Multi-Option Descending Clock Auction (Extended Abstract)

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ABSTRACT

A descending clock auction (DCA) is for buying items from multiple sellers. The literature has focused on the case where each bidder has two options: to accept or reject the offered price. However, in many settings—such as the FCC's imminent incentive auction—each bidder may be able to sell one from a *set of options*. We present a multi-option DCA (MDCA) framework where at each round, the auctioneer offers each bidder different prices for different options, and a bidder may find multiple options still acceptable. Setting prices during a MDCA is trickier than in a DCA. We develop a Markov chain model for the dynamics of each bidder's state (which options are still acceptable). We leverage it to optimize the trajectory of price offers to different bidders for different options. This is unlike most auctions which only compute the next price vector. Computing the trajectory enables better planning. We reoptimize the trajectory after each round. Each optimization minimizes total payment while ensuring feasibility in a stochastic sense. We also introduce percentile-based approaches to decrementing prices. Experiments with real FCC incentive auction interference constraint data show that the optimization-based approach dramatically outperforms the percentile-based approach—because it takes feasibility into account in pricing. Both pricing techniques scale to the large.

Keywords

Descending clock auction; incentive auction; spectrum auction; combinatorial auction; auction optimization; planned auction trajectory; pricing

1. INTRODUCTION

A *descending clock auction (DCA)* is for buying items from multiple sellers [2]. The DCA framework is agnostic to how offered prices are decremented across rounds. Doing that well is a key problem for which no solutions had been published until recently. Nguyen and Sandholm recently presented techniques for this [3]. That paper—and, to our knowledge, all other papers on incentive auctions and on the DCA to date [2, 4]—consider the setting where bidders have only two options, that is, either to sell or not.

In contrast, in many settings, each seller may be able to sell *one from a set of options*. The DCA can be generalized to this setting by offering each bidder a separate price for each of her options in each round. However, the problem of decreasing prices appropriately during the DCA is more intricate in this *multi-option DCA (MDCA)* setting.

We present an MDCA framework and price-decrementing techniques for it. The model captures a broad set of applications, including the imminent flagship application of DCAs, the FCC incentive auctions. The FCC announced that a multi-option DCA will be used for the reverse auction [1], but left open the important question of how prices will be decremented across rounds. Ours is, to our knowledge, the first paper on pricing techniques for multi-option DCAs.

We present our MDCA in the domain of the FCC incentive auction, where the number of options per bidder is at most three (plus the option of rejecting all three options), but the techniques can be directly extended to more options. Bidders (stations) that are currently in the UHF band, have four choices: go off-air, go down to LVH, go down to UVH, or reject all of these options. Similarly, stations currently in UVH (LVH) have three (two) choices. Rejecting all options leads to being allocated to a channel in the original band.

In each DCA round, the auctioneer offers each bidder prices for all the options for which the bidder is still active. Each bidder then decides which of those options are still acceptable. As long as a bidder is still active for an option, she enters the next round where the same process will be repeated for the remaining active options. If a bidder becomes inactive for all options, she needs to be allocated to her current band and is not paid anything.

2. SETTING & DECREMENTING PRICES

A key component of a DCA is how the prices offered to active bidders are initialized and decremented across rounds. The auctioneer needs to consider the tradeoff between minimizing payment to the accepted bidders and fulfillment of the target (e.g. repacking feasibility). How the prices are changed across rounds should depend on (a) the estimated value functions of the bidders, (b) the importance of the items for the target to be fulfilled and (c) the desired number of rounds. We provide an optimization model that does this. To this end, we need to calculate the (expected) number of stations that will finally end up in each band—which is non-trivial. We propose using Markov chains to model the dynamic of the station's state throughout the MDCA.

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For each station i and for each option $k \in \{1,2,3\}$, suppose the valuations v_{ik} are drawn from some distribution on support $[l_{ik}, u_{ik}]$. Assume that the auctioneer knows these distributions but not the valuations. We denote by $p_{ik},\ k \in \{1,2,3\}$, the offer price to station i for option k and by q_{ik} the probability that station i finds price p_{ik} acceptable for option k. For example, if the valuation distribution is uniform, $q_{ik} = \frac{u_{ik} - p_{ik}}{u_{ik} - l_{ik}}$. For convenience, we regard $\boldsymbol{q}$ as the decision variables that the auctioneer sets.

We define the state of a station to depend on which of the three options are still active. Figure 1 shows our Markov chain for a UHF station's state throughout the MDCA; Markov chains for UVH and LVH stations are analogous but simpler, and are omitted due to limited space.

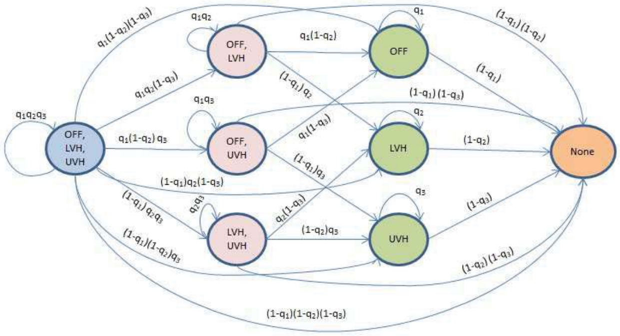

Figure 1: Markov chain for a UHF station.

THEOREM 1. *The state transition of the m-round MDCA with fixed acceptance probabilities* (q_1, q_2, q_3) *is equivalent to a single-round MDCA with acceptance probabilities* q_1^m, q_2^m, q_3^m.

The significance of Theorem 1 is that, instead of having to keep track of the transitions through m rounds, we can apply the change of variables presented in the theorem, and view the setting as a single-round MDCA. This helps simplify the expression of the state probabilities, and helps tame the complexity of the nonlinear model as we will show.

Optimization approach for price setting: We consider the setting where the auctioneer can offer different acceptance rates to different stations. Instead of seeking the same (q_1, q_2, q_3) for all stations, we aim to find the optimal (q_{i1}, q_{i2}, q_{i3}) for each station i. This gives flexibility and can lead to lower aggregate payment by the auctioneer.

Our OPT-SCHED model for minimizing expected payment while ensuring that the expected number of accepted bidders in each band does not exceed its target is

$$\min_{\boldsymbol{\kappa}} \sum_{i \in \mathcal{N}} f_i(\boldsymbol{\kappa_i})$$
$$\text{s.t.} \sum_{i \in \mathcal{N}} g_i(\boldsymbol{\kappa_i}) \le C_{UHF}, \sum_{i \in \mathcal{N}} h_i(\boldsymbol{\kappa_i}) \le C_{UVH}, \sum_{i \in \mathcal{N}} u_i(\boldsymbol{\kappa_i}) \le C_{LVH},$$

where $\kappa_{ik} = q_{ik}^m$ and $f_i(\boldsymbol{\kappa_i})$ is the expected payment to station i and where $g_i(\boldsymbol{\kappa_i}), h_i(\boldsymbol{\kappa_i}), u_i(\boldsymbol{\kappa_i})$ are the probabilities of station i ending up in UHF, UVH and LVH bands, respectively. The C's are the given targets. Here, we utilized Theorem 1 to transform from decision variable $\boldsymbol{q}$ to $\boldsymbol{\kappa}$ and show that the resulting model has a 4th-order polynomial on the objective function and 3rd-order polynomials on the constraints.

This problem has $3n$ continuous variables. Solving it directly is still not easy due to nonlinearity and nonconvexity. However, the problem has a separable objective and separable constraints. Hence we can apply Lagrangian relaxation.

Percentile-based approach for price setting: A simple way to adjust prices across MDCA rounds is to decrease each price by a given percentage—starting from the upper bound of the support of the station's valuation distribution for each of the options.

3. EXPERIMENTS

We tested our optimization-based price-decrementing technique against the percentile-based method using the real FCC constraint data. Since no incentive auctions have yet been conducted, we had to generate data on the bounds on the bidders' valuations. We did that based on population size covered by each license. (Results for the symmetric setting were very similar and are omitted due to limited space.)

We tried a number of possible acceptance probabilities in the range $\{0.97, 0.975, 0.98, 0.985, 0.99, 0.995, 1\}$ and report the sum of the prices of all active options of the second-to-last (i.e., last feasible) round for the asymmetric setting.[1] Figure 2 shows that OPT-SCHED yields lower final payment than the percentile-based approach—for all choices of the fixed per-round acceptance probability in the latter—on all instances. It results in 25% lower payment on average. It is able to reject high bids by taking into account feasibility considerations. It typically succeeds in proceeding through significantly more rounds before reaching infeasibility by more intelligently taking feasibility into account in the pricing.

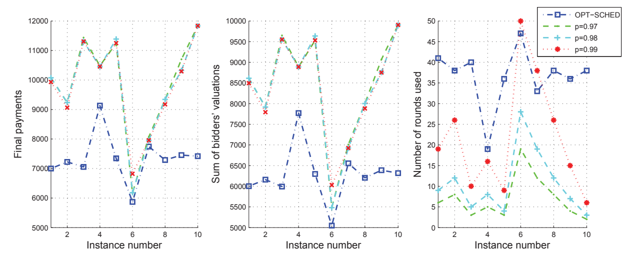

Figure 2: The sum of prices, the bidders' respective valuations, and the number of rounds used.

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[1]After the last round where the packing ceases to be feasible, the auctioneer has to decide an outcome for each bidder so as to minimize total payment. We developed a technique for this using decomposition of the station graph (into geographical regions) and Lagrangean relaxation.