Memory-Based Mechanisms for Economic Agents

(Extended Abstract)

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ABSTRACT

We investigate the relation between money and memory in computational systems. To do so, we introduce a model in which agents have a state associated with them that is known to those interacting with them. The joint states of agents who interact successfully change according to some prescribed probability distribution. We show that such mechanisms can in fact encode and generalize a rich variety of monetary mechanisms, while requiring very little memory per agent to represent state, possibly even a single bit. We explore how monetary considerations like the total amount of money apply in our model, and seek memory-based mechanisms that increase social welfare. We examine the natural encoding of a token-based system in memory, in which tokens are exchanged and conserved during each transaction. We find that mechanisms that use price discrimination or do not conserve tokens can provide higher social welfare.

CCS Concepts

•Theory of computation → Algorithmic mechanism design; Computational pricing and auctions;

Keywords

Economic mechanisms, Money, Memory

1. INTRODUCTION & MODEL

In this paper we generalize the idea of money to general states of memory [4]. Money acts to encode the debt that society owes to a particular individual who provided services to others: a memory state, which may reflect the number of tokens an agent is holding, is updated after each trade and serves to *remember* past behavior. While money is restricted to limited state transition rules (e.g., those that conserve the number of coins held by agents), we expand the exploration to other mechanisms as well. Our work, which is based on economic models for money [7, 5] thus contributes to the study of scrip systems [1, 2, 3, 6] and their use within computational systems.

The Model. Our model consists of a unit mass of non-atomic players that trade a service. Trade is performed in rounds, in which players are randomly paired off and assigned the role of 'buyer' or 'seller'. A successful transaction between the pair provides utility U to the buyer and costs the seller C units of utility (we assume $U > C > 0$) and future rewards are discounted by a factor of $0 < \gamma < 1$ for each round they are delayed: The utility of an agent is thus $\sum_{t=0}^{\infty} \gamma^t r_t$ where $r_t \in \{+U, 0, -C\}$. Note that due to the cost incurred by the seller, it is not in its interest to provide the service unless some future reward is expected.

Each player is associated with a memory cell that may be in one of the states in $\mathcal{T}$. We denote the fraction of players in state $n \in \mathcal{T}$ at round t by f_n^t. The initial distribution f_n^0 is provided by the designer of the system.[1]

The actions of agents are encoded as follows: When a buyer in state x meets a seller in state y its willingness to perform a trade is denoted by $B_{x,y} \in \{0,1\}$. Similarly $S_{x,y} \in \{0,1\}$ denotes a seller's willingness to trade. A transaction occurs iff $B_{x,y} \cdot S_{x,y} = 1$. If the transaction did not take place, the states of agents remain the same and they do not gain or lose any utility. If a transaction does take place, the agents transition to new states a, b correspondingly, with probability $P_{x,y}^{a,b}$, and rewards are given as described above. The probability distribution P effectively encodes the mechanism (incl. prices) and is assumed to be known.

The Solution Concept. We are eventually interested in equilibria of the system at which the fractions f_i^t remain constant, and in which the strategies of players are the best response. We restrict our interest to symmetric, pure, time-independent, sub-game perfect strategy profiles. One such equilibrium profile always exists: when no agents engage in trade ($\forall x, y \quad B_{x,y} = S_{x,y} = 0$). We will be interested in cases in which other equilibria also exist, specifically those that induce trade.

If one denotes by V_n^t the expected utility of playing strategy profile σ, we may write the utility of agents recursively using Bellman equations: Denoting $SB_{x,y} \equiv S_{x,y} \cdot B_{x,y}$, we obtain $V_n^t =$

$$\sum_i \tfrac{1}{2} f_i \left(SB_{i,n} \left(\sum_{j,k} P_{i,n}^{k,j} \gamma V_j^{t+1} - C \right) + (1 - SB_{i,n}) \gamma V_n^{t+1} \right) +$$
$$\sum_i \tfrac{1}{2} f_i \left(SB_{n,i} \left(\sum_{j,k} P_{n,i}^{j,k} \gamma V_j^{t+1} + U \right) + (1 - SB_{n,i}) \gamma V_n^{t+1} \right)$$

Incentive compatibility is then described as follows:

[1] We are interested in the behavior of f_n^t once equilibrium is reached, but issues pertaining to the amount of money in the system may sometimes be determined by initial conditions.

Appears in: *Proc. of the 16th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2017), S. Das, E. Durfee, K. Larson, M. Winikoff (eds.), May 8–12, 2017, São Paulo, Brazil.*
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$$B_{i,j} = \begin{cases} 1 & U + \sum_{n \in \mathcal{T}} \sum_{k \in \mathcal{T}} \gamma P_{i,j}^{n,k} V_n \geq \gamma V_i \\ 0 & otherwise \end{cases}$$

$$S_{i,j} = \begin{cases} 1 & -C + \sum_{n \in \mathcal{T}} \sum_{k \in \mathcal{T}} \gamma P_{i,j}^{n,k} V_k \geq \gamma V_j \\ 0 & otherwise \end{cases}$$

For the fraction of the population at each state to constant, we must have

$$f_k = \sum_{i,j \in \mathcal{T}} \frac{1}{2} f_i f_j (SB_{i,j} \sum_{a,b \in \mathcal{T}} P_{i,j}^{a,b} (\delta_{a,k} + \delta_{b,k}) + (1 - SB_{i,j})(\delta_{i,k} + \delta_{j,k}))$$

Where $\delta_{x,y} = 1$ if $x = y$ and 0 otherwise.

Social Welfare. We would ideally like to maximize the total welfare of the population. The utility of interacting pairs increases by $\gamma^{k-1}(U - C)$ for every transaction occurring in round k. Let D denote the expected fraction of pairs that transact in the population: $D = \frac{1}{2} \sum_{i,j \in \mathcal{T}} f_i f_j SB_{i,j}$ The social welfare is then proportional to this value.

Token-Conserving Mechanisms. Our model can be used to encode tokens that are conserved:

DEFINITION 1.1. *A state transition probability distribution P is* token-conserving *if for all x, y, a, b, if $P_{x,y}^{a,b} > 0$ then $x + y = a + b$.*

Intuitively, we regard different states as amounts of the same token, and all possible transitions conserve the total amount of tokens between the two players.

In token-conserving mechanisms, we can consider the notion of the total amount of money in the population $M = \sum_{n \in \mathcal{T}} n \cdot f_n$. We further refine the class of token conserving mechanisms to those that transfer at most a single token per transaction:

DEFINITION 1.2. Single-token transfer conserving mechanisms *are token-conserving mechanisms in which whenever $P_{x,y}^{a,b} > 0$, then $a \in \{x-1, x, x+1\}$. As a consequence, $b \in \{y-1, y, y+1\}$.*

We note that even in the context of mechanisms of the above form, it is possible to encode a notion of different "prices" by the probability that the token is transferred during the deal (i.e., a lower probability is similar to a lower price). We adopt the terminology of "price" in this context to refer to this probability (the precise mapping to price in regular contexts is not well defined, especially since we have departed from the quasi-linear utility model).

In most economies the price of a specific commodity is not dependent on the financial status of the buyer or of the seller. The following definition reflects this sort of notion.

DEFINITION 1.3. *A single-token transfer conserving mechanism is* non-discriminating *if the transition probabilities are independent of the value of x: $P_{x,y}^{a,b} = P_{x+1,y}^{a+1,b}$ (given that $x + 1, a + 1$ are states in $\mathcal{T}$).*

2. RESULTS

In the two-state model, we show that there exists a mechanism that induces trade.

THEOREM 1. *For $|\mathcal{T}| = 2$, There exist values of U, C and γ for which there are (token conserving) mechanisms and corresponding equilibria with positive social welfare. Furthermore, optimal social welfare in the two-state token-conserving mechanism is achieved when $f_0 = f_1 = 1/2$.*

Several empirical simulations and partial results lead us to the following conjecture.

CONJECTURE 1. *The mechanism for two states that maximizes social welfare is the conserving mechanism.*

Assuming this conjecture, the two-state case is in a sense solved by conserving tokens; thus we turn to three states, and prove the following.

THEOREM 2. *There exist mechanisms in three states that achieve better social welfare than the non-discriminating ones. In particular, price-discrimination - a higher probability of transfer for buyers with more tokens - is one of them.*

The effect of the price-discriminating mechanism is shown in Figure 1. In the figure, q_1 is the probability that a poor buyer (with one token) keeps the token after the transaction; a rich buyer (with two tokens) *always* loses a token.

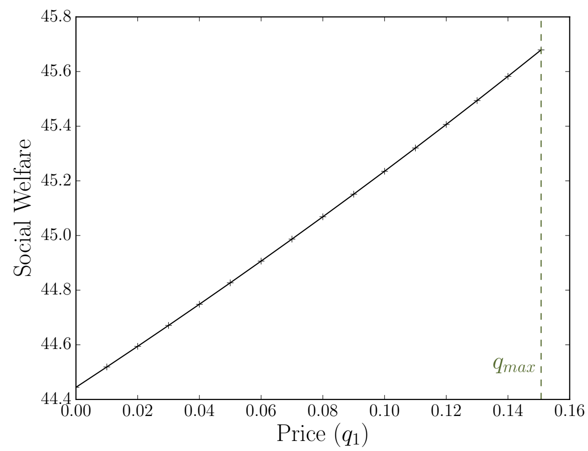

Figure 1: Social Welfare vs. Price with $U = 5$, $C = 3$, $\gamma = 0.99$. Beyond $q_1 \approx 0.1507$, some of the incentive compatibility inequalities fail.

To prove Theorem 2, we start by studying the conservation model, and show the following lemma.

LEMMA 3. *For non-discriminating single-token transfer with $p = 1$ in 3 tokens, the ratios yielding the best welfare are $f_0 = f_1 = f_2 = 1/3$.*

After empirically verifying that similar results occur with more states, we turn to scenarios with infinite memory.

THEOREM 4. *In a non-discriminating single token transfer mechanism in an infinite number of tokens, if there is positive social welfare, then there exists k such that $f_j = 0$ for all $j > k$.*

Thus in the case of non-discrimination, infinite memory degenerates to a finite model. We further show that the assumption of non-discrimination is necessary, and finish with a conjecture for future work.

CONJECTURE 2. *For any given U,C, γ, for every mechanism with an infinite number of states and a probability function P there exists a corresponding mechanism on a finite number of states that achieves at least the same social welfare.*

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