Coalitional Exchange Stable Matchings in Marriage and Roommate Markets

(Extended Abstract)

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ABSTRACT

We consider the stable roommates problem with respect to the stability based on exchange of agents. We present three natural variants of coalitional exchange stability and identify the relations between them. We also present a number of impossibility results. In particular, we show that even a (standard) exchange stable matching may not exist for dichotomous preferences. We prove that exchange stability has a fundamental incompatibility with weak Pareto optimality. We also prove that an exchange stable matching mechanism cannot be strategyproof.

Keywords

Game theory; matching markets; stability concepts; exchange stability

1. INTRODUCTION

In the stable marriage and stable roommate markets, stability of outcomes is typically defined by the absence of a pair of agents who have an incentive to leave their current assignment and pair up [8, 10, 11, 13]. The basic assumption is that agents have complete freedom to form new pairs. The assumption may not hold up if for instance $2n$ agents are paired up and sent to n rooms. In this case, two agents who are in different rooms may want to pair up but would balk at the idea of not living in a room. Alcalde [2] identified this problem and motivated the idea of *exchange stability* in matching markets especially when agents have property rights. The idea is that a roommate allocation is exchange stable if no two agents have an incentive to exchange their rooms.

Contributions.

It has previously been shown that an exchange stable roommate matching may not exist for strict preferences. We prove that a similar non-existence result also occurs for the case of dichotomous preferences for both marriage markets and roommate markets.

We focus on coalitional exchange stability that has previously only been briefly considered in the stable marriage setting. We present three natural notions of coalitional exchange stability for the stable roommate setting and identify their relations with each other. We also show that two of the notions are equivalent to standard coalition stability in the marriage setting.

A matching is weakly Pareto optimal if there exist no other matching in which each agent is strictly happier. We show that simple exchange stability is incompatible with weak Pareto optimality in the roommate setting: even if the set of exchange stable matchings is non-empty, the set could be disjoint from the set of weakly Pareto optimal matchings. The statement holds for both strict and dichotomous preferences.

We say a mechanism satisfies exchange stability or is exchange stable if for every preference profile that admits an exchange stable matching, it returns an exchange stable matching. We show that if there is a roommate matching mechanism that always returns an exchange stable matching if an exchange stable matching exists, then such a mechanism cannot be strategyproof.

2. PRELIMINARIES

A stable roommate instance consists of a set of agents N. We will assume that $|N|$ is even. In a *stable roommate (SR)* market or instance, each agent expresses a weak order over the rest of the agents. An outcome of the stable roommate instance is a matching which results in $|N|/2$ disjoint pairs of agents. For a matching, we will denote by $M(i)$ the matched partner of agent i.

A *stable marriage* market or instance consists of a set of men M and women W such that $|M| = |W|$. Each man expresses a weak order over W and each woman expresses a weak order over M. An outcome of the stable marriage instance is a matching in which each matched pair consists of a man and a woman. A marriage market can be viewed as a special case of a roommate instance in which each agent least prefers the members of its own gender.

We will denote the strictly preferred relation of agent i by $\succ_i$, the weakly preferred relation by $\succsim_i$, and the indifference relation by $\sim_i$. Note that $a \succ_i b$ if $a \succsim_i b$ but $b \not\succsim_i a$. Also note that $a \sim_i b$ if $a \succsim_i b$ and $b \succsim_i a$.

We define *dichotomous* preferences as those under which an agent can divide other agents into two indifference classes such that they strictly prefer all agents in one class over agents in the other class. Dichotomous preferences are especially important when agents have thresholds for their matching being good enough or not and they distinguish

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between preferred and not preferred matches.

We define strict preferences as when no agent is indifferent between two agents.

Definition 1 (Exchange Stability (ES)). *A matching M is* exchange stable *if there does not exist $i, j \in N$ and a matching M' such that $M(i) = M'(j)$, $M(j) = M'(i)$, $M'(i) \succ_i M(i)$, and $M'(j) \succ_j M(j)$.*

Informally speaking, a matching is *exchange stable* if no two agents can swap their matches to get better matches.

3. COALITIONAL EXCHANGE STABILITY

Coalition-exchange stability as introduced by Cechlárová [5] is a generalisation of exchange stability.

Definition 2 (Coalitional Exchange Stability). *A matching is* coalitional exchange stable (CES) *if it does not admit an* exchange-blocking coalition (EBC) *which is an ordered sequence of agents $\langle a_0, a_1, \cdots, a_{r-1}\rangle, r \geq 2$ where $M(a_{i+1}) \succ_{a_i} M(a_i)$ for all i (subscripts taken modulo r).*

This definition works well for marriage markets, but lends to an odd example for stable roommates.

Example 1. *Consider the following matchings of $(1, 2)$, $(3, 4)$ and $(5, 6)$ where $3 \succ_6 5$; $1 \succ_4 3$; $4 \succ_2 1$; and $5 \succ_3 4$.*

In this example, the ordered sequence $\langle 2, 3, 6, 4\rangle$ forms an exchange blocking coalition. Yet by rotating these agents through this order gives matchings of $(1, 4)$, $(2, 6)$ and $(5, 3)$. As agents 3 and 4 both left their original room, agents 2 and 6's moves are no longer individually useful to them as they have essentially swapped to be with each other. The distinctive feature in this case is that there is more than one person moving out of the same room.

One way to fix this would be to only allow one person to move from each room and another would be to simply ensure that each agent's final roommate is preferred by them to their original roommate, this leads to the following stability concept.

Definition 3 (1 per room). *A* 1 per room exchange blocking coalition (1PR-EBC) *is an ordered sequence of agents $C = \langle a_0, a_1, \ldots, a_{r-1}\rangle, r \geq 2$ where $M(a_{i+1}) \succ_{a_i} M(a_i)$ for all i (subscripts taken modulo r) and additionally for all $a \in C, M(a) \notin C$.[1] We say that a matching is* 1PR coalition-exchange stable (1PR-CES) *if it does not admit a 1PR-EBC.*

Another version of coalitional exchange stability is as follows.

Definition 4 (Final). *Define a* final exchange blocking coalition (F-EBC) *to be an ordered sequence of agents $C = \langle a_0, a_1, \ldots, a_{r-1}\rangle, r \geq 2$ where in the matching M' created by moving a_i to the room of a_{i+1} for all i, $M'(a_i) \succ_{a_i} M(a_i)$ for all i (subscripts taken modulo r). We say that a matching that does not admit a F-EBC is* final coalition-exchange stable (F-CES).

[1]We abuse notation to treat C as a set when the context is clear.

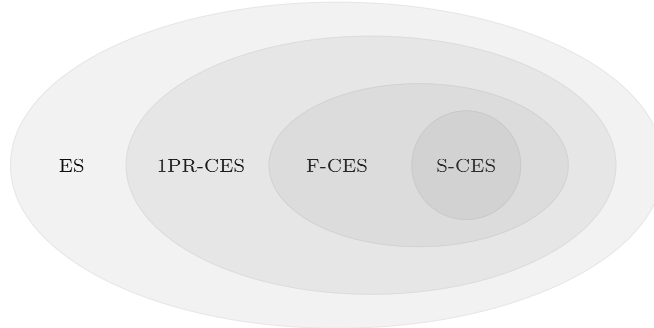

Figure 1: Relations between stability concepts. 1PR-CES and F-CES coincide for the marriage setting. Even the set of ES matchings can be empty.

F-EBC requires that for the rotation to be a blocking coalition, each agent's final roommate is preferred by them to their original one. We finally present a third variant of coalitional exchange stability.

Definition 5 (Shuffle). *Define a* shuffle exchange blocking coalition (S-EBC*) to be a set of agents $C = \{a_0, a_1, \ldots, a_{r-1}\}, r \geq 2$ where there is some deviation of agents in C such that for the new matching M' created from this deviation, $M'(a_i) \succ_{a_i} M(a_i)$ for all i (subscripts taken modulo r). We say that a matching that does not admit a S-EBC to be* shuffle coalition-exchange stable (S-CES).

Note that members of C may be matched with members outside C. Informally, S-CES requires that there is no subset of agents who can all be rearranged to be happier.

From these definitions, it is easy to see that any 1PR-EBC is also a F-EBC and any F-EBC is also a S-EBC. However in general, none of the CES concepts 1PR-CES, F-CES, and S-CES are equivalent for roommate markets. The relations between the stability concepts is depicted in Figure 1. For the marriage market, the original definition, 1PR-CES and F-CES are equivalent as there can only be one gender involved in a matched pair.

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