Influence-Based Opinion Diffusion

(Extended Abstract)

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1. INTRODUCTION

Understanding the dynamics of opinion among agents is an important question which has recently received large attention in the community of autonomous agents and multi-agent systems [2, 3, 4, 5, 6, 7, 8, 9]. It requires to model several parameters such as the different relations which exist between agents, the opinions agents have and the way they make them evolve. This present work inspires from [9] and propose a formal model of opinion diffusion in a population of agents by assuming that: (i) opinions are propositional formulas or, equivalently, sets of propositional interpretations; (ii) the population of agents is structured according to a binary relation of influence which relates two agents when one influences the other; (iii) any agent orders its influencers (i.e. agents which influence it) according to the strength of the influence relation; (iv) any agent changes its opinion by merging the opinions of its influencers, from the most influential one to the least; (v) there is a special formula, called *integrity constraint* which expresses something true in the world and which has to be taken into account by the merging operator.

2. INFLUENCE-BASED BELIEF REVISION GAMES

Consider a finite propositional language L. If φ is a formula of L, $Mod(\varphi)$ denotes the set of models of φ i.e., the set of interpretations in which φ is true. A multi-set of formulas $\{\varphi_1, ..., \varphi_n\}$ equipped with a total order $\prec$ s.t. $\varphi_i \prec \varphi_{i+1}$ $(i = 1...n-1)$ is called *an ordered multi-set of formulas* and denoted $\varphi_1 \prec ... \prec \varphi_n$.

DEFINITION 1 (IMPORTANCE-BASED MERGING OPERATOR). *An Importance-Based Merging Operator, is a function Δ, which associates a formula μ and a non empty ordered multi-set of consistent formulas $\varphi_1 \prec ... \prec \varphi_n$ with a formula denoted $\Delta_\mu(\varphi_1 \prec ... \prec \varphi_n)$ so that:*

$$Mod(\Delta_\mu(\varphi_1 \prec ... \prec \varphi_n)) = Min_{\leq_{d,\varphi_1 \prec ... \prec \varphi_n}} Mod(\mu)$$

with:

- $w \leq_{d,\varphi_1 \prec ... \prec \varphi_n} w'$ *iff* $[D(w,\varphi_1), ..., D(w,\varphi_n)] \leq_{lex} [D(w',\varphi_1), ..., D(w',\varphi_n)]$
- $[D(w,\varphi_1), ..., D(w,\varphi_n)]$ *is a vector whose* k^{th} *element is* $D(w,\varphi_k)$
- $D(w,\varphi) = min_{w' \in Mod(\varphi)} d(w,w')$*,* w *and* w' *being two interpretations and* d *is a pseudo-distance between interpretations.*
- $\leq_{lex}$ *is a lexicographic comparison of vectors of reals defined by:* $[v_1, ...v_n] \leq_{lex} [v'_1, ...v'_n]$ *iff (i)* $\forall k \ v_k = v'_k$ *or (ii)* $\exists k \ v_k < v'_k$ *and* $\forall j < k \ \ v_j = v'_j$

By definition, Importance-Based Merging Operators are Distance-Based Merging Operators [1].

In the following, we introduce Influence-Based Belief Revision Games and Influence-Based Belief Sequences. These definitions are adapted from [9].

DEFINITION 2 (INFLUENCE-BASED BELIEF REVISION GAME). *An Influence-Based Belief Revision Game (IBRG) is a quadruplet $G = (A, \mu, B, Inf)$ where: $A = \{1, ..., n\}$ is a finite set of agents; μ is a consistent formula of L; B is a function which associates any agent i of A with a consistent formula of L denoted for short B_i such that $B_i \models \mu$; Inf is a function which associates any agent i of A with a non-empty set of agents $\{i_1, ...i_{n_i}\}$ equipped with a total order $\prec_i$ s.t. $i_k \prec_i i_{k+1}$ for $i = 1...(n_i - 1)$. For short, we denote $Inf(i) = \{i_1 \prec_i ... \prec_i i_{n_i}\}$.*

A is the finite set of agents. The formula μ represents the information which is true in the world. It is called *integrity constraint*. For any agent i, the formula B_i represents its initial beliefs. It is called the *initial belief state of* i. We assume that agents are rational and thus that B_i is consistent and satisfies the integrity constraint μ. For any agent i, $Inf(i)$ is the non-empty set of agents i considers as influent i.e., i considers that its own opinion is influenced by the opinions of agents in $Inf(i)$. With the total order $\prec_i$, i ranks its influencers according to their degree of influence: for any agents j and k in $Inf(i)$, $j \prec_i k$ means that, according to i, its own opinion is more influenced by j's opinion than by k's opinion. Notice that i may or may not belong to $Inf(i)$.

DEFINITION 3 (INFLUENCE-BASED BELIEF SEQUENCE). *Consider an (IBRG) $G = (A, \mu, B, Inf)$ and an agent $i \in A$ with $Inf(i) = \{i_1 \prec_i ... \prec_i i_{n_i}\}$. The Influence-Based Belief*

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Sequence of i, denoted $(B_i^s)_{s\in\mathbb{N}}$, is defined by: $B_i^0 = B_i$ and $\forall s \in \mathbb{N}$, $B_i^{s+1} = \Delta_\mu(B_{i_1}^s \prec ... \prec B_{i_{n_i}}^s)$.

The Influence-Based Belief Sequence (or Belief Sequence for short) of agent i, $(B_i^s)_{s\in\mathbb{N}}$, represents i's belief state all along the game: B_i^0 is the initial belief state of i; B_i^s is the belief state of i after s moves i.e., the opinion of i after s steps. The evolution of i's opinion is done according to the importance-based merging operator Δ_μ: i's opinion at step s is the result of Δ_μ applied to the ordered multi-set of opinions: $B_{i_1}^s \prec ... \prec B_{i_{n_i}}^s$.

PROPOSITION 1. *In an (IBRG), the belief sequence of any agent is cyclic i.e., the belief sequence of any agent i is characterized by an initial segment $B_i^0...B_i^{b-1}$ and a belief cycle, $Cyc(B_i) = B_i^b...B_i^e$ which will be repeated up as infinitum.*

DEFINITION 4. *An (IBRG) $G = (A, \mu, B, Inf)$ converges iff $\forall i \in A \quad \mid Cyc(B_i) \mid = 1$.*

DEFINITION 5. *Let $G = (A, \mu, B, Inf)$ be an (IBRG) and φ a formula of L. φ is accepted by agent i of A iff for all $B_i^s \in Cyc(B_i)$, we have $B_i^s \models \varphi$. φ is unanimously accepted in G iff φ is accepted by all i in A. φ is majoritary accepted if the number of agents who accept it is strictly greater than the number of agents who do not.*

Let us now introduce three more definitions related to (IBRG). The first one is adapted from [8], the two others are original.

DEFINITION 6 (DAG WITH LOOPS). *From any (IBRG) $G = (A, \mu, B, Inf)$, we can build a graph whose nodes are agents of A and edges are $i \to j$ iff $i \in Inf(j)$. We say that G is a DAG with loops if this graph is a directed graph where the only permitted cycles are of type $i \to i$.*

DEFINITION 7 (DOGMATIC AGENT). *Consider the (IBRG) $G = (A, \mu, B, Inf)$ and $i \in A$. i is a dogmatic agent iff $Inf(i) = \{i\}$.*

An agent is dogmatic when it is not influenced by other agents. As a consequence, a dogmatic agent i never changes its opinion i.e., $\forall s \geq 0 \; B_i^s = B_i^0$.

DEFINITION 8 (SPHERE OF INFLUENCE OF AN AGENT). *Let $G = (A, \mu, B, Inf)$ an IBRG. Let $i \in A$. $Sphere(i) = \{j : Inf(j) = \{i \prec ...\}\} \cup \{j_k : \exists j_0...j_{k-1} \; \forall m = 1...(k-1) \; Inf(j_m) = \{j_{m-1} \prec ...\}$ and $j_0 = i\}$*

The following proposition identifies a case where IBRG converge.

PROPOSITION 2. *DAG with loops converges.*

The following proposition identifies some sufficient conditions for accepting a given formula.

PROPOSITION 3. *Let G be an IBRG and i a dogmatic agent. If $Sphere(i) = A$ then B_i is unanimously accepted. If $\mid Sphere(i) \mid > \frac{|A|}{2}$ then B_i is majoritary accepted.*

Obviously, if an agent is the most infuential one for any agent in the population, then its initial opinion is unanimously accepted. If an agent is the most infuential one for more than a half population, then its initial opinion is majoritary accepted.

3. CONCLUDING REMARKS

This work suggested to model influence-based diffusion of opinion with Influenced-Based Belief Revision Games. It also presented some conditions on the Influence-Based Revision Game which guarantee the convergence of the diffusion process or the emergence of a majoritary or unanimous opinion. This work can be extended in several directions.

First, it could be interesting to change our assumption about the way agents orders their influencers. In this present work, we have assumed that agents order their influencers according to a strict order. Thus, two agents cannot be considered as equally influential for a given agent. Changing this assumption will lead us to consider preorders instead of orders. Defining a merging operator which takes into account preorders is an open question.

Secondly, an interesting extension is to consider that there is no global integrity constraint but only some local ones, shared by the agents who, in some way, belong to a commn community. Studying the acceptability in such a context is challenging.

Finally, to make this work more realistic, we have to consider the fact that an agent may influence another one not for any type of information but for only some kind of information, related to a given topic. Extending this work to topic-dependent influence is the next step of our research.

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