Incorporating Chorus Line Effect into A Cucker-Smale System for Fast Manoeuvre Tracking

Extended Abstract

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1 INTRODUCTION

The two most popular flocking models for self-propelled agents are the Vicsek model [14] and the Cucker-Smale model [3]. They are fundamental models of alignment of the velocities of individual agents. The velocity of each agent is typically adjusted according to the average velocity of its topological or metric neighbourhood. For a flock to be stable, if a single agent abruptly changes its velocity, the interaction rule will keep the remainder of the flock to largely maintain its current velocity due to the law of averaging. However, analysis of observations of dunlin flocks by Potts [13] indicated that a sudden change in flight path could be initiated by one or a few birds, with the rest of the birds following in a coordinated movement. This change initially propagates through the flock slowly but subsequently accelerates to a high speed. Potts hypothesis is that the birds perceive the motion of the oncoming "manoeuvre wave" and time their own turn to match it. This has become known as the "chorus-line hypothesis". This ability for flocks to react quickly to sudden manoeuvres is important. It allows them to respond to the movements of a potential predator [10].

There have been some attempts in modelling this kind of phenomenon. Recent experimental data collected using high-speed camera have validated what these models have shown us by computer simulation [1]. Using a computational model called StarDisplay, the underlying wave speed for starling flocks could be studied [7, 8]. These studies concluded that only short range interactions are needed to generate such underlying waves. In [2], the propagation of density waves was derived with a pseudo-Hamiltonian based on the Vicsek model. While these models are useful for studying the propagating wave of movements, they cannot be translated into mathematical rules that the agents use to adjust their velocity, similar to those of Vicsek and Cucker-Smale models.

In this paper, we propose a way to incorporate the chorus-line effect into a standard Cucker-Smale model. Through computer simulations, we analyse the time it takes for the flock to realign to a sudden change in direction by one of the agents in the flock. Furthermore, we apply finite-time control to our proposed model to determine if additional gain could be obtained.

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2 CUCKER-SMALE MODEL WITH CHORUS-LINE EFFECT

Consider a group of five agents which are moving in a straight line with a velocity of $\bar{v}$ as shown in Figure 1. The agents are numbered from 1 to 5 from left to right. In this state, the agents are flocking. They continue to observe the movement of their neighbours within a certain distance, where the relative velocity is monitored. Now

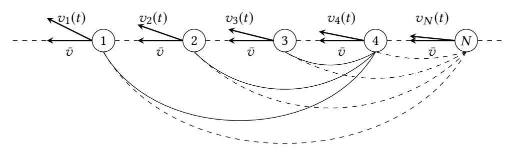

Figure 1: Realignment with chorus-line effect

assume that agent 1 abruptly changes its velocity to $v_1(t)$ and maintains this new velocity. The chorus line effect dictates that agent 2 changes its velocity upon observing the change in agent 1. To be able to change the velocity to be the same as agent 1 in a time τ, an acceleration of $(v_2 - v_1)/\tau$ would be needed. τ is known as the relaxation time which is the time required for an agent to return to the realignment velocity. In [6, 9], a social steering force is used to cause agents to slow down (e.g. to avoid collision) or to speed up (e.g. to catch up). This steering force is given by

$$f_{speed_i} = \frac{1}{\tau}(v_0 - v_i)e_{x_i} \tag{1}$$

where the τ represents the relaxation time, v_0 is cruise speed and v_i is the velocity of agent i, and $e_{x\,i}$ indicates its forward direction. The velocity of agent 3 will also change, not only because of agent 2 but also because it observed the change in agent 1, assuming it is within its monitoring distance. Thus its acceleration will be a sum of that caused by agent 1 as well as agent 2. This means that the acceleration of agent 3 will be larger than that of agent 2. Similarly, agent 4 accelerates due to the changes in velocities in agents 1, 2 and 3. In order to describe the relationship and movement of these agents, the agents are drawn in a line in Figure 1. In reality, they do not need to be in a line when they are flocking.

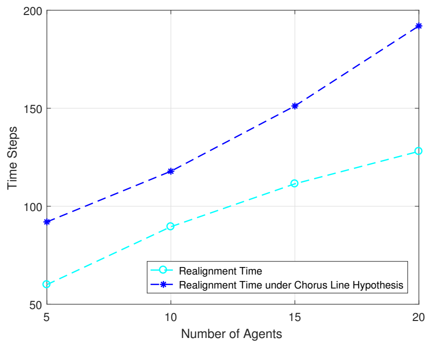

Figure 2: Realignment time for different number of agents

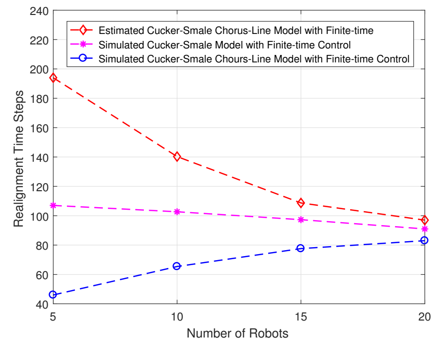

Figure 3: Computed and simulated realignment times

The standard Cucker-Smale alignment model is given by

$$\begin{cases} \dot{p_i} = & v_i \\ \dot{v_i} = & \frac{1}{N}\sum_{j=1}^{N} \psi(\|p_j - p_i\|)(v_j - v_i) \end{cases} \tag{2}$$

where p_i, v_i represent the position and velocity of agent i respectively. ψ is known as the communication rate function which is a positive decreasing function of the Euclidean distance between two agents.

The chorus-line effect can be incorporated into (2) by adding an additional term to $\dot{v_i}$, resulting in

$$\dot{v_i} = \frac{1}{N}\sum_{j=1}^{N} \psi(\|p_j - p_i\|)(v_j - v_i) + \sum_{j=1}^{N} \frac{1}{\tau_j}(v_j - v_i) \tag{3}$$

We shall now compare the time it takes for a flock to realign after an abrupt change in direction for the standard Cucker-Smale model and our proposed model by computer simulation. In these simulations, a group of agent is moving in one direction in a 2-dimensional space at the same speed v, the constant speed is 0.3. When one agent abruptly changes its heading by an angle of θ and maintain its movement in this new direction. The time it takes the rest of the flock to be realigned in this new direction is referred to as the *realignment time*. The relaxation time is assumed to be the same for all agents since the flock is assumed to be homogeneous and is set to 0.05. The system is considered to be in a flocking state when the average velocity $v_a \geq 0.99$ [12].

Simulation results for $\theta = \pi/3$ is shown in Figure 2 for flock sizes N of 5, 10, 15 and 20. These show that the realignment time for the our new proposed system is reduced comparing with standard Cucker-Smale model. Further simulations show that the realignment time increases significantly with the absolute amount of heading change. Results also seem to indicate that for larger flocks, the realignment time is less dependent on the relaxation time.

3 APPLYING FINITE-TIME CONTROL

Previous researches showed that flocking time can be reduced if finite-time control is used for the agents in a Cucker-Smale system [5, 11, 15]. To help us understand how efficient the Cucker-Smale system with chorus line effect is in terms of achieving realignment, finite-time control is applied to it to see if a shorter realignment time could be achieved. The finite-time controlled new model is given by replacing all instances of $(v_j - v_i)$ by $\mathrm{sgn}(v_j - v_i)^\gamma$ where $\mathrm{sgn}(x)^\gamma = \mathrm{sgn}(x)|x|^\gamma$ with the finite-time control parameter $0 < \gamma < 1$ [4]. Since we have two alignment terms in Cucker-Smale model with chorus line, we allow for two different control parameters θ and α.

In [5, 11, 15], an upper bound of the realignment time for the finite-time controlled Cucker-Smale system has been derived. A similar bound for the finite-time Cucker-Smale model with chorus line is established as the theorem in [11].

THEOREM 3.1. *The velocities of the autonomous agents in finite-time controlled Cucker-Smale system with chorus line converges to the same velocity in a finite amount of time with an upper bound T_f given by $T_f \leq \max\{T_1, T_2\}$. where*

$$\begin{aligned} T_1 &= C_1 N^{-\frac{\theta+1}{2}}, C_1 = \frac{2V(0)^{\frac{1-\theta}{2}}}{\psi^* \sqrt{2}^{\theta+1}(1-\theta)} \\ T_2 &= C_2 N^{-\frac{1+\alpha}{2}}, C_2 = \frac{2V(0)^{\frac{1-\alpha}{2}}}{M\sqrt{2}(1-\alpha)} \\ V(0) &= \sum_{i=1}^{N} \|v_i(0)\|^2 \end{aligned} \tag{4}$$

We shall now compare the realignment time given by Theorem 3.1 with that obtained by computer simulation. The parameters used in these simulations are $\theta = 1/2, \alpha = 1/2, \psi^* = 1$, and M depends on the relaxation time from $\sum_{i=1}^{N} \frac{1}{\tau_j} \geq M$. The remaining parameters are the same as those in the previous Section.

Computed and simulated results for flock sizes of 5, 10, 15 and 20 agents are shown in Figure 3. The upper curve shows the realignment time computed according to 3.1. In order to make comparisons with the simulated results, the time is converted to the equivalent number of time steps used in the simulations. The rest of the curves show the realignment times with the standard finite-time Cucker-Smale model, and finite-time to our proposed model. The graph shows that the upper bound of the realignment time is correct. It also shows that applying finite-time control to the Cucker-Smale model with chorus-line effect produces the best results. But the advantage provided by chorus-line effect is relatively small.

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