How is Cooperation/collusion Sustained in Repeated Multimarket Contact with Observation Errors?

(Extended Abstract)

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ABSTRACT

This paper analyzes repeated multimarket contact with observation errors where two players operate in multiple markets simultaneously. Multimarket contact has received much attention in economics, management, and so on. Despite vast empirical studies that examine whether multimarket contact fosters cooperation or collusion, little is theoretically known as to how players behave in an equilibrium when each player receives a noisy and different observation or signal indicating other firms' actions (*private* monitoring). To the best of our knowledge, we are the first to construct a strategy designed for multiple markets whose per-market equilibrium payoffs exceed one for a single market, in our setting. We first construct an entirely novel strategy whose behavior is specified by a non-linear function of the signal configurations. We then show that the per-market equilibrium payoff improves when the number of markets is sufficiently large.

Keywords

Game theory, repeated games, belief-free equilibrium

1. INTRODUCTION

This paper analyzes repeated multimarket contact with observation errors where two players operate in multiple markets simultaneously, e.g., [1]. A firm, e.g., Uber, provides its taxi service in multiple distinct markets (areas) and determines its price or allocation in each area, facing an oligopolistic competition, which is often modeled as a prisoners' dilemma (PD). To improve profits, it is inevitably helpful to realize how the firm's rival should behave in an equilibrium. However, despite vast empirical studies that have examined whether multimarket contact fosters cooperation or collusion, little is theoretically known as to how players behave in an equilibrium when each player receives a noisy observation or signal of other firms' actions.

This paper considers a realistic noisy situation where players do not share common information on each other's past history, i.e., *private* monitoring where each player may observe a different signal. For example, although a firm cannot directly observe its rival's action, e.g., prices, it can observe a noisy signal, e.g., its own sales amounts. Analytical studies on this class of games have not been very successful.

To the best of our knowledge, we are the first to construct a strategy designed for multiple markets whose per-market equilibrium payoffs exceed one for a single market. We construct an entirely novel strategy whose behavior is specified by a nonlinear function of the signal configurations.

2. MODEL

Two players play M PDs simultaneously in each period. In each PD, each player chooses either C (cooperation) or D (defection). The players can choose different actions over the M PDs, so that each player's action set in each period is $\{C, D\}^M$. In each PD, each player receives either a good signal g or a bad signal b. The pair of signals they privately receive in each PD follows a common symmetric probability distribution that depends entirely on the action pair of that PD. We denote it by $o(\omega_1, \omega_2|a_1, a_2)$, where $(\omega_1, \omega_2) \in \{g, b\}^2$ and $(a_1, a_2) \in \{C, D\}^2$. Since the signal distributions are described by one parameter, there exists $p \in (1/2, 1)$ such that for any i, (ω_1, ω_2), and (a_1, a_2),

$$\sum_{\omega_i \in \{g,b\}} o(\omega_i, \omega_j|a) = \begin{cases} p & \text{if } (a_i, \omega_j) \in \big\{(C, g), (D, b)\big\}, \\ 1-p & \text{otherwise.} \end{cases}$$

In each PD, player i's payoff depends only on his action and the signal of that PD. The payoff function is common to all PDs, denoted by $\pi_i(a_i, \omega_i)$. We are more interested in the expected payoff function

$$g_i(a_1, a_2) = \textstyle\sum_{(\omega_1,\omega_2)} \pi_i(a_i, \omega_i) o(\omega_1, \omega_2|a_1, a_2).$$

Their expected payoff functions are represented by the following payoff matrix:

	C	D
C	$1, 1$	$-y, 1+x$
D	$1+x, -y$	$0, 0$

We assume $x > 0$, $y > 0$ and $1 > x - y$, so that it indeed represents a PD.

All M PDs are played infinitely, in periods $t = 0, 1, 2, \ldots$. Player i's *private history* at the beginning of period $t \geq 1$ is an element of $H_i^t \equiv \big[\{C, D\}^M \times \{g, b\}^M\big]^t$. Let H_i^0 be an arbitrary singleton, and let $H_i = \cup_{t \geq 0} H_i^t$ be the set of player i's all private histories. Player i's strategy of this repeated game is a mapping from H_i to the set of all probability distributions over $\{C, D\}^M$. If the actual play of the repeated game is such that the action pair $\big(a_1^m(t), a_2^m(t)\big)$ is

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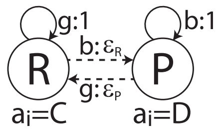

Figure 1: EV strategy

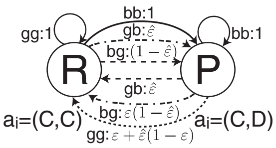

Figure 2: NTPD strategy

played in the m-th PD in period t for each m and t, player i's normalized average payoff is

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t\sum_{m=1}^{M}g_i\big(a_1^m(t),a_2^m(t)\big),$$

where $\delta \in (0,1)$ is their common discount factor. Let us herein focus on *belief-free equilibria*, which is a subclass of a *sequential equilibrium* [2]. A strategy pair is a belief-free equilibrium if for any $t \geq 0$, $h_1^t \in H_1^t$ and $h_2^t \in H_2^t$, each player i's continuation strategy given h_i^t is optimal against player j's continuation strategy given h_j^t.

Among belief-free equilibria, a benchmark strategy, which we call EV, is found by Ely and Välimäki [3] and is depicted in Figure 1. It is parameterized by two numbers, $\varepsilon_R \in [0,1]$ and $\varepsilon_P \in [0,1]$. A player first cooperates at state R, but after observing a bad signal, she defects at the next period with probability ϵ_R, or keep cooperation with $1-\epsilon_R$. Likewise, after she defects at P, if she observes a good signal, she returns cooperation with ϵ_P, or keep defection with $1-\epsilon_P$. They identify a sufficient condition for the existence of ε_R and ε_P in an equilibrium and derive the average payoff starting from state R is $V_R = 1 - \frac{(1-p)x}{2p-1}$.

3. NONLINEAR TRANSITION, PARTIAL DEFECTION STRATEGY

Let us introduce a novel class of strategies, which we call the *nonlinear transition, partial defection* (NTPD) strategy.

Definition 1 (NTPD strategy). *Given M PDs, an NTPD strategy for $M(\geq 2)$ PDs is a two-state automaton strategy, parameterized by an integer M_A such that $1 \leq M_A < M$ and two numbers $\varepsilon \in [0,1]$ and $\hat{\varepsilon} \in [0,1]$. Let $A = \{1,2,\dots,M_A\}$ and $B = \{M_A+1, M_A+2,\dots,M\}$.*

- *The state space is $\{R,P\}$, and R is the initial state.*
- *At state R, the player is prescribed to choose C in all PDs. At state P, she is prescribed to choose C in all PDs in A and D in all PDs in B.*
- *Suppose the current state is R and k is an integer between 0 and $M_B = M - M_A$. Then*
  1. *if b is observed among **all PDs in** $\mathbf{A}$ and there are k bad signals among the PDs in B, then the state shifts to P with probability $1-(M_B-k)\hat{\varepsilon}$ (and stays at R with the remaining probability).*
  2. *if g is observed among **some PD in** $\mathbf{A}$ and there are k bad signals among the PDs in B, then the state shifts to P with probability $k\hat{\varepsilon}$ (and stays R with the remaining probability).*
- *Suppose the current state is P and k is an integer between 0 and M_A. Then*
  1. *if g is observed among **all PDs in** $\mathbf{B}$ and there are k bad signals among the PDs in A, then the state shifts to R with probability $\varepsilon+\hat{\varepsilon}\big\{(1-\varepsilon)M_A - k\big\}$ (and stays P with the remaining probability).*
  2. *if b is observed among **some PD in** $\mathbf{B}$ and there are k bad signals among the PDs in A, then the state shifts to R with probability $(M_A-k)\hat{\varepsilon}$ (and stays P with the remaining probability).*

Figure 2 illustrates NTPD for two PDs. A player cooperates in all PDs in A at state P. Then, she always cooperates in all PDs in A whichever state she is in. For example, the transition probabilities from P to R are specified as follows. Their increase is constant for the number of bad signals from PDs in B. If she observes at least one bad signal from B, it is zero, otherwise, $\varepsilon - \hat{\varepsilon}\varepsilon M_A$. The transition probabilities decrease by $\hat{\varepsilon}$ in the number of bad signals k in A. For k bad signals from A, the transition probability from P to R is specified as $(M_A-k)\hat{\varepsilon}$ if she observes some b in B, or $\varepsilon + \hat{\varepsilon}\{(1-\varepsilon)M_A - k\}$ otherwise. We here mix $1-k\hat{\varepsilon}$ with $(M_A-k)\hat{\varepsilon}$ by the last parameter ε.

Theorem 1 (NTPD for M PDs). *There exist ε and $\hat{\varepsilon}$ such that the NTPD strategy pair is a belief-free equilibrium if*

$$\delta\Big[x(1-s^{M_A}) + s^{M_A-1}\Big\{M_B(p-s) - x(M_A-M_B)p - \frac{s^{M_B}(p-s)M_B y}{p^{M_B}-s^{M_B}}\Big\}\Big] \geq x(1+s^{M_A-1}M_B) \text{ and}$$

$$\delta\Big[(p^{M_B}-s^{M_B})\big\{M_B(p-s)+(M_A-M_B)x(s-s^{M_A})\big\} + M_B y(p-s)(1-s^{M_A}-s^{M_B})\Big] \geq M_A x(p^{M_B}-s^{M_B}) + M_B y(p-s)$$

hold. The average payoff starting from R is

$$V_R = M - \frac{\delta s^{M_A}(p-s)(M-V_P) + (1-\delta)(s-s^{M_A})M_B x}{(p-s)\big\{1-\delta(1-s^{M_A})\big\}},$$

where $V_P = M_A + \frac{pM_A x}{p-s} + \frac{s^{M_B}M_B y}{p^{M_B}-s^{M_B}}$.

We herein refer to $1-p$ as s for simplicity. Then, let us show that, if the numbers of PDs are sufficiently large, NTPD achieves a greater payoff than EV.

Corollary 1. *Fix x, y, and p. Suppose both M_A and M_B are sufficiently large and satisfy*

$$M_A p x - (M_B - 1)\{2p-1-(1-p)x\} \geq (1-p)y.$$

Then if NTPD is an equilibrium for sufficiently large δ, EV is equilibrium, but its payoff is smaller than that of NTPD.

In addition, our numerical analysis supports this result with a given discount factor ($\delta < 1$) and suggests that the transition probabilities to specify the behavior can still be improved. In future work, we would like to improve NTPD and to characterize an optimal equilibrium strategy class.

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