The Curse of Ties in Congestion Games with Limited Lookahead

Extended Abstract

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ABSTRACT

We introduce a novel framework to model limited lookahead in congestion games. Intuitively, the players enter the game sequentially and choose an optimal action under the assumption that the $k-1$ subsequent players play subgame-perfectly. Our model naturally interpolates between outcomes of greedy best-response ($k = 1$) and subgame-perfect outcomes ($k = n$, the number of players). We study the impact of limited lookahead (parameterized by k) on the stability and inefficiency of the resulting outcomes. As our results reveal, increased lookahead does not necessarily lead to better outcomes; in fact, its effect crucially depends on the existence of ties and the type of game under consideration.

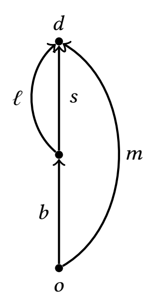
KEYWORDS

Congestion games; Limited backward induction; Subgame-perfect equilibrium; Greedy best-response

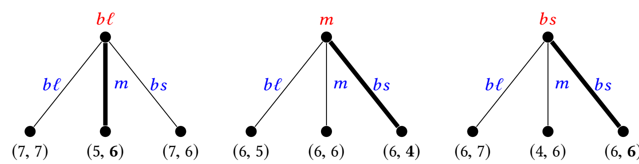
ACM Reference Format:
Carla Groenland and Guido Schäfer. 2018. The Curse of Ties in Congestion Games with Limited Lookahead. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10–15, 2018,* IFAAMAS, 3 pages.

1 INTRODUCTION

Consider the following situation, where two players want to travel from origin o to destination d in the (extension-parallel) graph on the right. They can take the metro m, which takes 6 minutes, or they can take the bike b and then walk either the long (but scenic) route ℓ, which takes 2 minutes, or the short route s, which takes 1 minute. There is only one bike: if only one of them takes the bike it takes 3 minutes; otherwise, someone has to sit on the backseat and it takes them 5 minutes. Both players want to minimize their own travel time.



Suppose they announce their decisions sequentially. There are two possible orders: either the red player 1 moves first or the blue player 2 moves first. We consider the *sequential-move version* of the game where player 1 moves first. There are three possible *subgames* that player 2 may end up in, for which the corresponding game trees are depicted below. A *strategy* for player 2 is a function $S_2 : \{b\ell, bs, m\} \to \{b\ell, bs, m\}$ that tells us which action player 2 plays given the action of player 1. Player 2 will always choose an action that minimizes his travel time and may break ties arbitrarily when being indifferent. The boldface arcs give a possible subgame-perfect strategy for player 2.



If player 2 fixes this strategy, then player 1 is strictly better off taking the bike and walking the long route (for a travel time of 5 compared to a travel time of 6 for the other cases). That is, the only subgame-perfect response for player 1 is $b\ell$. This shows $(b\ell, m)$ is a *subgame-perfect outcome*. However, this outcome is rather peculiar: Why would player 1 walk the long route if his goal is to arrive as quickly as possible? In fact, the outcome $(b\ell, m)$ is not *stable*, i.e., it does not correspond to a Nash equilibrium.

Subgame-perfect outcomes are introduced as a natural model for farsightedness [14, 15], or "full anticipation", and have been studied for various types of congestion games [2–4, 14]. Another well-studied notion in this context are outcomes of *greedy best-response* [6, 7, 11, 17], i.e., players enter the game one after another and give a best response to the actions played already, thus playing with "no anticipation". In the above example both (bs, m) and (bs, bs) are greedy best-response outcomes and they are stable. The example thus illustrates that full lookahead may have a negative effect on the stability of the outcomes. After a moment's thought, we realize that in the subgame-perfect outcome the indifference of player 2 is exploited (by breaking ties accordingly) to force player 1 to play a suboptimal action. Immediate questions that arise are: Does full lookahead guarantee stable outcomes if we adjust the travel times such that the players are no longer indifferent (i.e., if we make the game *generic*)? What is the lookahead that is required to guarantee stable outcomes? What about the inefficiency of these outcomes?

2 LOOKAHEAD OUTCOMES

We introduce k-lookahead outcomes as a model for situations which arise if players enter the game sequentially and anticipate the next k players. The cases $k = 1$ and $k = n$ correspond to the greedy best-response outcomes and subgame-perfect outcomes mentioned above, respectively. Intermediate lookahead ($1 < k < n$) might be useful when the availability of computational resources or information is limited.

We study the efficiency and stability of k-lookahead outcomes, where we call an outcome stable if it is a Nash equilibrium (NE). In order to assess the inefficiency of k-lookahead outcomes, we introduce the *k-Lookahead Price of Anarchy (k-LPoA)* which generalizes

*This work was done while the first author was a research intern at CWI and a student at the University of Amsterdam (UvA).

Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.), July 10–15, 2018, Stockholm, Sweden. © 2018 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). All rights reserved.

both the standard Price of Anarchy (PoA) [9] and the Sequential Price of Anarchy [14]. Paes Leme et al. [14] show that the Sequential Price of Anarchy can be much lower than the Price of Anarchy if the game is generic. On the other hand, this does not necessarily hold if the game is non-generic (see, e.g., [2, 3, 12]).

The idea of limited backward induction dates back to the 1950s [16] and has been studied in game-theoretic settings before (e.g. [1, 10, 13]). For example, Mirrokni et al. [13] introduce *k-lookahead equilibria* that incorporate various levels of anticipation as well. However, their 1-lookahead equilibria correspond to Nash equilibria rather than greedy best-response outcomes and none of the equilibria correspond to subgame-perfect outcomes.

We will require the following definitions. A *congestion game* is a tuple $G = (\mathcal{N}, R, (\mathcal{A}_i)_{i\in\mathcal{N}}, (d_r)_{r\in R})$ where $\mathcal{N} = \{1, \ldots, n\}$ is a finite set of players, R a finite set of *resources*, $\mathcal{A}_i \subseteq 2^R$ the *action set* of player i, and $d_r : \mathbb{N} \to \mathbb{R}_{\geq 0}$ a *delay function* ($r \in R$). Unless stated otherwise, we assume that d_r is non-decreasing. The cost function c_i of player $i \in \mathcal{N}$ is given by $c_i(A) = \sum_{r\in A_i} d_r(x(A)_r)$ where $x(A)_r = |\{i \in \mathcal{N} : r \in A_i\}|$. A congestion game is *symmetric* if $\mathcal{A}_i = \mathcal{A}_j = \mathcal{A}$ for all $i, j \in \mathcal{N}$. A congestion game G is *generic* if for all $N \subseteq \mathcal{N}$, $A, B \in \prod_{i\in N} \mathcal{A}_i$ and $j \in N$, $A_j \neq B_j$ implies $c_j(A) \neq c_j(B)$. An *order* on the players is a bijection $\sigma : \mathcal{N} \to [n]$. We denote the sequential-move version of G with respect to order σ by G^σ. The outcome on the equilibrium path of a subgame-perfect equilibrium in G^σ is an action profile of G and we refer to it as the *subgame-perfect outcome (SPO)*.

Our definition of k-lookahead outcome is given below; a more general definition can be found in the full version of the paper [8].

Definition 2.1. Let G be an n-player congestion game and let $k \in [n]$. Let G^k denote the same game with player set $\{1, \ldots, k\}$. An action profile A is a **k-lookahead outcome** of G if A_i equals the action B_i played by player i in some subgame-perfect outcome B of G'^k, where G' is the subgame of G induced by $(A_j)_{j<i}$.[1]

3 SYMMETRIC NETWORK CONGESTION GAMES

In a *symmetric network congestion game (SNCG)*, the common set of actions $\mathcal{A}$ is given by the set of all directed paths in a single-commodity network. A *series-parallel graph (SP-graph)* either consists of (i) a single arc, or (ii) two series-parallel graphs in parallel or series. An *extension-parallel graph (EP-graph)* either consists of (i) a single arc, (ii) two extension-parallel graphs in parallel, or (iii) a single arc in series with an extension-parallel graph. Fotakis et al. [6] show that each 1-lookahead outcome is a Nash equilibrium for SNCG on SP-graphs. We prove that the converse also holds for EP-graphs.

Theorem 3.1. *For every SNCG on an EP-graph, the set of 1-lookahead outcomes coincides with the set of Nash equilibria.*

3.1 Stability and inefficiency of generic games

As shown in the introduction, SPOs are not guaranteed to be stable for SNCGs on EP-graphs. However, stability is guaranteed if the game is generic.

[1] $G' = (\{i, \ldots, n\}, R, (\mathcal{A}_j)_{j\geq i}, (d'_r)_{r\in R})$ where $d'_r(y) = d_r(y + x((A_j)_{j<i})_r)$.

Theorem 3.2. *Let G be a generic SNCG on an EP-graph. Then for every k the set of k-lookahead outcomes coincides with the set of Nash equilibria. As a consequence, k-LPoA(G) = PoA(G).*

The result of Theorem 3.2 does not extend to series-parallel graphs.

Proposition 3.3. *For any SP-graph Γ that is not EP, there is a generic SNCG G on Γ such that the sets of 1-lookahead and n-lookahead outcomes are disjoint.*

Anticipation may still be beneficial for the first player.

Theorem 3.4. *Let G be a generic SNCG on an EP-graph. Let B be a subgame-perfect outcome with respect to the identity. Then $c_1(B) \leq \cdots \leq c_n(B)$. In particular, $c_1(B) \leq c_1(A)$ for any k-lookahead outcome A with respect to the identity.*

3.2 Inefficiency of non-generic games

By proving that each 1-lookahead outcome is a global optimum of Rosenthal's potential function, combined with a result of Fotakis [5, Lemma 3], we derive the following result.

Corollary 3.5. *For SNCGs on SP-graphs 1-LPoA $\leq$ PoS.*

This result only guarantees 1-lookahead outcomes to be optimal Nash equilibria for games with the worst Price of Stability (PoS); a procedure for finding an optimal Nash equilibrium for every SNCG on a SP-graph is an NP-hard problem [17].

Our result below shows it is no coincidence that the instable SPO of the introduction still had an optimal egalitarian social cost.

Theorem 3.6. *For every SNCG G on an EP-graph, each SPO A has optimal egalitarian social cost.*

4 EXTENSIONS

As for the introductory example, it can be shown that the curse of ties also applies to cost-sharing games and consensus games.

A *cost-sharing game* is a congestion game, where the delay functions are non-increasing. We show that there exist symmetric singleton congestion games with unstable subgame-perfect outcomes. On the other hand, the instability can be resolved for either symmetric or singleton cost-sharing games if no ties exist. We moreover identify a class of cost-sharing games for which the k-Lookahead Price of Anarchy increases monotonically and non-trivially with k.

In a *consensus game*, each player is a vertex in a weighted graph $\Gamma = (V, E, w)$ and can choose between actions L and R. The cost of player i in outcome A is given by the sum of the weights w_{ij} of all incident edges $ij \in E$ for which $A_i \neq A_j$. We show that subgame-perfect outcomes of consensus games can be unstable, but that all k-lookahead outcomes are optimal (in particular stable) if all players adopt a common tie-breaking rule.

While the focus in our paper is on congestion games, our notion of k-lookahead outcomes naturally extends to arbitrary normal-form games. It will be interesting to study k-lookahead outcomes for other classes of games. In particular, it would be interesting to further explore the relation between ties and anticipation within this framework.

Acknowledgements. We thank Pieter Kleer for helpful discussions. The full version of this paper is available on arXiv. [8].

REFERENCES

[1] V. Bilò, A. Fanelli, and L. Moscardelli. 2017. On lookahead equilibria in congestion games. *Mathematical Structures in Computer Science* 27, 2 (2017), 197–214.

[2] V. Bilò, M. Flammini, G. Monaco, and L. Moscardelli. 2015. Some Anomalies of Farsighted Strategic Behavior. *Theory of Computing Systems* 56, 1 (2015), 156–180.

[3] J. Correa, J. de Jong, B. Keijzer, and M. Uetz. 2015. The Curse of Sequentiality in Routing Games. In *Proceedings of the 11th International Conference on Web and Internet Economics (WINE)*. Springer, Berlin, Heidelberg, 258–271.

[4] J. de Jong. 2016. *Quality of equilibria in resource allocation games.* Ph.D. Dissertation. University of Twente.

[5] D. Fotakis. 2008. Congestion Games with Linearly Independent Paths: Convergence Time and Price of Anarchy. *Theory of Computing Systems* 47, 1 (2008), 113–136.

[6] D. Fotakis, S. Kontogiannis, and P. Spirakis. 2006. *Symmetry in Network Congestion Games: Pure Equilibria and Anarchy Cost.* Springer, Berlin, Heidelberg, 161–175.

[7] E. Gassner, J. Hatzl, S. O. Krumke, H. Sperber, and G. J. Woeginger. 2009. How hard is it to find extreme Nash equilibria in network congestion games? *Theoretical Computer Science* 410, 47 (2009), 4989–4999.

[8] C. Groenland and G. Schäfer. 2018. The Curse of Ties in Congestion Games with Limited Lookahead. (2018). http://arxiv.org/abs/1804.07107.

[9] E. Koutsoupias and C. Papadimitriou. 2009. Worst-case Equilibria. *Computer Science Review* 3, 2 (2009), 65–69.

[10] C. Kroer and T. Sandholm. 2015. Limited Lookahead in Imperfect-information Games. In *Proceedings of the 24th International Conference on Artificial Intelligence (IJCAI)*. AAAI Press, Buenos Aires, Argentina, 575–581.

[11] S. Kuniavsky and R. Smorodinsky. 2013. Greediness and equilibrium in congestion games. *Economics Letters* 121, 3 (2013), 499–503.

[12] I. Milchtaich. 1998. Crowding games are sequentially solvable. *International Journal of Game Theory* 27 (1998), 501–509.

[13] V. S. Mirrokni, N. Thain, and A. Vetta. 2012. A Theoretical Examination of Practical Game Playing: Lookahead Search. In *Proceedings of the Fifth International Symposium on Algorithmic Game Theory (SAGT)*. Springer-Verlag, Berlin, Heidelberg, 251–262.

[14] R. Paes Leme, V. Syrgkanis, and E. Tardos. 2012. The Curse of Simultaneity. In *Proceedings of the 3rd Innovations in Theoretical Computer Science Conference (ITCS '12)*. ACM, New York, NY, USA, 60–67.

[15] R. Selten. 1965. Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragetraegheit. *Zeitschrift fur die gesamte Staatswissenschaft* 12 (1965), 301–324.

[16] C. E. Shannon. 1950. Programming a Computer for Playing Chess. *Philosopical Magazine* 41, 314 (1950), 256–275.

[17] H. Sperber. 2010. How to find Nash equilibria with extreme total latency in network congestion games? *Mathematical Methods of Operations Research* 71, 2 (2010), 245–265.