Manipulation of Hamming-based Approval Voting for Multiple Referenda and Committee Elections

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ABSTRACT

Several methods exist for electing committees of representatives and conducting multiple referenda based on approval voting. Recently, a family of rules for approval-based voting using ordered weighted averaging was proposed in [1], ranging from a simple candidate-wise majority (minisum) to egalitarian rule (minimax). Even though the first rule is strategyproof and the second is not, due to its egalitarian nature, only a partial study on manipulation has been conducted for inbetween rules.

This paper investigates the manipulability of fair rules within this family. We first prove that all rules parameterized by fair (non-increasing) weight vectors are manipulable, except minisum, if we consider them either resolute with a tie-breaking mechanism or irresolute with classic extension principles. Then, we conduct an empirical study of the proportion of elections that are manipulable, showing that it increases based on the rule's fairness.

CCS Concepts

•**Computing methodologies** → **Multi-agent systems;**

Keywords

Approval voting; Manipulation; Minisum; Minimax; Ordered weighted averaging

1. INTRODUCTION

Multiple referenda and committee elections are similar problems. In the first situation, voters are making a collective decision over several binary issues, while in the second they are electing several winners from a set of candidates. In both cases, voters have to decide a common value for a binary variable that corresponds either to an issue (accepted or rejected) or a candidate (elected or rejected). There may be constraints on the set of feasible combined decisions, such as cardinality constraints (e.g. the elected committee must have between 6 and 10 members).

Approval voting is a well-known voting procedure used for conducting multiple referenda [4] and committee elections [5, 17]. Voters cast approval ballots that, in the context of a committee election, consist of a set of approved candidates, and in the context of a multiple referendum, a set of approved binary issues. From here on, we stick to committee election terminology, but everything naturally flows to multiple referenda where we replace candidates with issues and committees with bundles of issues. As previously reviewed in [17], there are several different ways of finding the winning outcome, i.e. the winning set of candidates. The most standard way, called *minisum*, consists of choosing candidates by their approval scores: if the size of the committee is not subject to any constraint (which will be the case in this paper), the candidates approved by a majority of voters are elected. Another way is *minimax* approval voting which, as argued in [5], makes decisions that are fairer than minisum (where fairness has to be understood in the Rawlsian sense): minimax selects a committee that minimizes the maximum over all the voters of the Hamming distance to a voter's ballot, seen as a binary vector (and equivalently, minimizes disagreement).

However, a strong objection to minimax is its extreme behavior. Its egalitarian nature gives a huge influence to the agent with the worst utility (the largest distance), even if everyone else agrees. Indeed, consider the following situation: 10 voters have to decide on two binary propositions, p_1 and p_2. Nine voters agree on assignment 11 (assigning both p_1 and p_2 to true), but the last voter prefers assignment 00. Instead of choosing the almost unanimous outcome, 11, minimax chooses an outcome that has a Hamming distance of 1 to any voter, for example, 01. Since the last voter can prevent an otherwise unanimous outcome from being chosen, she seems to have too much influence.

Related to this, minimax approval voting is not strategyproof [19], whereas minisum is. This should not be considered too strong an objection, since strategyproof ways of reaching collective decisions are the exception rather than the rule. However, the tremendous influence of the least happy voter implies that her vote can wield strong manipulation power, and this is a specificity of minimax approval voting.

A family of rules for committee elections and multiple referenda has been proposed in [1] for designing rules that lie between minisum and minimax in terms of fairness. Making use of *ordered weighted averaging* (OWA), these rules, denoted as **w**-Approval Voting (or **w**-AV), are parameterized by weight vector **w**, which weights voters by their *rank* when we order their Hamming distances to a given outcome, from

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the largest to the smallest. As an example, minimax corresponds to weight vector $(1, 0, \ldots, 0)$ since only the voter with the largest Hamming distance matters when computing the score of a committee. Given an election with n voters, minisum is parameterized by vector $(1/n, \ldots, 1/n)$ since all voters are equally taken into account.

Rules parameterized by non-increasing weight vectors are considered fair because they respect a basic fairness requirement: the Pigou-Dalton principle [8, 22]. Focusing on non-increasing weight vectors, we have a continuum of rules from minisum to minimax, which are considered fairer and fairer when they are closer to minimax and further from minisum.

From these constatations, we may wonder how the manipulation potential of minimax is related to its fairness property. An interesting question is the study of the manipulation of **w**-AV rules, depending on their fairness (seen as the proximity to minimax). A preliminary study [1] showed that a subset of these rules is manipulable, when considering them resolute with a linear order tie-breaking mechanism. The question of knowing if minisum is the only strategyproof resolute rule[1] in this context remains open. Furthermore, since minimax tends to elect a large number of winning committees, due to its low discriminative power, manipulation of its irresolute version is worth studying.

Contribution: The aim of this paper is to study the manipulability of the **w**-AV rules that fall between minisum and minimax. We first study fair resolute **w**-AV rules with a linear order tie-breaking mechanism and show that minisum is the only strategyproof rule of this family. Irresolute rules require more assumptions on how preferences over committees extend to preferences over subsets of committees. With classical extension principles (optimistic, pessimistic and Gärdenfors principles), minisum is again the only fair rule that is strategyproof. Moreover, if we assume that each voter derives utility from a committee that is proportional to the opposite of its Hamming distance, then any **w**-AV rule composed with the uniformly randomized tie-breaking mechanism is manipulable (in terms of expected utility), except minisum.

Then, through an experimental study, we explore the link between fairness and manipulability. The orness measure [25], an appealing concept of fairness, captures the proximity of a **w**-AV to minimax. We compute the average number of manipulable elections depending on the orness of the studied rule and observe that this number increases almost linearly from minisum to minimax.

Related Work: A few recent works use OWA operators in a voting context. One work [15] generalized positional scoring rules by weighting a candidate's scores obtained from different voters by their rank in the ordered list of these scores and studied the properties of these rules, before conducting an experimental study of manipulation, under several distributions of preferences. Two works used OWAs for defining generalizations of the Chamberlin and Courant proportional representation rule. In [24], a committee's score from a vote was computed by applying an OWA to the scores of the committee members, ranked by the voter's ballots, where the score of a committee from a collection of votes is simply the sum of the scores obtained from different voters. Another work [9] extended the Chamberlin and Courant rule in a different way: for each candidate, the OWA is applied to the scores obtained by the candidate for different voters. Last, another previous work [11] studied OWA-based scoring rules, among other natural classes of scoring rules. This work focuses on the axiomatic properties of such classes and the containment relations between them.

The series of works that study manipulation issues and strategic behavior in multi-winner elections (such as [20]) is another related research stream. The computational aspects of strategic behavior in standard multiwinner approval voting have been studied [3]. Two other works [7, 19] both study the conditions under which an approximation of minimax is sensitive to manipulation. Minimax approval voting has also been used in judgment aggregation [18]. In all these contexts, it seems appropriate to study rules that are less extreme than minimax while remaining fairer than minisum.

Outline: In the rest of our paper, we consider that no cardinality constraint exists on the size of the outcome. This assumption is especially justified in multiple referenda, where any proposition can be accepted or rejected, but also in some kinds of multiwinner election contexts, such as hall of fame elections. The outline of this paper is as follows. In Section 2, we formally define our framework and manipulation in this context, and address the manipulation of resolute **w**-AV rules in Section 3. The manipulation of irresolute (or non-resolute) rules is studied in Section 4. Section 5 addresses the experimental study of the evolution of manipulability when fairness is increased. Finally, we give several research directions in Section 6.

2. DEFINITIONS AND NOTATION

2.1 w-AV Rules

The exposition of **w**-AV rules is taken from a previous work [1].

We denote the set of voters by N, $|N| = n$, and the set of candidates by X, $|X| = m$. Approval profile P is a tuple $P = (P_1, ..., P_n)$, where $P_i \subseteq X$ denotes the approval ballot of voter i. An approval ballot is a subset of candidates approved by a voter. We can also represent approval ballot P_i as a binary vector in $\{0, 1\}^m$, where 1s indicate the candidates approved by voter i. In most of the proofs, we exploit the binary vector representation. An election under approval voting is specified by tuple (N, X, P).

There are several ways of using approval voting for committee elections; see [17] for a review. Two popular methods are *minisum*, which consists of electing the candidates who are approved by a majority of voters, and *minimax*. To define minimax, we first define the Hamming distance between two ballots P_i and P_j, as the total number of candidates on whom they differ: $d_{\mathcal{H}}(P_i, P_j) = |P_i \setminus P_j| + |P_j \setminus P_i|$. Minimax approval voting, introduced in [5], selects a committee S that minimizes $\max_{i \in N} d_{\mathcal{H}}(S, P_i)$.

We study a family of voting rules introduced in [1], that generalizes minisum and minimax. To define that family, they used Ordered Weighted Averaging operators (OWA) [25]. In this family, each rule is specified by a weight vector and selects an outcome that minimizes the weighted sum of Hamming distances, after ordering them in non-increasing order. Given a preference profile of n voters,

[1]Resolute minisum is strategyproof when ties are broken by a linear order over the committee and under the assumption of Hamming-consistent preferences, explained in Section 2.2.

$P = (P_1, ..., P_n)$, and a committee $S \subseteq X$, we denote by $\mathcal{H}(P, S)$ the n-dimensional vector that contains the Hamming distances of each P_i from S in non-increasing order, i.e. $(d_{\mathcal{H}}(P_i, S))_{i \in N}$ after ordering it in non-increasing order. Now consider $\mathbf{w} = \{\mathbf{w}_n\}_{n\in\mathbb{N}}$, a family of weight vectors such that for every number of voters $n \in \mathbb{N}$, $\mathbf{w}_n = (\mathbf{w}_n(1), \mathbf{w}_n(2), \ldots, \mathbf{w}_n(n)) \in [0, 1]^n$ and the coordinates of $\mathbf{w}_n$ sum up to 1. Weight $\mathbf{w}_n(i)$ is attached to the i-th largest Hamming distance of the selected outcome to the n voters. Then the voting rule, which we refer to as $\mathbf{w}$-*Approval Voting*, or $\mathbf{w}$-AV, is given as follows.

$\mathbf{w}$-AV: Given election (N, X, P) with n voters, select a subset of candidates S such that it minimizes dot product $\mathbf{w}_n \cdot \mathcal{H}(P, S)$.

In some settings, the size of the committee to be elected is fixed, but we focus on a case where the size is not fixed, which is especially justified in multiple referenda.

Recall that $\mathbf{w}$-AV is a generalization of both minisum and minimax, corresponding respectively to weight vectors $\mathbf{w}_n = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ and $\mathbf{w}_n = (1, 0, \ldots, 0)$.

To illustrate this family of rules, consider the following slightly modified example [1]:

EXAMPLE 1. *Six coauthors of a paper, $A_1, \ldots A_6$, have to decide whether they should use Dropbox (d or $\overline{d}$), have a physical meeting (m or $\overline{m}$) and whether they should continue to work and prove more results before submission (w or $\overline{w}$). Here are their votes:*

	d	m	w
A_1	1	1	0
A_2	0	1	0
A_3	1	0	0
A_4	1	1	1
A_5	1	1	1
A_6	0	0	1

Minisum selects a winning committee in $\{110, 111\}$*, and minimax selects a winning committee in* $\{010, 100, 111\}$*. Consider vector* $\mathbf{w} = (\frac{6}{21}, \frac{5}{21}, \frac{4}{21}, \frac{3}{21}, \frac{2}{21}, \frac{1}{21})$*, which can be seen as the Borda weight vector. Then corresponding* $\mathbf{w}$*-AV selects unique committee* 110*, where* $\mathcal{H}(P, 110) = (3, 1, 1, 1, 1, 0)$ *and* $\mathbf{w} \cdot \mathcal{H}(P, 110) = \frac{32}{21}$.

In this paper, we focus on OWA vectors that are *non-increasing*. This non-increasing condition is standard in the study of OWAs because it corresponds to a basic fairness requirement: the Pigou-Dalton principle [22, 8]. This principle simply states that the utility derived by a set of voters from a committee should be as equally distributed as possible, all things being equal otherwise. Strictly decreasing weight vectors satisfy the Pigou-Dalton principle, whereas non-increasing weight vectors only satisfy it in a weak sense. Arguably, minimax is the fairest and minisum the least fair of the rules satisfying the Pigou-Dalton principle, with a continuum inbetween. In Section 5, we introduce fairness as a measure of proximity to minimax.

2.2 Manipulation

To define manipulation in this setting, we need to know the preferences of the voters on all committees. However, we only have the top committee of each voter, corresponding to each approval ballot. We don't have to exhaustively express those preferences. We just assume that the smaller the Hamming distance is between a committee and a ballot, the more appreciated this committee is. Such preferences are called *Hamming-coherent*. Formally, given voter i and ballot P_i, for all $c, c' \subseteq X$,

$$d_{\mathcal{H}}(c, P_i) < d_{\mathcal{H}}(c', P_i) \Leftrightarrow c \succ_i c',$$

which means that i prefers c to c'. Notice that in most of our results, we only require the assumption that each voter strictly prefers his optimal committee to any other. Of course, this assumption is much weaker than the *Hamming-coherence* assumption.

Note that the Gibbard-Satterthwaite theorem ([23, 14]) does not apply in our setting since we are in a domain of Hamming-induced preferences. Not surprisingly, however, we show that most of the rules in this family are manipulable.

Given irresolute rule R, we study manipulation in three ways: 1) we "determinize" irresolute rule R with tie-breaking mechanism T to obtain resolute rule R^T; 2) we study manipulation of irresolute voting rules using an *extension principle*; 3) we "cardinalize" the preferences by assuming that the utility of a committee for a voter is (to a constant) the opposite of the Hamming distance to its optimal committee.

Given profile P and irresolute rule R, we denote by $R(P)$ the outcome of the rule on profile P. We also denote by P_{-i} the preferences of all voters besides i. Hence, we can also write P as (P_i, P_{-i}). Manipulability means that voter i has an incentive to unilaterally change her preference to reduce the distance of P_i from the outcome. Then we can formally define manipulation for resolute rule R^T:

DEFINITION 1. *Resolute rule R^T is manipulable if there exist profile P, voter i, and preference $P'_i \subseteq X$ such that:*

$$R^T(P'_i, P_{-i}) \succ_i R^T(P).$$

Manipulation for irresolute rules requires another assumption. Indeed, we have to compare non-empty subsets of winning committees and define how the preferences on the subsets of committees are related to the preferences on committees. To do so, we exploit extension principles, which extend preferences over committees to preferences over non-empty subsets of committees. Let $\mathcal{A}$ and $\mathcal{B}$ be two non-empty subsets of 2^X, and let $\succ_i$ be the preference relation of i on 2^X. Extension principle E turns $\succ_i$ on 2^X into preference relation $\succ_i^E$ on $2^{2^X} \setminus \emptyset$.

We study three well-known extension principles[2]: optimistic, pessimistic and Gärdenfors (see [13]).

Optimistic $\mathcal{A} \succ_i^O \mathcal{B}$ if and only if $\exists A \in \mathcal{A}$ such that $\forall B \in \mathcal{B}$, we have $A \succ_i B$. In other words, the best element of $\mathcal{A}$ is preferred to the best element of $\mathcal{B}$.

Pessimistic $\mathcal{A} \succ_i^P \mathcal{B}$ if and only if $\exists B \in \mathcal{B}$ such that $\forall A \in \mathcal{A}$, we have $A \succ_i B$. In other words, the worst element of $\mathcal{A}$ is preferred to the worst element of $\mathcal{B}$.

Gärdenfors $\mathcal{A} \succ_i^G \mathcal{B}$ if and only if (a) $\mathcal{A} \subseteq \mathcal{B}$ and $\forall A \in \mathcal{A}$, $\forall B \in \mathcal{B} \setminus \mathcal{A}$, we have $A \succ_i B$; or (b) $\mathcal{A} \supseteq \mathcal{B}$ and $\forall A \in \mathcal{A} \backslash \mathcal{B}, \forall B \in \mathcal{B}$, we have $A \succ_i B$; or (c) $\forall A \in \mathcal{A} \backslash \mathcal{B}$ $\forall B \in \mathcal{B} \setminus \mathcal{A}$, we have $A \succ_i B$.

[2]For a discussion of extension principles in social choice, see [2] and [6].

Example 2 illustrates these extension principles.

EXAMPLE 2. *Consider two candidates and the Hamming-coherent preference of voter 1:*

$$P_1 : 10 \succ 11 \sim 00 \succ 01.$$

First, use the optimistic and pessimistic principles to compare two subsets of committees, $\{10, 01\}$ *and* $\{11, 00\}$*. We obtain* $\{10, 01\} \succ_1^O \{11, 00\}$ *because the best choice of* $\{10, 01\}$ *is preferred to the best choice of* $\{11, 00\}$*. However, we have* $\{10, 01\} \prec_1^P \{11, 00\}$ *because the worst choice of* $\{11, 00\}$ *is preferred to the worst choice of* $\{10, 01\}$*.*

Now, by comparing the subsets $\{10, 11\}$ *and* $\{10, 01\}$ *with the Gärdenfors principle, we obtain* $\{10, 11\} \succ_1^G \{10, 01\}$ *because* 11 *is preferred to* 01*.*

Then, for extension principle E, profile P, and irresolute rule R, we define an E-manipulation of R as a manipulation by voter i whose preferences respect E, i.e., vote P'_i such that $R(P'_i, P_{-i}) \succ_i^E R(P)$.

DEFINITION 2. *Irresolute rule* R *is manipulable with respect to* E *if there exist profile* P*, voter* i*, and preference* $P'_i \subseteq X$ *such that:*

$$R(P'_i, P_{-i}) \succ_i^E R(P).$$

Finally, assume that the preferences are cardinals. We study rule $R^{\mathcal{R}}$ representing the composition of irresolute rule R and the uniformly randomized tie-breaking mechanism, $\mathcal{R}$. Furthermore, we consider that the utility of a voter whose favorite committee is x (when the outcome of the election is y), is $m - d_{\mathcal{H}}(x, y)$. Thus, the expected utility of voter i, denoted as $\overline{u_i}(P)$ with x as a favorite committee (when he votes sincerely) is:

$$\overline{u_i}(P) = \frac{1}{|R(P)|} \sum_{y \in R(P)} m - \mathcal{H}(x, y).$$

Then a manipulation for rule $R^{\mathcal{R}}$ by voter i is vote P'_i such that the expected utility of i in $P' = (P'_i, P_{-i})$ is strictly greater than its expected utility in P.

DEFINITION 3. *Irresolute rule* R *is manipulable with respect to* $\mathcal{R}$ *if there exist profile* P*, voter* i*, and preference* $P'_i \subseteq X$ *such that:*

$$\overline{u_i}(P'_i, P_{-i}) > \overline{u_i}(P).$$

In the context of Hamming-coherent preferences, we know that minimax is manipulable but not minisum (see [5, 19]). A previous argument [5] can easily be extended to any type of manipulation considered above. How does this result extend to $\mathbf{w}$-AV rules in general?

For readability, we introduce some notations. For committee c, we write $\mathcal{H}(c) = \mathcal{H}(c, P)$ and $D(c) = w \cdot \mathcal{H}(c, P)$ when there is no ambiguity about P or weight vector w. Moreover, for $i \leq j$, we define $W_{i \to j} = \sum_{k=i+1}^{j} w_k$, $W_{j \to i} = -W_{i \to j}$ and $W_{i \to i} = 0$. Finally, $(2^p\ 1^q\ 0^r)$ represents a vector that starts with p coordinates that equal 2, and then q coordinates equal 1 and $r = n - p - q$ coordinates equal 0.

3. RESOLUTE w-AV

In this section, we focus on resolute $\mathbf{w}$-AV^T rules obtained from the composition of an irresolute $\mathbf{w}$-AV rule and tie-breaking mechanism, T. To protect anonymity, tie-breaking mechanisms are usually linear priority order over committees. The main result of this section shows that any resolute $\mathbf{w}$-AV^T rule parameterized by a non-increasing weight vector is manipulable, except minisum.

THEOREM 1. *If* $\mathbf{w}$ *is non-increasing and* $\mathbf{w}$-AV^T *differs from minisum*^{T}*, then, for any* m*,* $\mathbf{w}$-AV^T *is manipulable.*

The proof of Theorem 1 is presented in Subsections 3.1 and 3.2, where the first considers an even number of voters and the second considers an odd number. These two cases lead to slightly different results. Indeed, when the number of voters is even, the smallest deviation from weight vector $\mathbf{w}_n = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ leads to a rule that is manipulable. When it is odd, some weight vectors close to $\mathbf{w}_n = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$ produce $\mathbf{w}$-AV rules that are equivalent to minisum, and therefore resistant to manipulation.

3.1 Even number of voters

First, we consider manipulation for non-increasing weight vectors with an even number of voters. The following proposition shows the manipulation of all non-increasing $\mathbf{w}$-AV^T in this case.

PROPOSITION 1. *If* n *is even,* $\mathbf{w}$ *is non-increasing and differs from the minisum weight vector; then, for any* $m \geq 2$*,* $\mathbf{w}$-AV^T *is manipulable.*

PROOF. The proof is a generalized example, providing a manipulation for all such weight vectors. Consider an even number of voters, $n = 2q$, and integer α, $0 \leq \alpha < q$. Profile P is composed of $(q - \alpha)$ votes 00, α votes 10 and q votes 01. The tie-breaking priority favors 01 over 00 and 00 over 11. Let $\mathbf{w}$ be such that $W_{0 \to \alpha} = W_{q \to q+\alpha}$, and $W_{0 \to \alpha+1} > W_{q \to q+\alpha+1}$. We have the following scores:

$$\begin{array}{ll}
\mathcal{H}(00) = (1^{q+\alpha} 0^{q-\alpha}) & D(00) = W_{0 \to q+\alpha} \\
\mathcal{H}(01) = (2^{\alpha} 1^{q-\alpha} 0^{q}) & D(01) = 2W_{0 \to \alpha} + W_{\alpha \to q} \\
\mathcal{H}(10) = (2^{q} 1^{q-\alpha} 0^{\alpha}) & D(10) = 2W_{0 \to q} + W_{q \to 2q-\alpha} \\
\mathcal{H}(11) = (2^{q-\alpha} 1^{q+\alpha}) & D(11) = 2W_{0 \to q-\alpha} + W_{q-\alpha \to 2q}.
\end{array}$$

Clearly, for $0 \leq \alpha < q$, we have $D(00) < D(11)$ and $D(01) < D(10)$ and $D(01) - D(00) = W_{0 \to \alpha} - W_{q \to q+\alpha}$. Then, the winner is 01 due to tie-breaking.
Now let one of the 00-voters vote 10. The new scores are:

$$\begin{array}{ll}
\mathcal{H}(00) & = (1^{q+\alpha+1} 0^{q-\alpha-1}) \\
\mathcal{H}(01) & = (2^{\alpha+1} 1^{q-\alpha-1} 0^{q}) \\
\mathcal{H}(10) & = (2^{q} 1^{q-\alpha-1} 0^{\alpha+1}) \\
\mathcal{H}(11) & = (2^{q-\alpha-1} 1^{q+\alpha+1}) \\
D(00) & = W_{0 \to q+\alpha+1} \\
D(01) & = 2W_{0 \to \alpha+1} + W_{\alpha+1 \to q} \\
D(10) & = 2W_{0 \to q} + W_{q \to 2q-\alpha-1} \\
D(11) & = 2W_{0 \to q-\alpha-1} + W_{q-\alpha-1 \to 2q}.
\end{array}$$

Clearly, for $0 \leq \alpha < q$, we still have $D(01) < D(10)$. If $\alpha < q - 1$, then $D(00) < D(11)$, and if $\alpha = q - 1$, then $D(00) = D(11)$, and the tie-breaking favors 00. Finally, we have $D(01) - D(00) = W_{0 \to \alpha+1} - W_{q \to q+\alpha+1}$. Then, the winner is 00 and the manipulation is successful. □

This result holds for any q, and by symmetry, for any linear order tie-breaking mechanism T. Furthermore, the result extends to any number of candidates by adding candidates that are unanimously approved (or disapproved).

This example also deals with weight vectors that are close to minimax. Indeed, when $\alpha = 0$, any weight vector $\mathbf{w}$ such that $w_1 > w_{q+1}$ induces a manipulation.

3.2 Odd numbers of voters

Next we study how this result extends to an odd number of voters by first considering a similar example as in the proof of Proposition 1 to prove the following proposition.

PROPOSITION 2. *If $n = 2q+1$, $\mathbf{w}$ is non-increasing and there exists α, $0 \le \alpha < q$, such that $W_{0\to\alpha} \le W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} > W_{q\to q+\alpha+2}$; then, for any $m \ge 2$, $\mathbf{w}$-AV^T is manipulable.*

PROOF. The proof resembles the proof of Proposition 1, adapted to an odd number of voters. Consider an odd number of voters, $n = 2q+1$, and integer α, $0 \le \alpha < q$. Profile P is composed of $(q-\alpha)$ votes 00, α votes 10 and $(q+1)$ votes 01. The tie-breaking priority favors 01 over 00 and 10, and 00 over 11. Let $\mathbf{w}$ be such that $W_{0\to\alpha} \le W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} > W_{q\to q+\alpha+2}$. The scores are:

$$\begin{aligned}
\mathcal{H}(00) &= (1^{q+\alpha+1}0^{q-\alpha})\\
\mathcal{H}(01) &= (2^{\alpha}1^{q-\alpha}0^{q+1})\\
\mathcal{H}(10) &= (2^{q+1}1^{q-\alpha}0^{\alpha})\\
\mathcal{H}(11) &= (2^{q-\alpha}1^{q+\alpha+1})\\
D(00) &= W_{0\to q+\alpha+1}\\
D(01) &= 2W_{0\to\alpha} + W_{\alpha\to q}\\
D(10) &= 2W_{0\to q+1} + W_{q+1\to 2q-\alpha+1}\\
D(11) &= 2W_{0\to q-\alpha} + W_{q-\alpha\to 2q+1}.
\end{aligned}$$

Clearly, for $0 \le \alpha < q$, we have $D(00) < D(11)$ and $D(01) \le D(10)$, and the tie-breaking favors 01 if $D(01) = D(10)$. Finally, $D(01) - D(00) = W_{0\to\alpha} - W_{q\to q+\alpha+1}$. Then, the winner is 01, due to tie-breaking if $W_{0\to\alpha} = W_{q\to q+\alpha+1}$. Now one of the 00-voters casts 10. The scores become:

$$\begin{aligned}
\mathcal{H}(00) &= (1^{q+\alpha+2}0^{q-\alpha-1})\\
\mathcal{H}(01) &= (2^{\alpha+1}1^{q-\alpha-1}0^{q+1})\\
\mathcal{H}(10) &= (2^{q+1}1^{q-\alpha-1}0^{\alpha+1})\\
\mathcal{H}(11) &= (2^{q-\alpha-1}1^{q+\alpha+2})\\
D(00) &= W_{0\to q+\alpha+2}\\
D(01) &= 2W_{0\to\alpha+1} + W_{\alpha+1\to q}\\
D(10) &= 2W_{0\to q+1} + W_{q+1\to 2q-\alpha}\\
D(11) &= 2W_{0\to q-\alpha-1} + W_{q-\alpha-1\to 2q+1}.
\end{aligned}$$

Clearly, for $0 \le \alpha < q$, we still have $D(01) < D(10)$. If $\alpha < q-1$, $D(00) < D(11)$, and if $\alpha = q-1$, then $D(00) = D(11)$, and the tie-breaking favors 00. Finally, we have $D(01) - D(00) = W_{0\to\alpha+1} - W_{q\to q+\alpha+2}$. Then, the winner is 00 and the manipulation is successful. $\square$

As before, this result holds for any q, and by symmetry, for any linear order tie-breaking mechanism T. Furthermore, the result extends to any number of candidates by adding candidates that are unanimously approved (or disapproved). This example also deals with weight vectors that are close to minimax.

However, scrutinizing the weight vectors that are close to $\mathbf{w}_n = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$, we encounter rules that are equivalent to minisum and thus resistant to manipulation when the number of voters is odd.

To specify these rules, let us first state the following technical proposition, for which we omit the proof due to space limitations.

PROPOSITION 3. *Consider a set of n integers, $\{a_i\}_{i=1\dots n}$, with odd n, such that $-\alpha \le a_i \le \alpha$ and $\sum_{i=1}^n a_i = -\alpha$, and a non-increasing vector $w = \langle w_i \rangle_{i=1\dots n}$. Then we have $\sum_{i=1}^n w_i * a_i \le \sum_{i=1}^{\lfloor n/2 \rfloor} \alpha * w_i - \sum_{i=\lfloor n/2 \rfloor+1}^{n} \alpha * w_i$.*

Now, given this property, we study the $\mathbf{w}$-AV rules that are close to minisum and show that the weight vectors that are too close to $\mathbf{w}_n = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$, i.e., where $W_{0\to q} < W_{q\to 2q+1}$, produce $\mathbf{w}$-AV rules that are equivalent to minisum.

PROPOSITION 4. *Given $n = 2q+1$ and non-increasing weight vector $\mathbf{w}$, if there exists approval profile P such that $\mathbf{w}$-$AV^T(P) \neq$ minisum$^T(P)$, then $W_{0\to q} \ge W_{q\to 2q+1}$.*

PROOF. Consider $n = 2q+1$, a non-increasing weight vector $\mathbf{w}$ and profile P such that $\mathbf{w}$-AV$^T(P) \neq$ minisum$^T(P)$. We denote $c = minisum^T(P)$ and $c' = \mathbf{w}$-AV$^T(P)$. Since the number of voters is odd, minisum has a unique winner; thus we have $minisum(c', P) - minisum(c, P) = \alpha$, for some $\alpha \ge 1$. Now we address the difference between the $\mathbf{w}$-AV scores of c and c':

$$D(c) - D(c') = \sum_{i=1}^{n} w_i * (\mathcal{H}(c)_i - \mathcal{H}(c')_i).$$

Note that the number of candidates that differ from c to c' is at most α. In fact, c contains all the candidates approved by a majority of voters; thus removing or adding a candidate from or to c (to obtain c') increases its minisum score by at least 1.

Then we prove that for all $1 \le i \le n$, we have $-\alpha \le (\mathcal{H}(c)_i - \mathcal{H}(c')_i) \le \alpha$. Given integer i such that $1 \le i \le n$, we call v the voter who corresponds to the i^{th} coordinates of $\mathcal{H}(c)$, and v' is the voter who corresponds to the i^{th} coordinates of $\mathcal{H}(c')$. With these notations, the previous remark implies that $d_{\mathcal{H}}(c, v') - d_{\mathcal{H}}(c', v') \ge -\alpha$. Moreover, without loss of generality (by symmetry), we assume that $d_{\mathcal{H}}(c, v) - d_{\mathcal{H}}(c, v') \ge 0$. Together, these two inequations imply that $d_{\mathcal{H}}(c, v) - d_{\mathcal{H}}(c', v') \ge -\alpha$, which is exactly $\mathcal{H}(c)_i - \mathcal{H}(c')_i \ge -\alpha$. Furthermore, by contradiction, assume that $d_{\mathcal{H}}(c', v') < d_{\mathcal{H}}(c, v) - \alpha$. Then, because of the previous remark, all voters v'' such that $d_{\mathcal{H}}(c, v'') \ge d_{\mathcal{H}}(c, v)$ also verify $d_{\mathcal{H}}(c', v'') > d_{\mathcal{H}}(c', v')$, which is a contradiction since v' is the voter corresponding to the i^{th} coordinates of $\mathcal{H}(c')$. Thus we have $d_{\mathcal{H}}(c', v') \ge d_{\mathcal{H}}(c, v) - \alpha$, which is exactly $\mathcal{H}(c)_i - \mathcal{H}(c')_i \le \alpha$.

In addition, since $minisum(c', P) - minisum(c, P) = \alpha$, we have $\sum_{i=1}^n (\mathcal{H}(c)_i - \mathcal{H}(c')_i) = -\alpha$. Then, we can apply the result of Proposition 3 and obtain

$$\sum_{i=1}^{n} w_i * (\mathcal{H}(c)_i - \mathcal{H}(c')_i) \le \sum_{i=1}^{q} \alpha * w_i - \sum_{i=q+1}^{n} \alpha * w_i.$$

However, since we have $\mathbf{w}$-AV$^T(P) = c'$, we know that $D(c) - D(c') \ge 0$. Thus $0 \le \sum_{i=1}^q \alpha * w_i - \sum_{i=q+1}^n \alpha * w_i$, which implies $W_{0\to q} \ge W_{q\to 2q+1}$. $\square$

Proposition 4 implies that if the weight vector is close to the minisum weight vector, i.e., if $W_{0\to q} < W_{q\to 2q+1}$, then $\mathbf{w}$-AV$^T(P)$ is equivalent to minisum^T and is therefore resistant to manipulation. Thus, the results of this section imply the following.

PROPOSITION 5. *If n is odd, $\mathbf{w}$ is non-increasing and $\mathbf{w}$-AV^T differs from minisum; then, for any m, $\mathbf{w}$-AV^T is manipulable.*

The conclusion of Sections 3.1 and 3.2 notes that as soon as we consider non-increasing weight vector $\mathbf{w}$ that does not induce minisum, the $\mathbf{w}$-AV^T rule is manipulable. In other words, since we introduce fairness in the $\mathbf{w}$-AV^T rules, we also introduce the potential for manipulation. The extent to which manipulation and fairness are related will be studied empirically in Section 5.

4. IRRESOLUTE w-AV

Now we investigate irresolute $\mathbf{w}$-AV, first with extension principles and then with a uniformly randomized tie-breaking mechanism (under the assumption of cardinal preferences).

4.1 Optimistic extension principle

Let us start with the optimistic principle to show that all the rules are O-manipulable, except minisum.

THEOREM 2. *If $\mathbf{w}$ is non-increasing and differs from the minisum weight vector, then $\mathbf{w}$-AV is O-manipulable.*

Again, we consider two cases to show this result. More precisely, we show that all $\mathbf{w}$-AV rules are O-manipulable as soon as $m \geq 2$ if the number of voters is odd. However, when it is even and $m = 2$, some rules may not be O-manipulable, but they become O-manipulable as soon as we consider three or more candidates.

4.1.1 Even number of voters

First, let us study O-manipulation with an even number of voters.

PROPOSITION 6. *If $n = 2q$ and $\mathbf{w}$ is non-increasing such that $w_1 > w_{q+1}$, then $\mathbf{w}$-AV is O-manipulable.*

PROOF. The proof is an example of manipulation that works for all such vectors. We consider an even number of voters, $n = 2q$, where P is composed of q votes 00, $(q-1)$ votes 01, and 1 vote 11. Let $\mathbf{w}$ be such that $w_1 > w_{q+1}$. We have the following scores:

$$\begin{array}{ll} \mathcal{H}(00) = (2\ 1^{q-1}0^q) & D(00) = 2W_{0\to 1} + W_{1\to q} \\ \mathcal{H}(01) = (1^{q+1}0^{q-1}) & D(01) = W_{0\to q+1} \\ \mathcal{H}(10) = (2^{q-1}1^{q+1}) & D(10) = 2W_{0\to q-1} + W_{q-1\to n} \\ \mathcal{H}(11) = (2^q1^{q-1}0) & D(11) = 2W_{0\to q} + W_{q\to n-1}. \end{array}$$

Since $w_1 > w_{q+1}$, we have $D(00) > D(01)$, which implies that 00 is not a winning committee.

Manipulation comes from one of the 00-voters who now votes 10. We obtain new scores:

$$\begin{array}{ll} \mathcal{H}(00) = (2\ 1^q0^{q-1}) & D(00) = 2W_{0\to 1} + W_{1\to q+1} \\ \mathcal{H}(01) = (2\ 1^q0^{q-1}) & D(01) = 2W_{0\to 1} + W_{1\to q+1} \\ \mathcal{H}(10) = (2^{q-1}1^q0) & D(10) = 2W_{0\to q-1} + W_{q-1\to n-1} \\ \mathcal{H}(11) = (2^{q-1}1^q0) & D(11) = 2W_{0\to q-1} + W_{q-1\to n-1}. \end{array}$$

Clearly, $D(00) = D(01)$ and $D(10) = D(11)$. If $q > 1$, $D(00) < D(11)$, and if $q = 1$, $D(00) = D(11)$. Either way, 00 is a winning committee and the O-manipulation is successful. $\square$

We failed to find an O-manipulation with two candidates, an even number of voters, and a weight vector such that $w_1 = w_{q+1}$. We conjecture that there is no such manipulation in that case. However, as soon as we consider three candidates, any non-increasing weight vector induces an O-manipulable rule.

PROPOSITION 7. *If $n = 2q$, $\mathbf{w}$ is non-increasing such that $w_1 > w_n$; then, for any $m \geq 3$, $\mathbf{w}$-AV is O-manipulable.*

The proof is an example of manipulation working for all such vectors, which again we omit due to space limitations.

4.1.2 Odd number of voters

For an odd number of voters, the situation is simpler since any fair rule that differs from minisum is manipulable, when considering two candidates.

PROPOSITION 8. *If $n = 2q + 1$, $\mathbf{w}$ is non-increasing such that $W_{0\to\alpha} < W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} \geq W_{q\to q+\alpha+2}$, $0 \leq \alpha < q$; then, for any $m \geq 2$, $\mathbf{w}$-AV is O-manipulable.*

PROOF. The proof comes from the manipulation example of Proposition 2. With irresolute $\mathbf{w}$-AV, such that $W_{0\to\alpha} < W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} \geq W_{q\to q+\alpha+2}$, $0 \leq \alpha < q$, committee 00 is not a winning committee before manipulation, but it becomes a winner after a 00-voter switches his vote to 10. Thus, it is an O-manipulation. $\square$

For the weight vectors that are closer to minisum weight vectors, we already know from Proposition 4 that if $W_{0\to q} < W_{q\to 2q+1}$, then there is no manipulation. However, Proposition 8 does not capture the rules that are close to minimax (when $w_{q+1} = 0$). Those rules are O-manipulable as shown in the following proposition, for which we omit the proof.

PROPOSITION 9. *If $n = 2q + 1$, $\mathbf{w}$ is non-increasing such that $w_1 \neq 0$ and $w_{q+1} = 0$; then, for any m, $\mathbf{w}$-AV is O-manipulable.*

The conclusion of optimistic manipulation is that minisum is the only fair rule (parameterized by a non-increasing weight vector) that is resistant to O-manipulation.

4.2 Pessimistic and Gärdenfors extension principles

In this section, we simultaneously study the pessimistic and Gärdenfors extension principles. Similar to the optimistic manipulation, we prove that all the fair rules are manipulable, except minisum.

THEOREM 3. *If $\mathbf{w}$ is non-increasing and differs from the minisum weight vector, then $\mathbf{w}$-AV is P- and G-manipulable.*

Again, we show this result in two steps, first with an even number of voters and then with an odd number.

4.2.1 Even number of voters

As in the optimistic manipulation, we show that most of the fair rules are manipulable when we consider two candidates or more. However, some rules require at least three candidates to be manipulable.

PROPOSITION 10. *If $n = 2q$, $\mathbf{w}$ is non-increasing such that $W_{0\to\alpha} = W_{q\to q+\alpha}$ and $W_{0\to\alpha+1} > W_{q\to q+\alpha+1}$, $0 \leq \alpha < q-1$; then, for any $m \geq 2$, $\mathbf{w}$-AV is P-manipulable and G-manipulable.*

PROOF. The proof is from the example of Proposition 1. With irresolute $\mathbf{w}$-AV, such that $W_{0\to\alpha} = W_{q\to q+\alpha}$ and $W_{0\to\alpha+1} \geq W_{q\to q+\alpha+1}$, $0 \leq \alpha < q-1$, committees 00 and 01 are winners before manipulation. Then when a 00-voter switches his vote to 10, committee 00 is the only winning committee. Thus, it is a P-manipulation and a G-manipulation. $\square$

Note this proposition does not capture the case where $\alpha = q-1$, and in that case, we failed to find any manipulation. We conjecture that there are no manipulations for two candidates in that case. However, with three candidates, there are manipulations for weight vectors such that $W_{0\to q-1} = W_{q\to 2q-1}$ and $W_{0\to q} > W_{q\to 2q}$.

PROPOSITION 11. *If $n = 2q$, $\mathbf{w}$ is non-increasing such that $W_{0\to q-1} = W_{q\to 2q-1}$ and $W_{0\to q} > W_{q\to 2q}$; then, for any $m \geq 3$, $\mathbf{w}$-AV is P-manipulable and G-manipulable.*

The proof is an example of a manipulation that we omit due to space limitations.

4.2.2 Odd number of voters

When considering an odd number of voters, we show that most of the fair rules are P-manipulable and G-manipulable when considering two or three candidates, and they are all manipulable with four or more candidates.

PROPOSITION 12. *If $n = 2q+1$, $\mathbf{w}$ is non-increasing such that $W_{0\to\alpha} \leq W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} > W_{q\to q+\alpha+2}$, $0 \leq \alpha < q-1$; then, for any $m \geq 2$, $\mathbf{w}$-AV is P-manipulable and G-manipulable.*

PROOF. The proof is from the manipulation example of Proposition 2. With irresolute $\mathbf{w}$-AV, such that $W_{0\to\alpha} \leq W_{q\to q+\alpha+1}$ and $W_{0\to\alpha+1} > W_{q\to q+\alpha+2}$, and with $0 \leq \alpha < q-1$, committee 01 is the winner, possibly with 00, before manipulation. Then when a 00-voter switches his vote to 10, committee 00 is the only winner. Thus, it is a P-manipulation and a G-manipulation. □

Note that this proposition states manipulation for vectors that are close to minimax. In fact, when $w_{q+1} = 0$, we obtain a P- and a G-manipulation, if $w_1 > 0$.

However, at the opposite side of the spectrum of weight vectors, this proposition does not capture the case where $\alpha = q-1$, and in that case we failed to find any manipulation with two candidates. We conjecture that there are no manipulations for two candidates in that case. However, with three candidates, there are manipulations for weight vectors such that $W_{0\to q} \leq W_{q\to 2q}$ and $W_{0\to q} > W_{q\to 2q+1}$. This result is obtained by Proposition 13, for which we omit the proof.

PROPOSITION 13. *If $n = 2q+1$, $\mathbf{w}$ is non-increasing such that $W_{0\to q-1} \leq W_{q\to 2q}$ and $W_{0\to q} > W_{q\to 2q+1}$; then for any $m \geq 3$, $\mathbf{w}$-AV is P-manipulable and G-manipulable.*

However, the weight vectors such that $W_{0\to q} = W_{q\to 2q+1}$ and $W_{2q+1} = 0$, are not captured by Proposition 13. In fact, those weight vectors lead to P- and G-manipulable rules only when the number of candidates is at least four; we omit the proof.

To conclude on extension principles, we also studied Fishburn and Kelly extension principles ([12, 16]). However, the results are heterogeneous and cannot be stated in a compact form. For example, minimax requires eight voters to be Kelly-manipulable with four candidates, but only seven voters with five candidates and five voters with six candidates. We will address this issue in future work.

4.3 Uniformly randomized w-AV

In this section, we briefly address the uniformly randomized $\mathbf{w}$-AV rules, denoted as $\mathbf{w}$-AV$^{\mathcal{R}}$. We show that all $\mathbf{w}$-AV$^{\mathcal{R}}$ rules are $\mathcal{R}$-manipulable by proving that a Gärdenfors manipulation implies a $\mathcal{R}$-manipulation in our setting.

THEOREM 4. *If $\mathbf{w}$ is non-increasing and differs from the minisum weight vectors, then $\mathbf{w}$-AV$^{\mathcal{R}}$ is $\mathcal{R}$-manipulable.*

PROOF. The proof is a study case, showing that a Gärdenfors manipulation implies a $\mathcal{R}$-manipulation, for a given profile. Consider profile P and G-manipulation P'_i from voter i whose favorite committee is x. Let $\mathcal{C}$ and $\mathcal{C}'$ be the subsets of committees such that $\mathcal{C} = \mathbf{w}\text{-AV}(P)$ and $\mathcal{C}' = \mathbf{w}\text{-AV}(P'_i, P_{-i})$. There are three cases to consider.

- $\mathcal{C}' \subseteq \mathcal{C}$. We know that for all $y \in \mathcal{C} \setminus \mathcal{C}'$ and for all $y' \in \mathcal{C}'$, $y' \succ_i y$, which implies that $m - d_{\mathcal{H}}(x, y') > m - d_{\mathcal{H}}(x, y)$. Thus, we have $\overline{u_i}(\mathcal{C}') > \overline{u_i}(\mathcal{C})$.
- $\mathcal{C} \subseteq \mathcal{C}'$. The argument resembles the previous case.
- Otherwise, as in the first case, we have $\overline{u_i}(\mathcal{C} \cap \mathcal{C}') > \overline{u_i}(\mathcal{C})$, and as in the second one, we have $\overline{u_i}(\mathcal{C}') > \overline{u_i}(\mathcal{C} \cap \mathcal{C}')$, and we obtain $\overline{u_i}(\mathcal{C}') > \overline{u_i}(\mathcal{C})$.

Thus, a Gärdenfors manipulation is a $\mathcal{R}$-manipulation for $\mathbf{w}$-AV$^{\mathcal{R}}$. □

The conclusion on irresolute manipulation resembles the resolute case: minisum is the only fair rule that is strategyproof.

5. EMPIRICAL RESULTS

In the previous sections, we proved that all the $\mathbf{w}$-AV rules parameterized by non-increasing weight vectors are manipulable, except minisum. However, we conjecture that the rules that are closer to minisum will be less often manipulable in expectation than those closer to minimax. We focus on resolute[3] manipulation.

We study resolute manipulation under two assumptions about the distribution of preferences: *impartial culture* distribution (IC) and *biased* distribution. In the context of approval ballots, impartial culture implies that any candidate has a probability of 0.5 to be approved by each voter. Impartial culture is the worst-case scenario where we don't assume anything about the preferences of the voters. In biased distribution[4], we randomly generate two approval probabilities, p_1 and p_2, for each candidate, uniformly between 1 and 0. The voters are then divided into three groups: 40% approve each candidate with probability p_1, another 40% with probability p_2, and the remaining 20% with probability 0.5. This biased distribution reflects a bipartite electorate with some uncertain voters.

An appealing way to characterize the fairness of a $\mathbf{w}$-AV is its orness measure, which calculates its proximity to minimax. Introduced in [25], the orness measure of an OWA operator with weight vector w is defined as:

$$orness(w) = \frac{1}{n-1}\sum_{j=1}^{n}(n-j)w_j.$$

[3]Ties are broken in the following way: between two committees, the prior is the one containing the most prior candidates based on lexicographic order.

[4]For example, this distribution has been used in previous work [19].

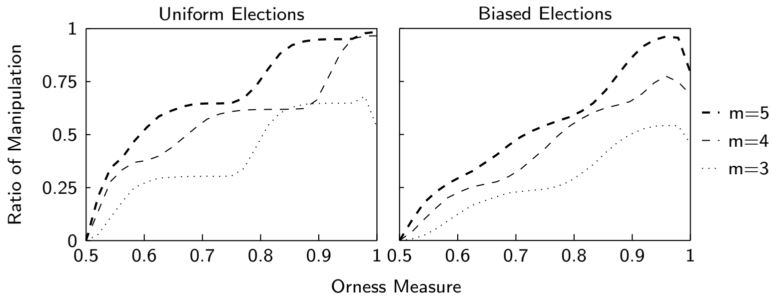

Figure 1: Proportion of manipulable elections depending on orness of $\mathbf{f^i}$-AV, where $n = 25$

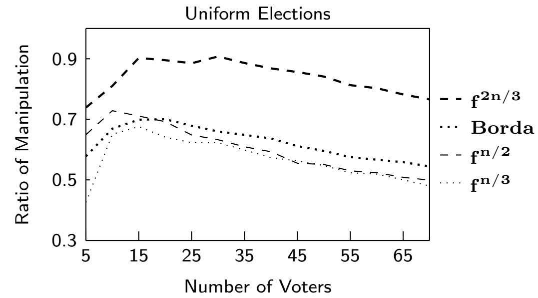

Figure 2: Proportion of manipulable elections depending on number of voters, where $m = 5$

Minimax, parameterized by weights $\mathbf{w} = (1, 0, \dots, 0)$, gets an orness of 1, while minisum, corresponding to the average operator, has an orness of 0.5. Arguably, OWA operators with an orness exceeding 0.5 are considered fair[5], and fairer when they get closer to 1. However, a given orness level does not characterize a unique weight vector.

We study a family of rules, $\mathbf{f^i}$-AV, defined in [1], which is parameterized by weight vector $f_n^i = \frac{1}{n-i}(1, \dots 1, 0, \dots, 0)$, where i is the number of 0. This family generalizes both minisum and minimax, corresponding respectively to f^0 and f^{n-1}. Given n voters, each $\mathbf{f_n^i}$ corresponds to a specific orness measure, which is $orness(\mathbf{f_n^i}) = \frac{n+i-1}{2(n-1)}$.

Figure 1 presents the proportion of manipulable uniform, respectively biased, elections depending on the orness measure of the $\mathbf{f_n^i}$-AV rules, when considering 25 voters and small numbers of candidates. We obtained these proportions by randomly generating a population of 10^4 elections[6] for each distribution and checking with a brute force algorithm whether there exists a manipulation by any voter. In both cases, we observe that the proportion of manipulable elections increases when the orness measure increases from minisum orness to minimax orness. The conjecture seems verified and a clear trade-off appears between fairness (proximity to minimax) and sensitivity to manipulation. However, we observe a slight fall for rules close to minimax in the case of biased elections, but notice that the proportion of manipulable elections remains high, which means that almost any election is potentially manipulable by at least one voter.

Now, we address study a particular $\mathbf{w}$-AV rule, the $\mathbf{B}$-AV rule, parameterized by a weight vector proportional to the Borda vector. For example, with six voters, the weight vector is $(\frac{6}{21}, \frac{5}{21}, \frac{4}{21}, \frac{3}{21}, \frac{2}{21}, \frac{1}{21})$. We compare it with three rules of the $\mathbf{f^i}$-AV family, for $i \in \{\frac{n}{3}, \frac{n}{2}, \frac{2n}{3}\}$. The $\mathbf{B}$-AV rule is interesting because it satisfies the Pigou-Dalton principle since its weight vector is strictly decreasing, whereas the $\mathbf{f^i}$-AV rules only satisfy it in a weak sense. Thus, we expect it to be more sensitive to manipulation than the $\mathbf{f^i}$-AV rules. However, the orness of $\mathbf{B}$-AV is 2/3 for any number of voters, whereas for $\mathbf{f^{n/3}}$-AV, $\mathbf{f^{n/2}}$-AV, and $\mathbf{f^{2n/3}}$-AV, it is respectively 0.67, 0.75, and 0.84 when considering 70 voters. This measure suggests that since the $\mathbf{B}$-AV rule is less fair than the three above $\mathbf{f^i}$-AV rules, it is less sensitive to manipulation.

Figure 2 shows the proportion of manipulable uniform elections depending on the number of voters for the four above rules with five candidates. We first observe that the $\mathbf{f^{2n/3}}$-AV rule has a proportion much higher than $\mathbf{B}$-AV, which tends to confirm the conjecture that manipulability increases with the orness. However, this result is tempered by the fact that, despite having quite different orness measures, $\mathbf{f^{n/2}}$-AV and $\mathbf{B}$-AV appear to be equally manipulable. Moreover, with 20 voters or more, $\mathbf{B}$-AV is slightly more manipulable than $\mathbf{f^{n/2}}$-AV. Finally, when the number of voters increases, the proportion of manipulable elections globally decreases. This result is not surprising since when the number of voters increases, each voter has a lower impact on the outcome. We also compared these rules with the biased elections model and obtained similar results with a noticeably faster decrease when the number of voters increases.

6. CONCLUSION

We analyzed the manipulation of a family of Hamming-based approval voting rules for committee elections and multiple referenda using ordered weighted averaging. This family ranges from a candidate-wise majority to a minimax with a continuum of rules inbetween, which are fairer when closer to minimax. We first proved that minisum is the only fair rule of this family to be strategyproof to resolute and irresolute manipulation, using extension principles or the uniformly randomized tie-breaking mechanism. Through an experimental study, we explored the link between manipulability and fairness, and showed that the rules that are closer to minimax are more sensitive to manipulation. This study could be extended to non-monotonic weight vectors, such as the Olympic weight vector that drops extreme scores, or to additional models of preferences, in particular to Candidate/Voter Interval profiles introduced in [10].

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[5]For a discussion on fairness in optimization, see [21].

[6]The size of the population leads to a margin of error strictly lower than 0.01 with a confidence of 95% for any point of any chart in this paper.

REFERENCES

[1] G. Amanatidis, N. Barrot, J. Lang, E. Markakis, and B. Ries. Multiple referenda and multiwinner elections using hamming distances: Complexity and manipulability. In *AAMAS*, pages 715–723, 2015.

[2] S. Barberà. *Strategy-proof social choice*, volume 2. Elsevier, 2010.

[3] N. Barrot, L. Gourves, J. Lang, J. Monnot, and B. Ries. Possible winners in approval voting. In *International Conference on Algorithmic Decision Theory*, pages 57–70. Springer, 2013.

[4] S. Brams, D. Kilgour, and W. Zwicker. Voting on referenda: the separability problem and possible solutions. *Electoral Studies*, 16(3):359–377, 1997.

[5] S. J. Brams, D. M. Kilgour, and M. R. Sanver. A minimax procedure for electing committees. *Public Choice*, 3-4(132):401–420, 2007.

[6] F. Brandt and M. Brill. Necessary and sufficient conditions for the strategyproofness of irresolute social choice functions. In *Proceedings of the 13th Conference on Theoretical Aspects of Rationality and Knowledge*, pages 136–142. ACM, 2011.

[7] I. Caragiannis, D. Kalaitzis, and E. Markakis. Approximation algorithms and mechanism design for minimax approval voting. In *AAAI*, pages 737–742, 2010.

[8] H. Dalton. The measurement of the inequality of incomes. *The Economic Journal*, 30(119):348–361, 1920.

[9] E. Elkind and A. Ismaili. Owa-based extensions of the chamberlin–courant rule. In *International Conference on Algorithmic DecisionTheory*, pages 486–502. Springer, 2015.

[10] E. Elkind and M. Lackner. Structure in dichotomous preferences. *arXiv preprint arXiv:1505.00341*, 2015.

[11] P. Faliszewski, P. Skowron, A. Slinko, and N. Talmon. Committee scoring rules: Axiomatic classification and hierarchy. In *IJCAI*, pages 250–256, 2016.

[12] P. C. Fishburn. Even-chance lotteries in social choice theory. *Theory and Decision*, 3(1):18–40, 1972.

[13] P. Gärdenfors. Manipulation of social choice functions. *Journal of Economic Theory*, 13(2):217–228, 1976.

[14] A. Gibbard. Manipulation of voting schemes. *Econometrica*, 41(4):587–602, 1973.

[15] J. Goldsmith, J. Lang, N. Mattei, and P. Perny. Voting with rank-dependent scoring rules. In *Proceedings of AAAI-14*, 2014.

[16] J. S. Kelly. Strategy-proofness and social choice functions without singlevaluedness. *Econometrica: Journal of the Econometric Society*, pages 439–446, 1977.

[17] D. M. Kilgour. Approval balloting for multi-winner elections. In *Handbook on Approval Voting*, chapter 6. Springer, 2010.

[18] J. Lang, G. Pigozzi, M. Slavkovik, and L. van der Torre. Judgment aggregation rules based on minimization. In *TARK*, pages 238–246, 2011.

[19] R. LeGrand, E. Markakis, and A. Mehta. Some results on approximating the minimax solution in approval voting. In *AAMAS*, pages 198:1–198:3, 2007.

[20] R. Meir, A. D. Procaccia, J. S. Rosenschein, and A. Zohar. Complexity of strategic behavior in multi-winner elections. *Journal of Artificial Intelligence Research*, 33:149–178, 2008.

[21] W. Ogryczak, H. Luss, M. Pióro, D. Nace, and A. Tomaszewski. Fair optimization and networks: A survey. *Journal of Applied Mathematics*, 2014, 2014.

[22] A. C. Pigou. *Wealth and welfare*. Macmillan and Company, limited, 1912.

[23] M. A. Satterthwaite. Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217, 1975.

[24] P. Skowron, P. Faliszewski, and J. Lang. Finding a collective set of items: From proportional multirepresentation to group recommendation. In *COMSOC*, 2014.

[25] R. R. Yager. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man, and Cybernetics*, 18:183–190, 1988.