793	0.00357
r	$-1 \leq r \leq 1$	0.0647	0.325	0.887	-0.562
α	$0 \leq \alpha$	2.14	1.38	7.97	0.723
β	$0 \leq \beta$	21.4	13.4	89.7	4.17

Table 2: Condition in highest and lowest networks

	Highest	2nd highest	3rd highest
$\overline{P_c}$	0.769	0.621	0.612
L	$L < 2.65$	$L \geq 2.65$	-
C	-	-	$C < 0.559$
r	$r \geq -0.261$	$r \geq -0.159$	$-0.146 \leq r < 0.339$
α	-	$\alpha < 2.90$	$\alpha < 3.05$
β	$\beta \geq 24.2$	$\beta \geq 24.2$	$15.0 \leq \beta < 24.2$

3.2 Discussion

First, we found that not very disassortative and very assortative networks keep cooperators. Rong et al. [3] showed that disassortative networks are needed for cooperation. However, they analyzed the networks with assortativity $r \geq 0$. We analyzed the networks with assortativity $r < 0$, and we found very disassortative networks cannot keep cooperators. Therefore, the condition does not contradict their findings.

Second, there is almost no influence by the cluster coefficient in this research. Assenze et al. [1] showed that the networks with a high cluster coefficient lead to cooperation achievement. However, they analyzed only cluster coefficient, and did not analyze the features that may correlate with a cluster coefficient. A cluster coefficient is likely to be correlated with average shortest-path length in the networks generated by the HK model. It is suggested that average shortest-path length is related cooperation, and the correlation between cluster coefficient and cooperation is a spurious correlation.

Finally, it can be said that the distributions with high β and low α lead to high cooperation. These distributions are power-law distributions. That is, the scale-free networks (those with power-law degree distributions) are required to achieve cooperation. The average of cooperation degree in $R^2 \geq 0.5$ is 0.331 and that in $R^2 \leq 0.5$ is 0.505. It suggests that the power-low distributions lead to higher cooperation.

4. CONCLUSION

In this paper, we analyzed the relation between cooperation degree and each network feature statistically by simulating the SPD game on generated networks with various structures. First, we estimated the cooperation degree from network features to show that they are sufficient to explain cooperation. Second, we clarified how and which features affect cooperation by decision tree analysis.

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