Optimal Pricing for Efficient Electric Vehicle Charging Station Management

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ABSTRACT

The rapid development of Electric Vehicles (EVs) seen in recent years has been drawing increasing attentions from the public, markets, decision-makers, and academia. Notwithstanding the progress, issues still remain. Because of the widely complained disadvantages of limited battery capacity and long charging time, charging convenience has become a top concern that greatly hinders the adoption of EVs. Specialized EV charging station, which provides more than 10 times faster charging speed than domestic charging, is therefore a critical element for successful EV promotion.

While most existing researches focus on optimizing spatial placement of charging stations, they are inflexible and inefficient against rapidly changing urban structure and traffic pattern. Therefore, this paper approaches the management of EV charging stations from the pricing perspective as a more flexible and adaptive complement to established charging station placement. In this paper, we build a realistic pricing model in consideration of residential travel pattern and EV drivers' self-interested charging behavior, traffic congestion, and operating expense of charging stations. We formulate the pricing problem as a mixed integer non-convex optimization problem, and propose a scalable algorithm to solve it. Experiments on both mock and real data are also conducted, which show scalability of our algorithm as well as our solution's significant improvement over existing approaches.

Categories and Subject Descriptors

[**Social and professional topics**]: Pricing and resource allocation

General Terms

Management, Design, Algorithms, Experimentation

Keywords

Electric Vehicle; Charging Station; Pricing; Game Theory

1. INTRODUCTION

Electric Vehicles (EVs) are welcoming a rapid development along with progresses of relevant technologies in recent years. As an eco-friendly substitute for traditional fuel-engined vehicle, EV is seen as a promising solution to the ever devastating energy crisis and environmental pollution around the globe, thus has drawn increasing attentions from the public, markets, decision-makers, and academia. Many countries and cities have proposed plans to promote EV usage or have been preparing to do so, providing a foreseeable vision that EV will become the major vehicle of private transportation sector in the near future [1]. Notwithstanding the progresses, challenges still remain. Limited battery capacity and long charging time, probably the most widely complained disadvantages, raise mileage anxiety and largely impair EV users' driving experience. As a result, charging convenience has become a top concern affecting potential users' choice between EV and traditional fuel-engined vehicle. Specialized EV charging stations, which provide more than 10 times faster charging speed than domestic charging, are therefore critical to the successful promotion of EV.

While there have been some existing work concerning the management of EV charging stations, they mostly focused on the spatial placement of charging stations. For example, Frad et al. studied the placement and capacity allocation of EV charging stations for an area of Lisbon with the emphasis of maximizing coverage of charging demands [2]. Wong et al. proposed a muti-objective planning model for the placement of EV charging stations in Chengdu, China, with a solution based on demand and usage of existing gas stations [3]. Chen et al. particularly considered EV users' costs for accessing charging stations, and minimizing the costs and penalizing unmet demand [4]. Moreover, He et al. and Xiong et al. took a more broad view and emphasized the impact on overall efficiency of transportation system when optimizing the placement [5, 6]. However, a major drawback of the existing work is that such once-for-all solutions can hardly adapt to rapidly changing urban structures. Development of local infrastructure, such as opening-up of a new hospital, shopping mall, school, or housing estate, can all fundamentally modify the residential traffic pattern, making it unbalanced against the existing charging network. Thus follow-up adjustments are expected but might be costly and inefficient if we only rely on optimizing the placement.

To adapt to the urban structure change as well as varying

charging demand, a practical solution as we propose in this paper is to leverage charging price to readjust EV users' charging behavior and improve efficiency of the charging network. Compared with placement, pricing is easily and immediately implementable without additional cost or waste of resources. Dynamic pricing schemes adapt to either long-term changes of travel demand caused by residential movements or short-term variances between peak and non-peak time, and serve as a flexible complement to existing charging station placement. Our goal is to optimize the pricing scheme to optimize the efficiency of charging stations, i.e., to minimize the additional cost caused by EV users' charging behavior, which is referred as social cost. There have been some works leveraging dynamic pricing to improve efficiency of public transportation systems, such as taxi systems [7, 8]. Some works have particularly focused on real-time pricing and charge-discharge policy for EV management [9, 10]. However, their aim is merely to balance electricity load in power grids, while traffic condition is not in their consideration. Moreover, their method cannot be incorporated with trivial modifications because the traffic condition deeply relates to EV users' self-interested charing behavior associated with a graph-based road network, which are all absent from existing work.

In this paper, we take a game-theoretic perspective and build the problem on a non-atomic congestion game played by EV users. The model incorporates the following key features: 1) EV users' self-interested charging behavior that they strategically choose the best charging plan (i.e., where to charge and how to reach the charge station) to minimize their costs including charging fees, traveling time, and queuing time; 2) EV users' traffic pattern with complex spatial variances; 3) Traffic congestion in the road network that is affected by both the EVs and other external vehicles; and 4) A budget constraint that ensures sufficient income to support sustainable operation of the charging network. Using this model, we formulate the EV charging station pricing problem as a mixed integer non-convex optimization problem, and propose a scalable algorithm to solve the problem, in particular to deal with the large strategy space of the EVs. Experiments on both mock and real data are also conducted, which show scalability of our algorithm as well as our solution's significant improvement in social cost over existing approaches. A concrete instance is also used to visualize the difference between our approach and existing approaches.

2. MOTIVATION

Singapore is a city country with a vehicle ownership of around $970,000$ on its small territory of 720 km^2. As a highly developed metropolis with open attitudes toward cutting edge technologies, yet a country with limited natural resource and energy supply, Singapore is actively seeking the possibility of mass adoption of EVs to support its sustainable development. Ever since 2011, its authorities have started an EV test-bed to study the feasibility of EVs on its road. More recently, in the Government's sustainable blueprint to guide the country's development over the next 15 years launched in 2014, Singapore has even planned to lead an EV-sharing project to make the new technology even more
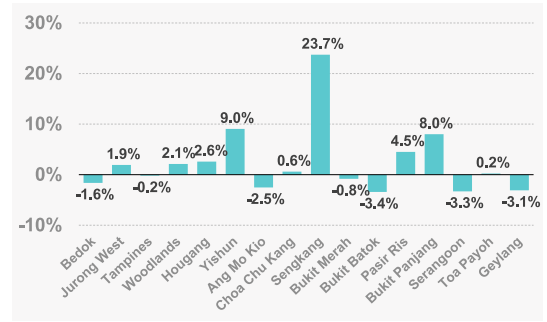

Figure 1: Population growth rate of major residential zones of Singapore from 2010 to 2015

convenient and environmentally-friendly.

Indeed, the relatively short driving distances on the small territory and the advanced power grid of Singapore make EVs a good option for this city. However, there are also many difficulties that require every step taken to be carefully planned. Because of the land scarcity and the fact that roads have already taken up 12 percent of Singapore's total land area, there is limited room for further expansion of Singapore's road network. This leaves Singapore a very high road density of $4.8km/km^2$, and a transportation system that is highly sensitive to any changes to the current transportation mode. Besides, Singapore is undergoing a rapid change in residential pattern along with its continuing development. As shown in Figure 1, population growth varies significantly among major residential zones of Singapore, indicating similar significant changes in residential traffic pattern. A sustainable plan therefore needs to be compatible to the current system while adaptable to future changes, to ensure a smooth transition toward the new EV-led transportation mode. This motivates our work and offers us a concrete study case.

3. PRELIMINARY

In this section, we introduce some notations and definitions that will be helpful to scenario visualization and be used in formulating the problem.

3.1 Notations

Considering the residential distribution of the studied city (e.g., Singapore), we divide the region to be analyzed (whole or part of the city) into a set $\mathcal{Z}$ of zones. There are roads linking the zones. Without loss of generality, we assume that there is at most one link between a pair of zones representing the average connectivity between them, and denote the set of links as $\mathcal{E}$ and the road network as a graph $\mathcal{G} = (\mathcal{Z}, \mathcal{E})$. In each zone $i \in \mathcal{Z}$, there are γ_i EVs owned by the residents who have some chance to charge in the charging stations. The γ_i EVs are furthermore classified into K_i groups according to their travel patterns, i.e., their daily travel routine as a set of most frequently visited zones. Each travel pattern is a set of connected zones that they visit daily. We denote by γ_{ij} the number of EVs in each group, and by $\mathcal{P}_{ij}$ their pattern for $j \in \mathcal{K}_i = \{1, \ldots, K_i\}$. The union of all patterns is denoted by $\mathcal{P} = \bigcup_{i \in \mathcal{Z}, j \in \mathcal{K}_i} \mathcal{P}_{ij}$.

Given the number τ_i (≥ 0) of chargers in each zone i, our goal is to calculate the charging rate (i.e., per unit electricity price) x_i to be set at each zone, such that the social cost (to be defined later) is minimized. Accordingly, we denote the set of feasible price as $\mathcal{X}$.

3.2 Factors Affecting EVs' Decision

EVs make decisions about where to charge and how to reach the charging zone according to the estimated charging cost, which consists of three parts: charging fee, travel cost and queuing cost. The charging fee is the variable to be optimized in this work. In the following, we introduce the definition of travel cost and queuing cost.

Travel Cost. There are furthermore two kinds of travel cost: 1) *cost on link*, i.e., cost for travelling between zones, and 2) *cost on node*, i.e., cost for traveling within the zone where the EV charges. The cost on a link depends on the length and the traffic congestion level of this link. Generally, more vehicles on the road results in higher congestion level, and larger road capacity leads to lower congestion. We adopt a widely used linear model of traffic congestion taken as the ratio of the traffic flow to the road capacity (both in the number of vehicles). Thus, for a link $(i, i') \in \mathcal{E}$, given the length $d_{ii'}$, the capacity $C_{ii'}$ and traffic flow $f^0_{ii'} + f_{ii'}$ (we distinguish flow of EVs heading for charging, i.e., $f_{ii'}$, with flow of other vehicle, i.e., $f^0_{ii'}$, which is assumed to be constant), the traffic congestion $\alpha_{ii'}$ of link (i, i') is presented in Eq. (1) [11]. Travel cost on (i, i') is defined as a function of its length and traffic congestion as Eq. (2).

$$\alpha_{ii'} = \frac{f^0_{ii'} + f_{ii'}}{C_{ii'}} \qquad (1)$$

$$t_{ii'} = d_{ii'} \alpha_{ii'} \qquad (2)$$

Meanwhile, when an EV chooses to charge in a zone i, there is extra travel cost, i.e., cost on node, as she drives off the main road to access the charging station within zone i. Considering that a zone includes more internal roads (than in between two zones) and that the EVs coming for charging does not have to traverse all of them, we add a discount factor ζ to denote the EVs' influence on congestion in the zone. In this case, a similar function as Eq. (2) is used for the extra travel cost in the charging zone:

$$t_i = d_i \frac{f^0_i + \zeta f_i}{C_i}, i \in \mathcal{Z}, \qquad (3)$$

where d_i denotes the radius of the zone, f^0_i is the normal traffic amount, f_i is the total number of EVs that choose to charge in zone i and C_i is the capacity of the zone regarding all its travel network.

Queuing Cost. The queuing cost depends on the number of chargers in the charging station and the number of EVs that come to the charging station. We use a linear model to denote the relationship among them as Eq. (4). Recalling that τ_i denotes the number of chargers in zone i, let q_i be the queuing cost and f_i be the number of EVs charging there, then we have

$$q_i = \frac{f_i}{\tau_i}, i \in \mathcal{Z}. \qquad (4)$$

4. EVS' CHARGING BEHAVIOR

From a game-theoretic perspective [12], when we optimize charging station management, we need to take into account the strategic behavior of EV owners. Namely, they are self-interested and profit-driven, such that they will respond to our pricing with the best charging strategy to minimize their charging cost. Next, we explicitly explain the charging game - the model of EVs' charging behavior. To distinguish a zone that EVs reside in from one EVs charge in, we use i and z respectively to denote a zone in the follows.

4.1 Charging Strategy

Each EV chooses a pure charging strategy and the strategies of the EVs in the same zone of the same travel pattern form a distribution over their strategy space. A pure charging strategy is to choose a zone with charging stations installed, and an additional travel path from a zone on her daily routine to the charging zone and back if the chosen zone is not in the EV's travel pattern. Thus a pure charging strategy can be denoted as a tuple **s** with

$$\mathbf{s} = \begin{cases} \{z\}, z \in \mathcal{P}_{ij} \\ \{z', (z, z'), (z', z) \mid z \in \mathcal{P}_{ij}, z' \in \mathcal{Z} \setminus \mathcal{P}_{ij}, \} \end{cases}, \qquad (5)$$

where both (z, z') and (z', z) are in set $\mathcal{E}$. Note that by Eq. (5), we only consider EVs' charging zones inside or adjacent to zones in their travel pattern and assume that they do not charge in farther places. This is because the distance to those places is usually much farther than a single hop, thus causing higher travel cost and is unlikely to happen in reality. Experimental results in Section 6.3 verify that this assumption is reasonable. We then use $\mathcal{S}_{ij}$ to denote the strategy space for the EVs in zone i of travel pattern j. Furthermore, $\mathcal{S} = \bigcup_{i \in \mathcal{Z}, j \in \mathcal{K}_i} \mathcal{S}_{ij}$ is used to denote the strategy space union for all the EVs in the studied region. For example, there are 6 zones illustrated in Figure 2, and
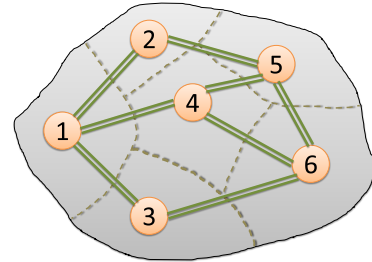

Figure 2: Zone division illustration

we suppose there are chargers in all zones. If a group of EVs' daily travel routine includes zones 1, 4 and 6, their strategy space includes these 8 pure strategies:

$$\mathbf{s}_1 = \{1\}, \quad \mathbf{s}_2 = \{4\}, \quad \mathbf{s}_3 = \{6\}, \quad \mathbf{s}_4 = \{2, (1, 2), (2, 1)\},$$

$$\mathbf{s}_5 = \{3, (1, 3), (3, 1)\}, \quad \mathbf{s}_6 = \{5, (4, 5), (5, 4)\},$$

$$\mathbf{s}_7 = \{5, (6, 5), (5, 6)\}, \quad \mathbf{s}_8 = \{3, (6, 3), (3, 6)\}.$$

When the charging zone is in their routine, there is no links in the strategy (e.g., $\mathbf{s}_1$, $\mathbf{s}_2$ and $\mathbf{s}_3$). Otherwise (e.g., $\mathbf{s}_4$ to

$\mathbf{s}_8$), there are two additional links specifying a round trip to the charging zone.

EVs choose charging strategy according to charging cost, including charging fee, travel cost and queuing cost. In reverse, their strategies also influence congestion level and queueing time, i.e., travel cost and queuing cost. We denote the strategy distribution of EVs of group j in the zone i as $\mathbf{p}_{ij}$, with each $p_{ij}(\mathbf{s})$ denoting the proportion of γ_{ij} EVs using pure strategy $\mathbf{s}$ in the group of EVs' strategy space $\mathcal{S}_{ij}$.

Given a strategy profile $\mathbf{P} = \{\mathbf{p}_{ij}\}$, the number of EVs in each charging station $z \in \mathcal{Z}$ and the number of EVs on each link $e \in \mathcal{E}$ can then be seen as functions of the $\mathbf{P}$ as:

$$f_z(\mathbf{P}) = \sum_{\mathbf{s} \in \mathcal{S}: z \in \mathbf{s}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i: \mathbf{s} \in \mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \qquad (6)$$

$$f_e(\mathbf{P}) = \sum_{\mathbf{s} \in \mathcal{S}: e \in \mathbf{s}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i: \mathbf{s} \in \mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \qquad (7)$$

Similarly, travel cost t for link e and zone z, and queuing cost q for zone z are respectively defined as:

$$t_e(\mathbf{P}) = \frac{d_e}{C_e} \left(f_e^0 + \sum_{\mathbf{s} \in \mathcal{S}: e \in \mathbf{s}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i: \mathbf{s} \in \mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \right), \qquad (8)$$

$$t_z(\mathbf{P}) = \frac{d_z}{C_z} \left(f_z^0 + \zeta \sum_{\mathbf{s} \in \mathcal{S}: z \in \mathbf{s}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i: \mathbf{s} \in \mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \right), \qquad (9)$$

$$q_z(\mathbf{P}) = \frac{1}{\tau_z} \sum_{\mathbf{s} \in \mathcal{S}: z \in \mathbf{s}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i: \mathbf{s} \in \mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}). \qquad (10)$$

Apart from the above costs, EVs also consider their charging fees, denoted by x_z, which vary at different zones. The weights ω_1, ω_2, and ω_3 are assigned to the three types of costs respectively. Thus, given the electricity price x_z in zone z, the charging cost of an EV using charging strategy $\mathbf{s}$ under strategy profile $\mathbf{P}$ is:

$$C_{ij}(\mathbf{P}, \mathbf{s}) = \sum_{z \in \mathbf{s}} \left(\omega_1 q_z(\mathbf{P}) + \omega_2 t_z(\mathbf{P}) + \omega_3 x_z \right) + \sum_{e \in \mathbf{s}} \omega_2 t_e(\mathbf{P}) \qquad (11)$$

for all zone $i \in \mathcal{Z}$, group $j \in \mathcal{K}_i$ and pure strategy $\mathbf{s} \in \mathcal{S}_{ij}$.

4.2 Equilibrium

We adopt Nash equilibrium pin *non-atomic* congestion game as our solution concept. A non-atomic congestion game is one that is played by an uncountably large number of players (which is exactly the case in our problem, with around $30,000$ EVs in the system), so that each agent's effect on the congestion level is negligibly small. It is widely used to model congestion scenario with a large number of agents, which is exactly our case. In an equilibrium state, no EV can decrease her charging cost by unilaterally changing her charging strategy. Specifically, for each EV group j in zone i, the charging cost of all pure strategies that are used with non-zero probability are the same and the minimal, i.e.,

$$C_{ij}(\mathbf{P}, \mathbf{s}) \leq C_{ij}(\mathbf{P}, \mathbf{s}') \quad \forall \mathbf{s} \in \mathcal{S}_{ij}, \ p_{ij}(\mathbf{s}) > 0 \qquad (12)$$

In this case, no EV has incentive to unilaterally change her charging strategy.

4.3 Pricing Problem for EV Charging Station Management

As we have mentioned before, our goal is to minimize the social cost, denoted as SC. Specifically, we consider the extra social cost incurred by EVs' charging behavior, which is measured with the congestion experienced by all EVs in charging stations and extra congestion caused by EVs' charging behavior for *all* vehicles in the road network, i.e.,

$$SC = \nu_1 \sum_{z \in \mathcal{Z}} f_z(\mathbf{P}) q_z(\mathbf{P}) + \qquad (13)$$

$$\nu_2 \sum_{z \in \mathcal{Z}} \left((f_z^0 + f_z(\mathbf{P})) t_z(\mathbf{P}) - f_z^0 t_z(\mathbf{0}) \right) + \qquad (14)$$

$$\nu_2 \sum_{e \in \mathcal{E}} \left((f_e^0 + f_e(\mathbf{P})) t_e(\mathbf{P}) - f_e^0 t_e(\mathbf{0}) \right), \qquad (15)$$

where the first component represents queuing cost for EVs in all zones, weighted with ν_1; and the second and the third components respectively represent additional travel cost in each zone and on each link for all vehicles, weighted with ν_2. It suffices for us to formulate the pricing problem for EV charging station management, which turns out to be a non-convex optimization problem **PCS** as follows.

PCS:
$$\min_{\mathbf{x}, \mathbf{P}} \quad SC \qquad (16)$$

$$\text{s.t.} \quad p_{ij}(\mathbf{s}) C_{ij}(\mathbf{P}, \mathbf{s}) \leq p_{ij}(\mathbf{s}) C_{ij}(\mathbf{P}, \mathbf{s}'),$$
$$\forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s}, \mathbf{s}' \in \mathcal{S}_{ij} \quad (17)$$

$$\sum_{z \in \mathcal{Z}} f_z(\mathbf{P}) x_z \geq B \qquad (18)$$

$$x_z \in \mathcal{X}, \qquad \forall z \in \mathcal{Z} \qquad (19)$$

$$p_{ij}(\mathbf{s}) \geq 0, \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s} \in \mathcal{S}_{ij} \qquad (20)$$

$$\sum_{\mathbf{s} \in \mathcal{S}_{ij}} p_{ij}(\mathbf{s}) = 1, \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i \qquad (21)$$

Note that Eq. (17) functions as the equilibrium criteria in Eq. (12), i.e., when $p_{ij}(\mathbf{s}) = 0$, it holds unconditionally, and when $p_{ij}(\mathbf{s}) > 0$, it is equivalent to $C_{ij}(\mathbf{P}, \mathbf{s}) \leq C_{ij}(\mathbf{P}, \mathbf{s}')$; Eq. (18) is a budget constraint requiring that the income of all charging station can at least cover their management and operation expenses; we suppose the charging rate in each zone is selected from a price set $\mathcal{X}$; and the last two constraints are to bound the p variables.

5. COMPUTING OPTIMAL PRICE

In this section, we present our algorithm for problem **PCS**. Problem **PCS** is a non-convex quadratic optimization problem, with its objective function SC being quadratic, and the first constraint (i.e., Eq. (17)) being non-convex. Besides, the scale of problem **PCS** is very large because EVs have many travel patterns and, moreover, each pattern may contain many zones, which amounts to a large strategy space and a similarly large set of variables for problem **PCS**. Therefore, problem **PCS** is hard to solve (particularly hard to scale up). To resolve the problem, we first rewrite constraint (17), and reformulate **PCS** to the following *binary*

programming **PCS**-*binary* with an additional set of binary variables $\mathbf{y} = \langle y_{ij}(\mathbf{s}) \rangle$.

PCS-*binary*:

$$\min_{\mathbf{x},\mathbf{P},\mathbf{y}} \quad SC \tag{22}$$

$$\text{s.t.} \quad y_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}) \le y_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}'),$$
$$\forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s}, \mathbf{s}' \in \mathcal{S}_{ij} \tag{23}$$

$$p_{ij}(\mathbf{s}) \le y_{ij}(\mathbf{s}), \ \forall i \in \mathcal{Z}, j \in \mathcal{K}_i, \mathbf{s} \in \mathcal{S}_{ij} \tag{24}$$

$$y_{ij}(\mathbf{s}) \in \{0,1\}, \ \forall i \in \mathcal{Z}, j \in \mathcal{K}_i, \mathbf{s} \in \mathcal{S}_{ij} \tag{25}$$

$$\text{Eqs. } (18) - (21)$$

Eqs. (23)–(24) are modified from Eq. (17) with the auxiliary variable $\mathbf{y}$. As we can see, $y_{ij}(\mathbf{s})$ is an indicator for whether $\mathbf{s}$ can be used with *non-zero* probability in the solution, i.e., when $y_{ij}(\mathbf{s}) = 0$, we have (by Eq. (24)) $0 \le p_{ij}(\mathbf{s}) \le 0 \Rightarrow p_{ij}(\mathbf{s}) = 0$, and when $y_{ij}(\mathbf{s}) = 1$, we have $0 \le p_{ij}(\mathbf{s}) \le 1$. Therefore, Eqs. (23)–(24) are equivalent to (17), and **PCS**-*binary* is equivalent to **PCS**. To solve **PCS**-*binary*, a brute-force way is to exhaustively try all the 0/1 combinations in the feasible space $\{0,1\}^{|\mathbf{y}|}$ for $\mathbf{y}$. When $\mathbf{y}$ is fixed, **PCS**-*binary* becomes a quadratic programming with linear constraints, which is relatively easy to solve. We then propose a Strategy Space Generation Algorithm (*SSGA*) to speed up the brute-force search, which is sketched with Algorithm 1.

SSGA starts with an initialized vector $\mathbf{y}^*$ (Line 1) and, repeatedly, solves **PCS**-*binary* with $\mathbf{y}^*$, and updates $\mathbf{y}^*$ with two key procedures Rule A (Lines 6–8) and Rule B (Lines 11–15), until no update is made on $\mathbf{y}^*$ in some iteration (Line 16). Specifically,

- Rule A disables strategies $\mathbf{s}$ that are chosen with very small probability (i.e., $p_{ij}(\mathbf{s}) < \delta$ with $0 < \delta \ll 1$) by setting $y_{ij}(\mathbf{s}) = 0$, so that they will not be used in the next iteration. The intuition behind Rule A is that when $p_{ij}(\mathbf{s})$ is very closed to 0, setting it to 0 will not cause much change to the cost of the EVs, but can significantly expand the feasible space as the associate constraint in Eq. (23) is relaxed.

- Rule B checks if there are any unused strategies that could potentially lower EVs' cost, and enables them by setting $y_{ij}(\mathbf{s}) = 1$ when they are found. Intuitively, these newly enabled strategies are EVs' better responses for the current strategy profile.

Finally, Proposition 1 shows that *SSGA* always converges to a Nash equilibrium. The price $\mathbf{x}^*$ it returns is thus the optimal price under the equilibrium.

PROPOSITION 1. *The algorithm SSGA always converges to an equilibrium charging strategy profile.*

PROOF. Recalling that we use *Rule B* to validate whether or not we need to keep on the iteration of upgrading the set of used pure strategies and *Rule B* is exactly using the equilibrium criteria as stated in Eq.(12), we can always ensure the strategy distribution $\mathbf{P}$ in the solution is an equilibrium. □

Algorithm 1: SSGA

1 $\mathbf{y}^* \leftarrow$ Initialize as a binary vector;
2 $\mathbf{x}^* \leftarrow$ *Null*;
3 repeat
4 $\langle \mathbf{x}^*, \mathbf{P}^* \rangle \leftarrow$ Fix $\mathbf{y}$ to $\mathbf{y}^*$ and solve **PCS**-*binary*;
5 $\mathbf{y}' \leftarrow \mathbf{y}^*$;
    /* -------------- Rule A -------------- */
6   **for** *each* $i \in \mathcal{Z}$, $j \in \mathcal{K}_i$, $\mathbf{s} \in \mathcal{S}_{ij}$ **do**
7   **if** $p_{ij}^*(\mathbf{s}) < \delta$ **then**
8 $y_{ij}^*(\mathbf{s}) \leftarrow 0$;
9   **if** $\mathbf{y}^* \ne \mathbf{y}'$ **then**
10   **goto** Line 4;
    /* -------------- Rule B -------------- */
11   **for** *each* $i \in \mathcal{Z}$, $j \in \mathcal{K}_i$ **do**
12 $C_{ij}^{min} \leftarrow \arg\min_{\mathbf{s}:\, \mathbf{s} \in \mathcal{S}_{ij} \wedge p'_{ij}(\mathbf{s}) > 0} C_{ij}(\mathbf{P}^*, \mathbf{s})$;
13   **for** *each* $\mathbf{s} \in \mathcal{S}_{ij}$ **do**
14   **if** $y'_{ij}(\mathbf{s}) = 0$ *and* $C_{ij}(\mathbf{P}^*, \mathbf{s}) \le C_{ij}^{min}$ **then**
15 $y_{ij}^*(\mathbf{s}) \leftarrow 1$;
16 until $\mathbf{y}^* = \mathbf{y}'$;
17 return $\mathbf{x}^*$;

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6. EXPERIMENTAL RESULTS

In this section, experimental results are provided to verify the optimality and scalability of the proposed approach *SSGA*, and to present the improvement of traffic system performance provided by our approach. All computations are performed on a 64-bit machine with 16 GB RAM and a quad-core Intel i7-4770 3.4 GHz processor. All standard optimization problems, such as Line 4 of Algorithm 1, are solved with KNITRO 9.0.0.

Data of Singapore. We divide Singapore into 23 zones as Figure 3 according to the official planning-area information [13] and other geographic information. According to statistic data from the Department of Statistics, Ministry of Trade and Industry, Singapore [14], 23% of the residents usually drive to work. Besides, there are more than $972,000$ vehicles in year 2014, among which more than $600,000$ are cars or station-wagons. We suppose 5% of the $600,000$ vehicles are EVs that charge in the charging game, then the total number of $30,000$ EVs are assigned to different zones according to the residential population distribution. The traffic flow and road capacity for each zone and link are estimated according to Google real-time traffic map. The price set is set as $\mathcal{X} = \{1, 1.5, 2, 2.5, 3\}$, according to the charging fee of charging stations in the U.S. [15]. The discount factor in computing the traffic congestion inside a zone is set as $\zeta = 0.5$. The weights for different parts in social cost and EVs' charging cost are set as: $\nu_1 = 0.8$, $\nu_2 = 0.2$; $\omega_1 = 0.1$, $\omega_2 = 0.3$ and $\omega_3 = 0.6$. The reason for this setting is to make sure that charging fee, travel cost and queuing cost are comparable. Except in Section 6.1, where we use a set of mock data of the traffic network and

EV population to test the solution quality and scalability of *SSGA*, all other experiments are based on the data of Singapore.
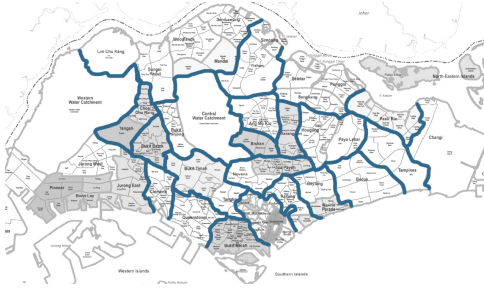

Figure 3: Singapore map and zone division

Initializing the Binary Indicators for *SSGA*. The initial value of the indicators in Line 1 of the algorithm *SSGA* significantly influences the accuracy and speed of the approach. We apply the following method for initializing starting indicators for *SSGA*. We first compute the estimate charging cost of each pure strategy assuming that there is only one EV that charges in the charging zone of that pure strategy and ignoring the charging fee. Formally, The estimate charging cost $\widetilde{C}_{ij}(\mathbf{s})$ for EVs of each pattern j in each zone i is computed as

$$\widetilde{C}_{ij}(\mathbf{s}) = \sum_{z \in \mathbf{s}} \left(\omega_1 \frac{1}{\tau_z} + \omega_2 \frac{d_z}{C_z}(f_z^0 + \zeta) \right) + \sum_{e \in \mathbf{s}} \omega_2 \frac{d_e}{C_e}(f_e^0 + 1). \tag{26}$$

Then we select the strategies whose estimated charging costs are no more than twice of the minimum of them and set their indicators as 1. That is to say, we set the indicator $y_{ij}(\mathbf{s}) = 1$ for all the $\mathbf{s} \in \mathcal{S}_{ij}$ with $\widetilde{C}_{ij}(\mathbf{s}) \leq 2 \min_{\mathbf{s}' \in \mathcal{S}_{ij}} \widetilde{C}_{ij}(\mathbf{s}')$.

Virtual Charging Station Placement for Experiments on Data of Singapore. Since the charging station network in Singapore are not settled yet, we use virtual placements of EV charging stations for our experiments. For the total number of 30,000 EVs in the charging game, we assign a total number of 2,000 chargers to the charging stations in the region. Three kinds of placement are used according to the following rules, respectively.

A1 Placement according to population distribution. Namely, the chargers are distributed proportionally according to the population in each zone, i.e., $\tau_z \propto \gamma_z, \forall z \in \mathcal{Z}$.

B1 Placement according to current gas station distribution. This rule refers to the current gas station distribution in Singapore and assign chargers proportionally according to the number of gas stations in each zone. Formally, assume that there are τ_z^{Gas} gas stations in zone z, the number of chargers in this zone $\tau_i \propto \tau_z^{Gas}$.

C1 Placement considering traffic congestion. The number of chargers in a zone is set proportional to the inverse of the normal congestion (regardless of the charging EVs) inside the zone, i.e., $\tau_z \propto \frac{C_z}{f_z^0}, \forall z \in \mathcal{Z}$.

6.1 Solution Quality and Scalability of SSGA

For experiments in this part, we generate a set of mock data, because we need problems of different scales to verify the optimality and scalability of *SSGA* through comparing with **PCS**.

Mock Data. We generate mock data using a Java program. First, the number of zones n is specified and the budget is set as $100n$. The traffic network is randomly generated by building a two-way link between any pair of zones with probability $4.5/n$. After the construction of the travel network, we randomly set the number of travel patterns in each zone as one of the elements in the set $\{1, 2, 3\}$ and we randomize the number of EVs of each pattern between 50 and 100. The traffic capacity and external flow (i.e., f_z^0 and f_e^0) in zones and links are randomized as integers in $[140, 160]$ and $[100, 200]$, respectively. For the charging stations' location and size, we use two methods to set them up. In the following are the two **charging station placement plans on mock data**.

A2 We randomly choose some of the zones and assign 10 chargers to each of them. The expectation of the number of zones with chargers is $n/2$.

B2 We first calculate an index value θ_z for each zone $z \in \mathcal{Z}$ as following

$$\theta_z = \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i : z \in \mathcal{P}_{ij}} \gamma_{ij}.$$

This index value reflects how many EVs visit zone z frequently. Then we assign a number $5n$ of chargers to the zones proportionally, i.e., $\tau_z \propto \theta_z, \forall z \in \mathcal{Z}$.

To test the optimality and scalability of *SSGA*, we generate different problems with n ranging from 5 to 12. Since the travel network, travel patterns and charging stations are randomly generated, the size of strategy space (i.e., $|\mathcal{S}|$), as well as the average size of strategy space for each travel pattern of EVs (i.e., $|\mathcal{S}|/|\mathcal{P}|$), are also randomized and does not have to increase with n (refer to "$|S|/\#$Pattern" curve in Figure 4(a) for the variation trend). We then solve **PCS** and *SSGA* based on the above described mock data and the corresponding charging station placement **A2** and **B2**.

Experimental results are shown in Fig 4. When we increase the number of zones, the size of the strategy space also increases accordingly. In Figure 4(a) and Figure 4(c), the running time of both approaches under different charging station placement plans is respectively described. It is shown that the running time of **PCS** does not monotonously increase with the number of zones, but corresponds to the "$|S|/\#$Pattern" curve. This is because the latter decides the complexity of the problem. For better visualization, we depict the relationship between running time and the average strategy space $|\mathcal{S}|/|\mathcal{P}|$ in Figure 4(b), from which we can see that the running time increases accordingly when $|\mathcal{S}|/|\mathcal{P}|$ increases. In Figure 4(c), as we are using the charging station plan **B2**, the number of zones with chargers increases, so does the problem scale. In this case, **PCS** cannot handle the problem even for a small graph. Obviously, our approach *SSGA* drastically decreases the running time. As we can see from Figure 4(d), *SSGA* always results in very close optimal
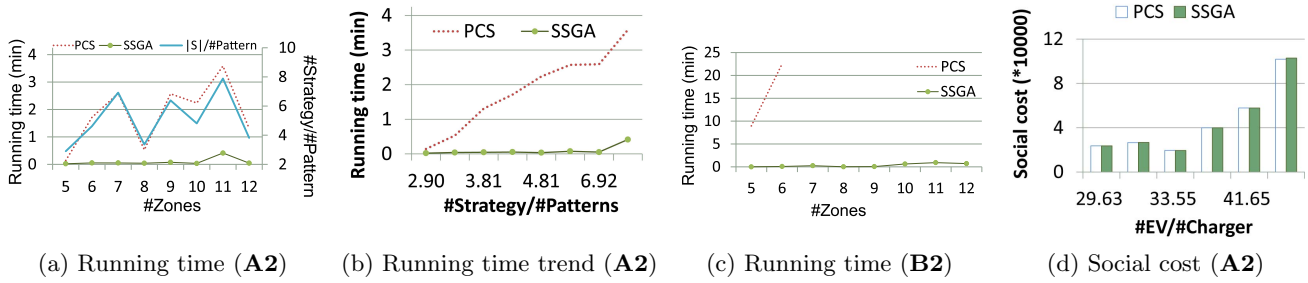
(a) Running time (**A2**)   (b) Running time trend (**A2**)   (c) Running time (**B2**)   (d) Social cost (**A2**)

Figure 4: Optimality and scalability of *SSGA* comparing with **PCS**: (a) illustrates the running time of **PCS** and *SSGA* on the primary y axis, while on the secondary y axis showing the total number of pure strategies divided by the total number of EV patterns (i.e., the average size of strategy space for each pattern of EVs) corresponding to the charging station placement **A2**; (b) shows the relationship between the running time and the average size of strategy space for each pattern of EVs; (c) depicts running time results corresponding to the charging station placement **B2**; and (d) depicts the social cost of both **PCS** and *SSGA* for placement **A2**;

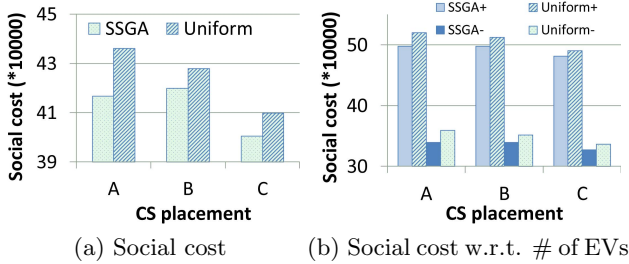(a) Social cost   (b) Social cost w.r.t. # of EVs

Figure 5: Comparing *SSGA* with uniform pricing under different charging station placement plans **A1**, **B1** and **C1**: (a) the number of EVs is estimated as we stated before; and (b) we increase and decrease the number of EVs by 10%.

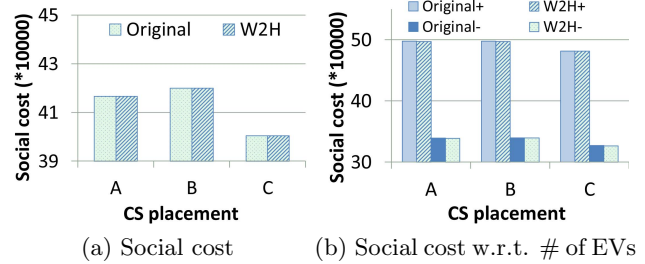(a) Social cost   (b) Social cost w.r.t. # of EVs

Figure 6: Social cost of *SSGA* on original data and with two-step hop charging strategies: (a) the number of EVs is estimated as we stated before; and (b) the number of EVs in each travel pattern is increased/decreased by 10%.

social cost to **PCS**. Note that the social cost increases with the number of EVs divided by the number of chargers, i.e., the average number of EVs that a charger needs to serve.

6.2 Advantages over Uniform Pricing

We apply our approach to the data of Singapore and compare our pricing policy with the benchmark - uniform pricing (i.e., set charging rate in all the charging stations as the same), which represents no utilization of pricing measure for improving traffic system performance. We then conduct experiments according to different charging station placement plans **A1** to **C1** for both *SSGA* and uniform pricing. In Figure 5(a), the number of EVs in each zone is set as we stated before. We can see that *SSGA* largely decreases the social cost, especially when the social cost is higher. We then increase/decrease the number of EVs in different extents to see what is the difference between the two methods when the EV density in the region is different. In Figure 5(b), the legends "*SSGA*+" and "Uniform+" refer to the results when the number of EVs is increased by 10%; similarly, the legends "*SSGA*-" and "Uniform-" refer to the results when the number of EVs is decreased by 10%. We find that when the number of EVs increases, the advantage of distinct pricing computed by *SSGA* also increases.

6.3 Two-step Hop Charging Strategies

In this part, we release the assumption that EVs only charge in her routine zones or adjacent ones of them to see what will happen to the optimal social cost and the equilibrium strategy profile. We choose zone 1 for the test by adding up all the 2-step hop charging strategies $\mathbf{s}'$ for the each patterns j EVs in zone 1:

$$\mathbf{s}' = \{z'', (z, z'), (z', z''), (z'', z'), (z', z)\}, \qquad (27)$$

where z is a zone in the travel pattern $\mathcal{P}_{1j}$, but z' and z'' are not. We then conduct experiments on original data and data with varied number of EVs (increas/decreas by 10% as we used in Section 6.2). We find that although the two-step hop strategies are added to the EVs' strategy space, they are never used and the optimal social cost never changes, which is shown in Figure 6. Furthermore, we find that the charging cost of those two-step hop strategies is much larger than those employed strategies. We conclude that it is reasonable to ignore these strategies.

6.4 Adaption to Population Change

The population in cities changes in amount as well as in distribution along with the city development. A concrete example is what we show in Section 2 about Singapore. Once

the charging stations are settled in the city, it is costly to modify their layout although that the traffic system performance will decrease along with city development, which directly leads to citizens' travel pattern change. Thus we propose to use adaptive dynamic pricing to accommodate changes in population density and travel patterns, thus to mitigate the traffic congestion and decrease the social cost.
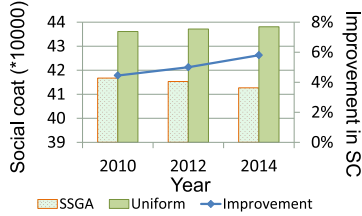

Figure 7: The optimal social cost of *SSGA* and uniform pricing, as well as the system efficiency improvement along with time.

We first arrange the charging stations according to plan **A2** for Singapore in year 2010. Based on that charging station placement, we then conduct experiments for population distribution in year 2012 and 2014, respectively. The results in social cost of both *SSGA* and uniform pricing are depicted in Figure 7, where the x axis denotes the year, and the primary and secondary y axis denote social cost and the percentage of improvement in social cost, respectively. As we can see, *SSGA* is quite adaptive to the population density change and it always decreases the social cost by a considerable amount. Furthermore, the "Improvement" curve shows that the decrease in social cost (i.e., improvement in system performance) increases with time. It turns out that when the system degenerates, *SSGA* performs even better.

6.5 Sensitivity and Robustness

In the above sections, the number of EVs in each zone of each pattern is accurately estimated. Considering that there might be some deviation between estimation and true values, we test the sensitivity of our approach regarding the number of EVs and consider the case that the estimated number of EVs is not accurate. We first compute the optimal price **x** according to our estimation. Then we compute the social cost with fixed **x** and EV number deviation by letting $\gamma_i' = \gamma_i(1 \pm \varepsilon)$ with $\epsilon = 5\%$ or 10%.

As we can see in Figure 8, *SSGA* always achieves better performance (i.e., results in lower social cost) than uniform pricing even when the estimation is not precise. Thus our pricing policy is robust regarding to the uncertainty of estimation of the number of EVs. Besides, according to the social cost of uniform pricing and the "Improvement" curve, we can see that when the social cost is higher, *SSGA* actually outperforms uniform pricing more.

7. CONCLUSION & APPLICATION

In this paper, we take a game-theoretic perspective to study the EV charging station pricing problem motivated by

(a) # of EVs decrease by 5% (b) # of EVs decrease by 10%

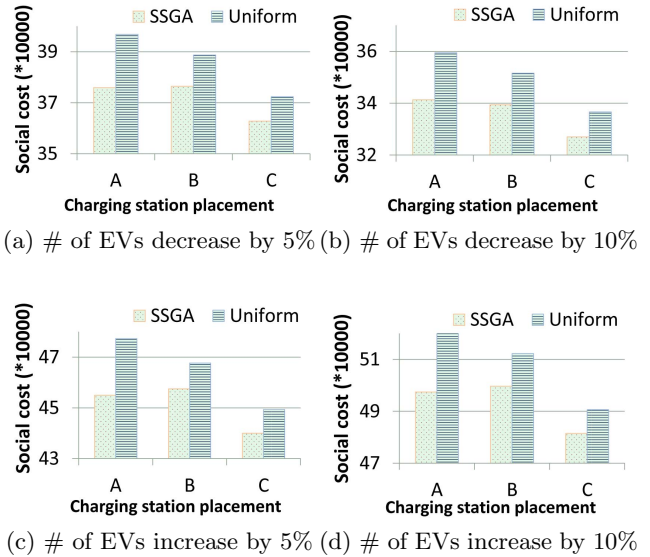(c) # of EVs increase by 5% (d) # of EVs increase by 10%

Figure 8: Robustness test of *SSGA* comparing with uniform pricing regarding to deviation in estimation of the number of EVs: (a) the actual number of EVs of each pattern is 5% less than the estimation; (b) the actual number of EVs of each pattern is 10% less than the estimation; (c) the actual number of EVs of each pattern is 5% more than the estimation; and (d) the actual number of EVs of each pattern is 10% more than the estimation.

the practical need of EV promotion in Singapore. Our first contribution in this paper is a novel pricing model that comprehensively incorporates EV users' self-interested charging behavior and their various traffic patterns, traffic congestion contributed by EVs and other non-EV vehicles in the road network, as well as the financial concern for a sustainable operation of the charging network. The second contribution is the algorithm, *SSGA*, to solve the mixed integer non-convex optimal pricing problem, which features two key rules that guarantee efficient converging to equilibrium solution and drastically improves the running time perfomance. The final contribution is our extensive experiments and results which demonstrate our approach in several aspects, including solution quality, scalability and robustness. Moreover, we compare our approach with uniform pricing and demonstrate how and to what extent *SSGA* can help with improving the traffic system efficiency and decreasing social cost caused by EV owners' charging behavior. Our approach can be applied in various modern cities like the motivated example Singapore to manage the charging stations in the future. We are actively approaching authorities of Singapore to look for such potential application.

8. ACKNOWLEDGEMENT

REFERENCES

[1] Farzad Rajaei Salmasi. Control strategies for hybrid electric vehicles: Evolution, classification, comparison, and future trends. *Vehicular Technology, IEEE Transactions on*, 56(5):2393–2404, 2007.

[2] I. Frade, R. Anabela, G. Goncalves, and A. P. Antunes. Optimal location of charging stations for electric vehicles in a neighborhood in lisbon, portugal. *Transportation Research Record: Journal of the Transportation Research Board*, 2252:91–98, 2011.

[3] H. Wang, Q. Huang, C. Zhang, and A. Xia. A novel approach for the layout of electric vehicle charging station. In *Proceedings of International Conference on Apperceiving Computing and Intelligence Analysis (ICACIA)*, pages 64–70, 2010.

[4] T. D. Chen, K. M. Kockelman, M. Khan, et al. The electric vehicle charging station location problem: a parking-based assignment method for seattle. In *Proceedings of 92nd Annual Meeting of the Transportation Research Board (TRB)*, 2013.

[5] F. He, D. Wu, Y. Yin, and Y. Guan. Optimal deployment of public charging stations for plug-in hybrid electric vehicles. *Transportation Research Part B: Methodological*, 47:87–101, 2013.

[6] Yanhai Xiong, Jiarui Gan, Bo An, Chunyan Miao, and Ana L. C. Bazzan. Optimal electric vehicle charging station placement. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 2662–2668, 2015.

[7] Jiarui Gan, Bo An, Haizhong Wang, Xiaoming Sun, and Zhongzhi Shi. Optimal pricing for improving efficiency of taxi systems. In *Proceedings of the 23rd International Joint Conference on Artificial Intelligence (IJCAI)*, pages 2811–2818, 2013.

[8] Jiarui Gan, Bo An, and Chunyan Miao. Optimizing efficiency of taxi systems: Scaling-up and handling arbitrary constraints. In *Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 523–531, 2015.

[9] Sara Deilami, Amir S Masoum, Paul S Moses, and Mohammad AS Masoum. Real-time coordination of plug-in electric vehicle charging in smart grids to minimize power losses and improve voltage profile. *Smart Grid, IEEE Transactions on*, 2(3):456–467, 2011.

[10] Keiichiro Hayakawa, Enrico H. Gerding, Sebastian Stein, and Takahiro Shiga. Online mechanisms for charging electric vehicles in settings with varying marginal electricity costs. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 2610–2616, 2015.

[11] Robert L Bertini. You are the traffic jam: an examination of congestion measures. *The 85th Annual Meeting of Transportation Research Board*, 2006.

[12] Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V Vazirani. *Algorithmic game theory*. Cambridge University Press, 2007.

[13] Wikipedia. Urban planning areas in singapore. `https://en.wikipedia.org/wiki/Urban_planning_areas_in_Singapore`, 2015.

[14] Singapore. General household survey 2005 statistical release 2: Transport, overseas travel, households and housing characteristics. `http://www.singstat.gov.sg/publications/publications-and-papers/GHS/ghrs2`, 2005.

[15] Chargepoint. Find charging spots. `https://na.chargepoint.com/charge_point`, 2015.