Optimal Multiphase Investment Strategies for Influencing Opinions in a Social Network

Extended Abstract*

Swapnil Dhamal
Samovar, Télécom SudParis, CNRS, Université Paris-Saclay
Evry, France

Walid Ben-Ameur
Samovar, Télécom SudParis, CNRS, Université Paris-Saclay
Evry, France

Tijani Chahed
Samovar, Télécom SudParis, CNRS, Université Paris-Saclay
Evry, France

Eitan Altman
INRIA Sophia Antipolis-Méditerranée
Valbonne, France

ABSTRACT

We study the problem of two competing camps aiming to maximize the adoption of their respective opinions, by optimally investing in nodes of a social network in multiple phases. The final opinion of a node in a phase acts as its biased opinion in the following phase. Using an extension of Friedkin-Johnsen model, we formulate the camps' utility functions, which we show to involve what can be interpreted as multiphase Katz centrality. We hence present optimal investment strategies of the camps, and the loss incurred if myopic strategy is employed. Simulations affirm that nodes attributing higher weightage to bias necessitate higher investment in initial phase. The extended version of this paper analyzes a setting where a camp's influence on a node depends on the node's bias; we show existence and polynomial time computability of Nash equilibrium.

ACM Reference Format:
Swapnil Dhamal, Walid Ben-Ameur, Tijani Chahed, and Eitan Altman. 2018. Optimal Multiphase Investment Strategies for Influencing Opinions in a Social Network. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10–15, 2018,* IFAAMAS, 3 pages.

1 INTRODUCTION

We consider two competing camps with positive and negative opinion values (referred to as good and bad camps respectively), aiming to maximize the adoption of their respective opinions in a social network. With the opinion adoption quantified as the sum of opinion values of all nodes [20, 21], the good camp aims to maximize this sum while the bad camp aims to minimize it. Since nodes update their opinions based on their neighbors' opinions [1, 15], a camp would want to influence the opinions of influential nodes by investing on them. Thus given a budget constraint, the strategy of a camp comprises of: how much to invest and on which nodes, in presence of a competing camp which would also invest strategically.

Motivation. In Friedkin-Johnsen model of opinion dynamics [16, 17], every node holds a bias in opinion. This bias plays a critical role in determining a node's final opinion, and consequently the opinions of its neighbors, and hence that of its neighbors' neighbors, and so on. If nodes give significant weightage to their biases, the camps would want to influence these biases. This could be achieved by campaigning in multiple phases, wherein a node's opinion at the conclusion of a phase would act as its biased opinion in the next phase. With the possibility of multiphase campaigning, a camp could not only decide which nodes to invest on, but also how much to invest in each phase (hence, how to split its budget across phases).

Related Work. Problems related to maximizing opinion adoption in social networks have been extensively studied in the literature [15, 22, 25]. A primary task in such problems is to determine influential nodes, which has been an important research area in the multiagent systems community [12, 19, 26, 27]. The competitive setting has resulted in several game theoretic studies [2, 4, 18]. Specific to analytically tractable models such as DeGroot and Friedkin-Johnsen, there have been studies to determine optimal investments on influential nodes [5, 14, 21]. Our work extends these studies to multiple phases by determining the influential nodes in different phases, and how much they should be invested on in a given phase.

There have been a few studies on adaptive selection of influential nodes in multiple phases [3, 8, 23, 28, 29, 31, 32]. A survey of such adaptive methodologies is presented in [30]. An empirical study on optimal budget splitting between two phases is presented in [13], which is extended to multiple phases in [9]. While the reasoning behind using multiple phases in these studies is to adaptively select nodes based on previous observations, we use them for influencing nodes' biases; this necessitates a very different treatment.

2 OUR MODEL

We represent social network as a weighted directed graph, with set of nodes N. Our model can be viewed as a multiphase extension of [10]. **Table 1** presents the notation. In our setting, the bias of node i in phase q is $v_i^{(q-1)}$, which is the opinion value of node i at the conclusion of phase $q-1$. Since the influence of good camp on node i in phase q would be an increasing function of its investment $x_i^{(q)}$ and weightage $w_{ig}^{(q)}$, we assume the influence to be $+w_{ig}^{(q)}x_i^{(q)}$ so as to maintain linearity of Friedkin-Johnsen model. Similarly, $-w_{ib}^{(q)}y_i^{(q)}$ is the influence of bad camp (negative opinion) on node i. Considering budget constraints, the camps should invest in the p phases such that $\sum_{q=1}^{p}\sum_{i\in N} x_i^{(q)} \le k_g$ and $\sum_{q=1}^{p}\sum_{i\in N} y_i^{(q)} \le k_b$.

Let $\mathbf{w}$ be the matrix consisting of weights w_{ij}. Let $\mathbf{v^{(0)}}$, $\mathbf{v^{(q)}}$, $\mathbf{w^0}$, $\mathbf{w_g}$, $\mathbf{w_b}$, $\mathbf{x^{(q)}}$, $\mathbf{y^{(q)}}$ be the vectors consisting of elements $v_i^{(0)}$, $v_i^{(q)}$, w_{ii}^0,

*The extended version of this paper is available at https://arxiv.org/pdf/1804.06081

Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.), July 10–15, 2018, Stockholm, Sweden.

Table 1: Notation

$v_i^{(0)}$	initial biased opinion of node i prior to the dynamics
$v_i^{(q)}$	opinion value of node i at the conclusion of phase q
w_{ii}^0	weightage attributed by node i to its bias in a phase
w_{ij}	weightage attributed by node i to the opinion of node j
$w_{ig}^{(q)}$	weightage attributed by node i to good camp in phase q
$w_{ib}^{(q)}$	weightage attributed by node i to bad camp in phase q
$x_i^{(q)}$	investment made by good camp on node i in phase q
$y_i^{(q)}$	investment made by bad camp on node i in phase q
k_g	budget of the good camp
k_b	budget of the bad camp

$w_{ig}, w_{ib}, x_i^{(q)}, y_i^{(q)}$, respectively. Vectors $\mathbf{x^{(q)}}, \mathbf{y^{(q)}}, \mathbf{v^{(q-1)}}$ are static throughout a phase q, while $\mathbf{v^{(q)}}$ gets updated in the dynamics. Let $\circ$ denote Hadamard product: $(\mathbf{a} \circ \mathbf{b})_i = a_i b_i$. Hence, generalizing the Friedkin-Johnsen update rule to multiphase setting and accounting for camps' investments, the update rule in phase q is:

$$\forall i \in N : \; v_i^{(q)} \leftarrow w_{ii}^0 v_i^{(q-1)} + \sum_{j \in N} w_{ij} v_j^{(q)} + w_{ig} x_i^{(q)} - w_{ib} y_i^{(q)}$$

$$\iff \mathbf{v^{(q)}} \leftarrow \mathbf{w^0} \circ \mathbf{v^{(q-1)}} + \mathbf{w}\mathbf{v^{(q)}} + \mathbf{w_g} \circ \mathbf{x^{(q)}} - \mathbf{w_b} \circ \mathbf{y^{(q)}}$$

With $\sum_{j \in N} |w_{ij}| < 1$, the dynamics in phase q converges to [11]:

$$\mathbf{v^{(q)}} = (\mathbf{I} - \mathbf{w})^{-1}(\mathbf{w^0} \circ \mathbf{v^{(q-1)}} + \mathbf{w_g} \circ \mathbf{x^{(q)}} - \mathbf{w_b} \circ \mathbf{y^{(q)}}) \quad (1)$$

3 PROBLEM FORMULATION

We first derive an expression for $\sum_{i \in N} v_i^{(p)}$, the sum of opinion values of the nodes at the end of terminal phase p. Let $(\mathbf{I}-\mathbf{w})^{-1} = \Delta$. Let $r_i^{(1)} = \sum_{j \in N} \Delta_{ji}$ and $r_i^{(t)} = \sum_{j \in N} r_j^{(t-1)} w_{jj}^0 \Delta_{ji}$. That is, $\mathbf{r^{(1)}} = \Delta^T \mathbf{1}$ and $\mathbf{r^{(t)}} = \Delta^T (\mathbf{r^{(t-1)}} \circ \mathbf{w^0})$. It can be shown that, premultiplying Equation (1) by $\mathbf{1}^T$ for $q = p$, and solving the recursion, we get:

$$\sum_{i \in N} v_i^{(p)} = \sum_{i \in N} r_i^{(p)} w_{ii}^0 v_i^{(0)} + \sum_{q=1}^{p} \sum_{i \in N} r_i^{(p-q+1)} (w_{ig} x_i^{(q)} - w_{ib} y_i^{(q)}) \quad (2)$$

Multiphase Katz Centrality. $r_i^{(1)} = \left((\mathbf{I} - \mathbf{w}^T)^{-1}\mathbf{1}\right)_i$ resembles Katz centrality of node i [24], capturing its influencing power over other nodes in a single phase setting (corresponds to terminal phase in multiphase setting). However, the effectiveness of node i with t phases to go ($r_j^{(t)}$), depends on its influencing power over those nodes j (Δ_{ji}), which would give good weightage to their bias in the next phase (w_{jj}^0), and also have good effectiveness in the next phase with $t-1$ phases to go ($r_j^{(t-1)}$). This is captured by $r_i^{(t)} = \sum_{j \in N} r_j^{(t-1)} w_{jj}^0 \Delta_{ji}$. Since $r_i^{(t)}$ quantifies i's influence looking t phases ahead, it can be interpreted as the t-phase Katz centrality.

The Problem. Here $(\mathbf{x^{(q)}})_{q=1}^p$ and $(\mathbf{y^{(q)}})_{q=1}^p$ are the respective strategies of the good and bad camps. Given an investment strategy profile $\left((\mathbf{x^{(q)}})_{q=1}^p, (\mathbf{y^{(q)}})_{q=1}^p\right)$, let $u_g\left(\left((\mathbf{x^{(q)}})_{q=1}^p, (\mathbf{y^{(q)}})_{q=1}^p\right)\right)$ be the utility of good camp and $u_b\left(\left((\mathbf{x^{(q)}})_{q=1}^p, (\mathbf{y^{(q)}})_{q=1}^p\right)\right)$ be the utility of bad camp. The good camp aims to maximize (2), while the bad camp simultaneously aims to minimize it. Hence the problem is:

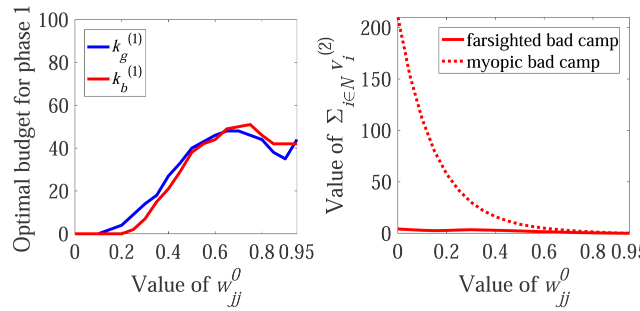
Find Nash equilibrium, given that

$$u_g\left(\left((\mathbf{x^{(q)}})_{q=1}^p, (\mathbf{y^{(q)}})_{q=1}^p\right)\right) = \sum_{i \in N} v_i^{(p)} \;,\; u_b\left(\left((\mathbf{x^{(q)}})_{q=1}^p, (\mathbf{y^{(q)}})_{q=1}^p\right)\right) = -\sum_{i \in N} v_i^{(p)}$$

subject to

$$\sum_{q=1}^{p} \sum_{i \in N} x_i^{(q)} \le k_g \,,\, \sum_{q=1}^{p} \sum_{i \in N} y_i^{(q)} \le k_b \,,\, \forall q \in \{1, \ldots, p\} \; \forall i \in N : x_i^{(q)}, y_i^{(q)} \ge 0$$

Optimal Investment Strategies. Since the optimization terms with respect to different variables are decoupled in Equation (2), the optimal strategies of camps are mutually independent. For the good camp, we order the terms $\{w_{ig} r_i^{(p-q+1)}\}_{i \in N, q=1,\ldots,p}$ in descending order. If the investment allowed per node is unbounded, its optimal strategy is to invest k_g on node i^* in phase q^*, where $(i^*, q^*) = \arg\max_{(i,p)} w_{ig} r_i^{(p-q+1)}$ (no investment if this value is non-positive). If the investment per node is bounded by $\mathcal{U}$, the (i, p) pairs are chosen one-by-one according to the aforementioned descending ordering, and invested on with $\mathcal{U}$ each, until budget k_g is exhausted. The optimal strategy of the bad camp is analogous.



(a) Optimal budget splits (b) Myopic vs farsighted

Figure 1: Results illustrating the effects of w_{jj}^0 (NetHEPT)

Simulation Results. For 2 phases on NetHEPT dataset (15,233 nodes) [6, 7, 25], Figure 1(a) presents optimal budget allotted for phase 1 as a function of w_{jj}^0 (assuming equal $w_{jj}^0, \forall j \in N$) with $k_g = k_b = 100$ ($\mathcal{U} = 1$ and $v_i^{(0)} = 0, \forall i \in N$). Detailed simulation setup is provided in [11]. For low w_{jj}^0, the optimal strategy of camps is to invest almost entirely in phase 2, since the effect of phase 1 would diminish considerably in phase 2. The value $r_i^{(2)} = \sum_{j \in N} r_j^{(1)} w_{jj}^0 \Delta_{ji}$ would be significant only if i influences nodes j with significant values of w_{jj}^0. So investing in phase 1 would be advantageous only if nodes have significant w_{jj}^0. The slight non-monotonicity of plots is explained in [11]. General observations indicate that a high range of w_{jj}^0 makes it advantageous for camps to invest in phase 1, so as to effectively influence the biases in phase 2. The reasoning generalizes to more than 2 phases. Figure 1(b) illustrates the loss incurred by bad camp when it is myopic (perceiving its utility as $-\sum_{i \in N} v_i^{(1)}$ instead of $-\sum_{i \in N} v_i^{(2)}$), while the good camp is farsighted (considering no bound on investment per node). A myopic bad camp would invest its entire budget in phase 1, and with the same reasoning as above, it would incur more loss for lower values of w_{jj}^0 (ref. [11] for details).

EXTENDED VERSION [11] analyzes a setting where a node attributes higher weightage to the camp more aligned with its bias. The camps' strategies are no longer mutually independent; we show existence and polynomial time computability of Nash equilibrium.

REFERENCES

[1] Daron Acemoglu and Asuman Ozdaglar. 2011. Opinion dynamics and learning in social networks. *Dynamic Games and Applications* 1, 1 (2011), 3–49.

[2] Aris Anagnostopoulos, Diodato Ferraioli, and Stefano Leonardi. 2015. Competitive influence in social networks: Convergence, submodularity, and competition effects. In *14th International Conference on Autonomous Agents & Multiagent Systems*. IFAAMAS, 1767–1768.

[3] Ashwinkumar Badanidiyuru, Christos Papadimitriou, Aviad Rubinstein, Lior Seeman, and Yaron Singer. 2016. Locally adaptive optimization: Adaptive seeding for monotone submodular functions. In *27th Annual Symposium on Discrete Algorithms*. SIAM, 414–429.

[4] Shishir Bharathi, David Kempe, and Mahyar Salek. 2007. Competitive influence maximization in social networks. In *3rd International Workshop on Web and Internet Economics*. Springer, 306–311.

[5] Kostas Bimpikis, Asuman Ozdaglar, and Ercan Yildiz. 2016. Competitive targeted advertising over networks. *Operations Research* 64, 3 (2016), 705–720.

[6] Wei Chen, Chi Wang, and Yajun Wang. 2010. Scalable influence maximization for prevalent viral marketing in large-scale social networks. In *16th International Conference on Knowledge Discovery and Data Mining*. ACM, 1029–1038.

[7] Wei Chen, Yajun Wang, and Siyu Yang. 2009. Efficient influence maximization in social networks. In *15th International Conference on Knowledge Discovery and Data Mining*. ACM, 199–208.

[8] José Correa, Marcos Kiwi, Neil Olver, and Alberto Vera. 2015. Adaptive rumor spreading. In *11th International Conference on Web and Internet Economics*. Springer, 272–285.

[9] Swapnil Dhamal. 2018. Effectiveness of diffusing information through a social network in multiple phases. *Arxiv Preprint:1802.08869* (2018).

[10] Swapnil Dhamal, Walid Ben-Ameur, Tijani Chahed, and Eitan Altman. 2017. Optimal investment strategies for competing camps in a social network: A broad framework. *Arxiv Preprint:1706.09297* (2017).

[11] Swapnil Dhamal, Walid Ben-Ameur, Tijani Chahed, and Eitan Altman. 2018. Optimal multiphase investment strategies for influencing opinions in a social network. *Arxiv Preprint:1804.06081* (2018).

[12] Swapnil Dhamal, Prabuchandran K J, and Y Narahari. 2015. A multi-phase approach for improving information diffusion in social networks. In *14th International Conference on Autonomous Agents & Multiagent Systems*. IFAAMAS, 1787–1788.

[13] Swapnil Dhamal, Prabuchandran K J, and Y Narahari. 2016. Information diffusion in social networks in two phases. *IEEE Transactions on Network Science and Engineering* 3, 4 (2016), 197–210.

[14] Pradeep Dubey, Rahul Garg, and Bernard De Meyer. 2006. Competing for customers in a social network: The quasi-linear case. In *2nd International Workshop on Web and Internet Economics*. Springer, 162–173.

[15] David Easley and Jon Kleinberg. 2010. *Networks, crowds, and markets: Reasoning about a highly connected world*. Cambridge University Press.

[16] Noah Friedkin and Eugene Johnsen. 1990. Social influence and opinions. *Journal of Mathematical Sociology* 15, 3-4 (1990), 193–206.

[17] Noah Friedkin and Eugene Johnsen. 1997. Social positions in influence networks. *Social Networks* 19, 3 (1997), 209–222.

[18] Javad Ghaderi and R Srikant. 2014. Opinion dynamics in social networks with stubborn agents: Equilibrium and convergence rate. *Automatica* 50, 12 (2014), 3209–3215.

[19] Amer Ghanem, Srinivasa Vedanarayanan, and Ali Minai. 2012. Agents of influence in social networks. In *11th International Conference on Autonomous Agents & Multiagent Systems-Volume 1*. IFAAMAS, 551–558.

[20] Aristides Gionis, Evimaria Terzi, and Panayiotis Tsaparas. 2013. Opinion maximization in social networks. In *2013 International Conference on Data Mining*. SIAM, 387–395.

[21] Michel Grabisch, Antoine Mandel, Agnieszka Rusinowska, and Emily Tanimura. 2018. Strategic influence in social networks. *Mathematics of Operations Research* 43, 1 (2018), 29–50.

[22] Adrien Guille, Hakim Hacid, Cécile Favre, and Djamel Zighed. 2013. Information diffusion in online social networks: A survey. *ACM SIGMOD Record* 42, 1 (2013), 17–28.

[23] Thibaut Horel and Yaron Singer. 2015. Scalable methods for adaptively seeding a social network. In *24th International Conference on World Wide Web*. ACM, 441–451.

[24] Leo Katz. 1953. A new status index derived from sociometric analysis. *Psychometrika* 18, 1 (1953), 39–43.

[25] David Kempe, Jon Kleinberg, and Éva Tardos. 2003. Maximizing the spread of influence through a social network. In *9th International Conference on Knowledge Discovery and Data Mining*. ACM, 137–146.

[26] Weihua Li, Quan Bai, Tung Doan Nguyen, and Minjie Zhang. 2017. Agent-based influence maintenance in social networks. In *16th International Conference on Autonomous Agents & Multiagent Systems*. IFAAMAS, 1592–1594.

[27] Ramakumar Pasumarthi, Ramasuri Narayanam, and Balaraman Ravindran. 2015. Near optimal strategies for targeted marketing in social networks. In *14th International Conference on Autonomous Agents & Multiagent Systems*. IFAAMAS, 1679–1680.

[28] Aviad Rubinstein, Lior Seeman, and Yaron Singer. 2015. Approximability of adaptive seeding under knapsack constraints. In *16th Conference on Economics and Computation*. ACM, 797–814.

[29] Lior Seeman and Yaron Singer. 2013. Adaptive seeding in social networks. In *54th Annual Symposium on Foundations of Computer Science*. IEEE, 459–468.

[30] Yaron Singer. 2016. Influence maximization through adaptive seeding. *ACM SIGecom Exchanges* 15, 1 (2016), 32–59.

[31] Guangmo Tong, Weili Wu, Shaojie Tang, and Ding-Zhu Du. 2016. Adaptive influence maximization in dynamic social networks. *IEEE/ACM Transactions on Networking* 25, 1 (2016), 112–125.

[32] Jing Yuan and Shaojie Tang. 2017. No time to observe: Adaptive influence maximization with partial feedback. In *26th International Joint Conference on Artificial Intelligence*. 3908–3914.