The Generalized N&K Value: An Axiomatic Mechanism for Electricity Trading

Extended Abstract

Mark Alexander Burgess
Australian National University
Canberra, ACT
markburgess1989@gmail.com
mark.burgess@anu.edu.au

Archie C. Chapman
University of Sydney
Sydney, NSW
archie.chapman@sydney.edu.au

Paul Scott
Australian National University
Canberra, ACT
paul.scott@anu.edu.au

ABSTRACT

We introduce an axiomatic solution concept for generalized non-cooperative games called the *generalized N&K value*. It is applied as a mechanism for determining appropriate financial transfers over DC optimal power flow (DC-OPF) instances in an electrical network. The generalized N&K value rewards network participants in proportion to the competitive position of the coalitions that they could be part of. In this way it reflects the relative bargaining powers of the electrical network participants.

KEYWORDS

Auctions and mechanism design; game theory for practical applications; noncooperative games: theory & analysis

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1 INTRODUCTION

For an electrical network, what electrically-feasible power flow and budget-balanced financial transactions should occur between the network participants when they each have different preferences? This is a long-standing question.

A well known approach is called *locational marginal pricing* (LMP), under which real-time prices vary between locations/areas in the network in proportion to the marginal cost of the electricity supplied to the location/area. This measure reflects the cost of network transmission losses and congestion factors [9–11]. LMP is grounded in the marginalist principle of economics, which is readily observed in large markets. However when the electrical interactions between individual participants are strongly coupled, it is not obvious that such a principle provides a sensible description of either a natural or an idealized market.

In many real-world settings there is a strategic interaction between electricity network participants. As such, we turn to *game theory* to analyze the setting and develop a more appropriate market pricing rule [7, 12]. We derive a novel solution concept called the *Generalized Neyman and Kohlberg value*, or *GNK value* for short, from four suitable axioms. This new solution concept stems from comparing the relative bargaining positions of coalitions of players in the context of a generalized non-cooperative game. We show how it defines a mechanism that allocates payments between network participants proportional to their competitive position for monetary compensation within an electricity network, and we present a method for calculating it for networks under DC-approximation.

The main contributions of this paper are:

- We derive and justify the GNK value, by extending the 'the Value' concept to the space of generalized games,
- We present a solution method for calculating the GNK value with linear action-space constraints and convex player preferences (suitable for DC approximated networks)

This paper is not about finding a point of optimal control between consumers, prosumers, devices etc. [1, 2, 8], but rather about developing a method of determining economically reasonable payments between such agents about this point, in an electrical network.

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2 DERIVING THE GNK VALUE

There have been many attempts to answer the question of what co-operative outcome *should* occur in the context of a non-cooperative TU game. One well known answer is the *Nash bargaining solution* between two players [6], which Harsanyi [4] extended to arbitrary numbers of players when game payoffs are transferable. Harsanyi's solution was derived from a simple set of axioms by Neyman and Kohlberg (N&K) [5]; our derivation mirrors the steps in theirs.

We consider N&K's *coalitional game of threats* which is defined by a pair $d = \langle N, v\rangle$ where:

- $N = \{1, \dots, n\}$ is a finite set of *players* or *agents*, and
- $v : 2^N \to \mathbb{R}$ is a *characteristic function* or 'threat' with

$$v(S) = -v(N \setminus S) \quad \forall S \subseteq N.$$

N&K's key result was to prove that if $\mathbb{D}$ is the set of all such games, then there exists a unique mapping $\psi : \mathbb{D} \to \mathbb{R}^n$ that satisfies the following four axioms:

- **Efficiency**: $\sum_i \psi(\langle N, v\rangle)_i = v(N)$
- **Symmetry**: If two players i and j are substitutes, such that: if $v(S \cup i) = v(S \cup j)\ \forall S \subseteq N \setminus \{i, j\}$, then $\psi(\langle N, v\rangle)_i = \psi(\langle N, v\rangle)_j$
- **Null Player**: If a player i is a null player (i.e. $v(S \cup i) = v(S)\ \forall S \subseteq N$) then $\psi(\langle N, v\rangle)_i = 0$
- **Additivity**: for any v_1 and v_2 that: $\psi(\langle N, v_1 + v_2\rangle) = \psi(\langle N, v_1\rangle) + \psi(\langle N, v_2\rangle)$

Letting agent i's element of ψ be denoted by ψ_i, this mapping is:

$$\psi_i(\langle N, v\rangle) = \frac{1}{n}\sum_{k=1}^{n} v_{i,k} = \frac{1}{n}\sum_{k=1}^{n}\frac{1}{\binom{n-1}{k-1}}\sum_{\substack{S:i\in S\\ |S|=k}} v(S) \tag{1}$$

where $v_{i,k}$ is the average value of $v(S)$ for all coalitions of size k that include i.

We define the *threat* or the *advantage* of a coalition $v(S)$, in the context of a *generalized non-cooperative game*, which is a game where the strategies available to one player may be restricted by the strategy choice of others. A generalized non-cooperative game consists of a triplet $G = \langle N, A, g\rangle$ in which:

- $N = \{1, \ldots, n\}$ is a finite set of players,
- $A \subseteq \prod_{i\in N} A_i$ is a set of all possible joint strategies, where A_i denotes the set of strategies of player $i \in N$, and A is a subset of their product space,
- $\{u_i(a) : A \to \mathbb{R}\}_{i\in N}$ is a set of functions of payoffs to the players when joint strategy $a \in A$ is executed.

We define the payoff "threat" or "advantage" of a coalition $S \subseteq N$ in this context (letting $A^S = \prod_{i\in S} A^i$), taking into account the constraints that apply to the joint action space as:

$$v(S) = \frac{1}{2}\max_{\substack{x\in A^S\\ \text{s.t.}\exists y,(x,y)\in A}}\ \min_{\substack{y\in A^{N\setminus S}\\ \text{s.t.}(x,y)\in A}}\left(\sum_{i\in S} u_i(x,y) - \sum_{i\in N\setminus S} u_i(x,y)\right) + \frac{1}{2}\min_{\substack{y\in A^{N\setminus S}\\ \text{s.t.}\exists x,(x,y)\in A}}\ \max_{\substack{x\in A^S\\ \text{s.t.}(x,y)\in A}}\left(\sum_{i\in S} u_i(x,y) - \sum_{i\in N\setminus S} u_i(x,y)\right) \tag{2}$$

Where $(x, y) \in A$ Denotes a partition of the joint action between two coalitions S and $N \setminus S$ respectively.

The expression in (2) represents the expectation of the competitive advantage (or threat) that a coalition has over its complement in a generalized strategy space under a fair coin-toss of who chooses their strategies first. The formulation of ψ, per (1), with this characteristic function (2) is our GNK value. It is a novel extension of existing work to the space of generalized games.

2.1 How the GNK value applies to networks

The players with payoffs and strategies can be interpreted as network participants with quantifiable utilities and electrical actions. As $v(N) = \max_{a\in A}(\sum_{i\in N} u_i(a))$ represents the maximum achievable sum utility that the network participants can achieve[1], the GNK value ψ splits this amount between the participants by the efficiency axiom. The participants execute electrical actions that achieve this maximal sum, and ψ is implemented by executing required (budget balanced) financial transfers.

3 GNK VALUE FOR DC POWER FLOWS

An electricity network of consumption/generation between participants under DC-approximation can be modeled as a generalized game. Hence the GNK value can be applied to this game and we give the details of a computational method used.

We consider an electrical network to have a set of buses B with power consumption at each bus p_i and voltage phase-angle θ_i (for all $i \in B$). The buses are connected by lines $C \subseteq B \times B$ with susceptance $b_{i,j}$ and power flow $p_{i,j}$ (power from i to j, for $(i, j) \in C$, and with $p_{i,j} = -p_{j,i}$). In this context, DC power-flow constraints are expressed as follows [3]:

$$\begin{array}{ll}\text{Variables:} & p_i(i \in B),\ \theta_i(i \in B),\ p_{i,j}((i,j) \in C)\\ \text{constraints:} & \begin{array}{ll} p_i^l \le p_i \le p_i^u & p_{i,j} = -b_{i,j}(\theta_i - \theta_j)\ \forall (i,j) \in C\\ p_{i,j}^l \le p_{i,j} \le p_{i,j}^u & p_j = \sum_{(i,j)\in C} p_{i,j}\ \forall j \in B\end{array}\end{array} \tag{3}$$

Where $p_i^l, p_i^u, p_{i,j}^l, p_{i,j}^u$ are the upper and lower bounds on power consumption/generation and line limits respectively. We eliminate redundant variables (the θ_i and $p_{i,j}$) and to ease presentation we use the functions h and g to represent the resulting linear constraints:

$$\begin{array}{ll}\text{Variables:} & p_i(i \in B)\\ \text{constraints:} & h_j(p_1, p_2, \ldots) = 0\ \forall j \quad g_k(p_1, p_2, \ldots) \le 0\ \forall k\end{array} \tag{4}$$

The participants on each bus are treated as individual players in a game (i.e. $N = B$), and the power consumption of that bus is the respective player's strategy space (i.e. $A_i = [p_i^l, p_i^u]$). Hence the DC constraints (equations 3 or 4) define the space of jointly executable strategies (generalized strategy space A). We assume that there is a utility (or payoff) associated with the power consumption for each player, $u_i(p_i)$, Hence the situation is as a generalized game.

Since the GNK value is quite difficult to solve directly, we reformulate (2) to be more amenable to standard optimization software by transforming the inner maximization/minimization constraints into KKT conditions. We replaced the constraint $(x, y) \in A$ with the constraint that the (x, y) satisfy the conditions for local minima/maxima (i.e. that they be *KKT points*) in the same space: [2]

$$2v(S) = \max_{\substack{p_i\\ i\in S}}\left[\begin{array}{c}(\sum_{i\in S} u_i(p_i) - \sum_{i\notin S} u_i(p_i))\\ \text{s.t.}\forall i\notin S\ -\frac{du_i}{dp_i} = \sum_j(\lambda_{h_j}\frac{\partial h_j}{p_i}) + \sum_k(\lambda_{g_k}\frac{\partial g_k}{p_i})\\ \forall j\ h_j(p_1,p_2,\ldots)=0\\ \forall k\ (1-Z_k)\bar{\lambda}_{g_k} \ge \lambda_{g_k} \ge 0\\ \forall k\ \underline{g_k} Z_k \le g_k(p_1,p_2,\ldots) \le 0\end{array}\right] + \min_{\substack{p_i\\ i\notin S}}\left[\begin{array}{c}(\sum_{i\in S} u_i(p_i) - \sum_{i\notin S} u_i(p_i))\\ \text{s.t.}\forall i\in S\ \frac{du_i}{dp_i} = \sum_j(\lambda_{h_j}\frac{\partial h_j}{p_i}) + \sum_k(\lambda_{g_k}\frac{\partial g_k}{p_i})\\ \forall j\ h_j(p_1,p_2,\ldots)=0\\ \forall k\ (1-Z_k)\bar{\lambda}_{g_k} \ge \lambda_{g_k} \ge 0\\ \forall k\ \underline{g_k} Z_k \le g_k(p_1,p_2,\ldots) \le 0\end{array}\right] \tag{5}$$

Where $\bar{\lambda}_{g_k}$ and $\underline{g_k}$ are the estimated upper and lower bounds on the KKT multipliers and constraint function respectively. This reformulation is amenable for calculation by optimization solvers and has been used to calculate the GNK value for DC networks. Where it has been demonstrated that the GNK value:

- Doesn't feature discontinuous congestion prices, unlike LMP;
- Is always budget balanced, unlike LMP.

However:

- It is significantly more difficult to compute than LMP;
- It is not incentive compatible (nor is LMP).

[1] calculating $v(N)$ is an OPF problem as there are no variables to minimize over in (2)

[2] By doing this we tacitly we assume that $u_i(p_i)$ is continuously differentiable, and satisfy some regularity conditions. Furthermore the constraints under DC-approximation are linear and hence define a convex polygon and if we assume that the functions $u_i(p_i)$ are weakly-concave then any of the KKT points will be equal in value to the global minima (or maxima). Hence the inner maximization (or minimization) over the KKT points can be ignored

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