# Learning Game-theoretic Models from Aggregate Behavioral Data with Applications to Vaccination Rates in Public Health

Extended Abstract

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## ABSTRACT

In this paper, we undertake the challenging task of uncovering independencies of public-health behavioral data on populations' vaccination rates collected by government officials in the United States. We use computational game theory to model such data as the result of distributed decision-making at the reported granularity level (e.g., nations and states). To achieve our task, we posit the view of aggregated behavioral data as jointly randomized, or mixed, strategies of multiple agents. We propose a novel general machine-learning approach to learn game-theoretic models within a given hypothesis class of games from any potentially noisy dataset of mixed strategies. We illustrate our framework using publicly available data on vaccination rates in the continental USA.

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## 1 INTRODUCTION

The USA's *Center for Disease Control and Prevention (CDC)* collects and reports aggregate data about vaccination rates, along with standard deviations, for each state yearly. Each vaccination percentages represent the state-wide behavior of the people living in the State. Alternatively, we can view each state's vaccination percentage as a proxy measure of the state government's achievement from effort to raise its population vaccination rate for some disease or epidemic (e.g., influenza and Ebola). We can view those vaccination rates of the states as the joint-behavior of the states (i.e., the outcome of their efforts). Given these state vaccination probabilities, we want to understand how the epidemic vaccination decisions of the states affect each other by modeling the strategic interaction as vaccination games or $\alpha$-IDS games (defined in Section 3).

*Contribution, Related Work, and Preliminary.* We view these probabilities collectively as possibly approximate mixed-strategy Nash equilibrium (MSNE) to account for noises. We (1) propose a machine learning (generative) framework to learn a game given behavioral data; (2) use our framework to derive a heuristic to learn $\alpha$-IDS games given the CDC vaccination data; and (3) experimentally show that our framework is effective for learning $\alpha$-IDS games.

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The closest work to ours is that of Honorio and Ortiz [12], in which they provide a general machine-learning framework to learn the structure and parameters of games from discrete behavioral data (e.g., "Yes/No"-type responses). Moreover, they demonstrate their framework on learning some classes of games. We refer the reader to the related-work section of Honorio and Ortiz [12] for a more detailed discussion. For brevity, all other previous methods assume that the actions and payoffs are observable in the data [5–7, 10, 15–17] while others are interested in predicting future behavior from the past behavior (system dynamics) [13, 18].

We refer the reader to [9] for an introduction to basic concepts in game theory. Let $V = \{1, 2, \ldots, n\}$ be a *set of players/agents*. For each $i \in V$, let $A_i$ be the *set of actions/pure-strategies* available to $i$, and $A = \times_{i \in V} A_i$ the *set of joint-actions/joint-pure-strategies*. Denote by $u_i : A \to \mathbb{R}$ the *payoff of i* for each joint-action in $A$. Similarly, let $X_i$ be the *set of mixed-strategies of i*, and $X \equiv \times_{i \in V} X_i$ be the *set of joint-mixed-strategies*, which is a probability simplex over $A_i$. Given the context of this paper and the application domain (i.e., the CDC dataset), we assume that $A_i = \{0, 1\}$ for all $i$, so that we can represent $X_i = [0, 1]$, and interpret each $x_i \in X_i$ as *the probability that agent i plays* $a_i = 1$. We also assume that for all $i$, the maximum and minimum payoff value of $u_i$ is 1 and 0, respectively. We denote the set of all $\epsilon$-MSNE of a game $\mathcal{G}$ as $\mathcal{NE}^\epsilon(\mathcal{G})$. A 0-MSNE is an *exact* MSNE, which always exists for any non-cooperative game [14].

*We refer the readers to the authors' webpages for the full version.*

## 2 A FRAMEWORK TO LEARN GAMES

Motivated in part by the CDC data, we propose a generative model of behavioral data over the set of *mixed-strategy* (as in [12]). Let $\mu$ be the *Borel measure.* (See [1] for an introduction to measure-theoretic concepts.) More formally, the *probability density function (PDF)* $f$ for the generative model with parameters $(q, \mathcal{G}, \epsilon)$ over the hypercube of joint-mixed-strategies $[0, 1]^n$ is

$$f_{(q,\mathcal{G},\epsilon)}(x) \equiv q\frac{\mathbb{1}[x \in \mathcal{NE}^\epsilon(\mathcal{G})]}{\mu(\mathcal{NE}^\epsilon(\mathcal{G}))} + (1-q)\frac{\mathbb{1}[x \notin \mathcal{NE}^\epsilon(\mathcal{G})]}{1-\mu(\mathcal{NE}^\epsilon(\mathcal{G}))}, \quad (1)$$

for all $x \in [0, 1]^n$. The below lemma shows Eqn. 1 is well-defined.

Lemma 2.1. *The set $\mathcal{NE}^\epsilon(\mathcal{G})$ is Borel $\mu$-measurable for any game $\mathcal{G}$ and any $\epsilon \geq 0$. For any $\epsilon > 0$, we have $\mu(\mathcal{NE}^\epsilon(\mathcal{G})) > 0$*

For Eqn. 1 to be valid, if $\epsilon = 1$ or $\mathcal{NE}^\epsilon(\mathcal{G}) = [0, 1]^n$, then we impose $q = 1$.

*Learning Games via Maximum-Likelihood.* We present a way to infer games from behavioral data on mixed strategies. Let $\pi^\epsilon(\mathcal{G})$ be the *true proportion of $\epsilon$-MSNE* in the game $\mathcal{G}$ where $\pi^\epsilon(\mathcal{G}) \equiv \mu(\mathcal{NE}^\epsilon(\mathcal{G}))$ . Given a dataset $D = \{x^{(1)}, \ldots, x^{(m)}\}$, where each

$x^{(l)} \sim f_{q,\mathcal{G},\epsilon}$, i.i.d., let $\widehat{\pi}^{\epsilon}(\mathcal{G})$ be the *empirical proportion of $\epsilon$-MSNE*: $\widehat{\pi}^{\epsilon}(\mathcal{G}) \equiv \frac{1}{m}\sum_{l=1}^{m} \mathbb{1}\left[x^{(l)} \in \mathcal{NE}^{\epsilon}(\mathcal{G})\right]$. We denote the *Kullback-Leibler (KL) divergence* between two Bernoulli distributions with parameters $p_1, p_2 \in (0,1)$ by $\mathrm{KL}(p_1 \| p_2)$.

Proposition 2.2. **(Maximum-likelihood Estimation)** *The tuple $(\hat{\mathcal{G}}, \widehat{q}, \widehat{\epsilon})$ is a* maximum likelihood estimator (MLE), *with respect to dataset D, for the parameters of the generative model $f_{(q,\mathcal{G},\epsilon)}$, as defined in Eqn. 1 if and only if (iff) $\widehat{q} = \widehat{\pi}^{\widehat{\epsilon}}(\widehat{\mathcal{G}})$, and $(\widehat{\mathcal{G}}, \widehat{\epsilon}) \in$* $\arg\max_{(\mathcal{G},\epsilon)} \mathrm{KL}(\widehat{\pi}^{\epsilon}(\mathcal{G}) \| \pi^{\epsilon}(\mathcal{G}))$.

Dealing with $\pi^{\epsilon}(\mathcal{G})$ directly would require us to compute all $\epsilon$-MSNE of $\mathcal{G}$; computing only one $\epsilon$-MSNE is PPAD-hard in general [3, 4]. The following lemma provides bounds on the KL divergence.

Lemma 2.3. *Given a game $\mathcal{G}$ with $0 < \pi^{\epsilon}(\mathcal{G}) < \widehat{\pi}^{\epsilon}(\mathcal{G})$ and $\mu(\mathcal{NE}^{\epsilon}(\mathcal{G})) \in (0,1)$, we have*

$$-\widehat{\pi}^{\epsilon}(\mathcal{G}) \log \pi^{\epsilon}(\mathcal{G}) - \log 2 < \mathrm{KL}(\widehat{\pi}^{\epsilon}(\mathcal{G}) \| \pi^{\epsilon}(\mathcal{G})) < -\widehat{\pi}^{\epsilon}(\mathcal{G}) \log \pi^{\epsilon}(\mathcal{G}).$$

From the above, it is easy to see that when $\pi^{\epsilon}(\mathcal{G})$ is "low enough," we can obtain an approximation to the MLE by simply maximizing $\widehat{\pi}^{\epsilon}(\mathcal{G})$ only: i.e., $\arg\max_{\mathcal{G}} \mathrm{KL}(\widehat{\pi}^{\epsilon}(\mathcal{G}) \| \pi^{\epsilon}(\mathcal{G})) \approx \arg\max_{\mathcal{G}} \widehat{\pi}^{\epsilon}(\mathcal{G})$.

## 3 APPLICATION: GENERALIZED IDS GAMES

In $\alpha$-IDS games [2] with $n$ state-agents, each state-agent $i$ determines whether or not to invest in protection (against epidemics). We denote $a_i = 1$ if $i$ invests and $a_i = 0$ if $i$ does not invest and let $x_i$ be the probability that $a_i = 1$. We let $x = (x_1, ..., x_n)$ to be the joint-mixed strategy profile of all agents and $x_{-S}$ to be the profile of all agents that are not in $S$. There is a cost of investment $C_i$ and loss $L_i$ associated with the bad event occurring, either through a direct or indirect (transferred) contamination. We denote by $p_i$ the probability that agent $i$ will experience the bad event from a direct contamination and by $q_{ji}$ the probability that agent $i$ will experience the bad event due to transfer exposure from agent $j$. The parameter $\alpha_i \in [0,1]$ specifies the probability that agent $i$'s investment will not protect $i$ against transfers of a bad event. Given the parameters, the *expected cost function of agent i* is $M_i(x_i, x_{-i})$

$$\equiv x_i[C_i + \alpha_i r_i(x_{-i}) L_i] + (1 - x_i)[p_i + (1 - p_i) r_i(x_{-i})] L_i,$$

where $r_i(x_{-i}) \equiv 1 - s_i(x_{-i})$ and $s_i(x_{-i}) \equiv \prod_{j \neq i}(x_j + (1-x_j)(1-q_{ji}))$ are $i$'s overall risk and safety functions, respectively. By definition, an $\epsilon$-MSNE $x$ of an $\alpha$-IDS game satisfies

$$M_i(x_i, x_{-i}) - \epsilon \le M_i(0, x_{-i}) \;\&\; M_i(x_i, x_{-i}) - \epsilon \le M_i(1, x_{-i}). \quad (2)$$

*Learning.* We approximate our MLE objective by maximizing the number of $\epsilon$-MSNE in the data when the true proportion of $\epsilon$-MSNE of the game is less than the empirical proportion of $\epsilon$-MSNE in the dataset *We empirically observe that the true proportion of $\epsilon$-MSNE in $\alpha$-IDS games is very low.* This would justify Lemma 2.3 and our method.

We subdivide the optimization by first optimizing over $\mathcal{G}$, and then optimizing over $\epsilon$. We use an upper bound by applying Eqn. 2. Then, we approximate the indicator function in the upper bound using a sigmoid function, which is the standard approach leading to the BackProp algorithm in neural networks [11].

Using standard primal-dual optimization and regularization techniques, we obtain and solve a non-linear program (using gradient-ascent/descent optimization) subject to the respective constraints on the variables. The process terminates when the objective function satisfies some condition and after exceeding some threshold based on the total running time (i.e., ≈ 5 hours for the CDC dataset).

## 4 EXPERIMENTS ON VACCINATION DATA

Viewing each State as a player in the game, we interpret the vaccination percentages as mixed-strategies and generate 1500 samples i.i.d. according to an $n$-variate jointly-independent Gaussian PDF, where $n = 48$, with the joint mean and standard deviations given by each State's reported vaccination rate and standard deviation in the CDC 2009-2010 US States H1N1 data [1]. This is our way to account for the noise in the data. We impose an *a priori* bias for learning where only neighboring states may transfer the virus.

*Learned $\alpha$-IDS Games.* Although game parameters themselves are not our main interest, we highlight similar observations on 10 learned games because they provide anecdotal validation.
***Players' Characteristics.*** All of the players have strategic substitutability behavior – this happens if $\alpha_i < 1 - p_i$ for each player $i$. In Figure 1, the x-axis denotes the $\alpha$ values of the players, the y-axis denotes the $1 - p$ values of the players, and the line is the equation $\alpha = 1 - p$. The plot is scaled to capture the $\alpha$ and $1 - p$ values. The plot illustrates that our learning formulation produces values of the parameters that are consistent with vaccination scenarios, in which $\alpha < 1 - p$.

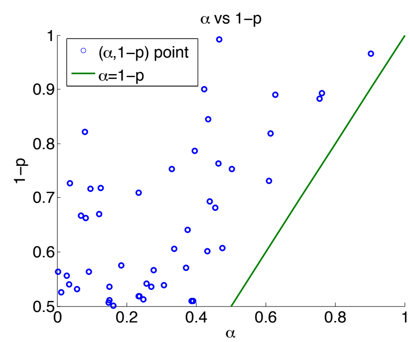


**Figure 1: Players' Characteristics of a Learned Game.**

***Player's Best-Response Correspondences.*** All of the players have non-trivial best-response – players do not have any "obvious" dominant strategies.
***Players' Transfer Risks.*** The transfer risks of the players are not random – they are correlated to the training examples.
***Players' Equilibrium Behavior.*** Given the learned games, we run a version of some learning-heuristics/regret-minimization [8], in which we use the mean vaccination rates as the initial mixed-strategy profile to compute $\epsilon$-MSNE in these games.

It turns out that the mean vaccination-rates given in the CDC data is an 0.35-MSNE of the learned game. We are able to find an exact MSNE which is also a PSNE after trying many initial mixed-strategies that are drawn uniformly at random.

[1] https://www.cdc.gov/flu/fluvaxview/reportshtml/reporti0910/resources/2009-10_coverage.xlsx

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