Coordination of Electric Vehicle Aggregators: A Coalitional Approach

Alvaro Perez-Diaz
University of Southampton
a.perez-diaz@soton.ac.uk

Enrico Gerding
University of Southampton
eg@ecs.soton.ac.uk

Frank McGroarty
University of Southampton
f.j.mcgroarty@soton.ac.uk

ABSTRACT

Given the rapid rise of electric vehicles (EVs) worldwide, and the ambitious targets set for the near future, the smart charging of an EV fleet must be seen as a priority. Specifically, we study a scenario where EV charging is managed through self-interested EV aggregators (*e.g.* car parks or electricity suppliers) who compete in the day-ahead market in order to purchase the electricity needed to meet their clients' requirements. In order to reduce electricity costs and lower the impact on electricity markets, we study the possibility of inter-aggregator cooperation. Specifically, we model the system as a coalitional game and prove that the resulting game is superadditive and balanced, hence having a non-empty core. However, due to the game not being convex, the Shapley value is not guaranteed to lie in the core. As an alternative, we propose employing the payment mechanism provided by the least-core, which we show to be in the core in our setting. Furthermore, a realistic empirical evaluation is presented, using real market and driver data from the Iberian Peninsula. The simulations show that large payment reductions can be achieved when using the coordination mechanism. Moreover, we show that the individual payments of the least-core are very close to the Shapley value, suggesting that the payment mechanism is both fair and stable.

KEYWORDS

electric vehicles; cooperative game theory; Shapley value; least-core

ACM Reference Format:

Alvaro Perez-Diaz, Enrico Gerding, and Frank McGroarty. 2018. Coordination of Electric Vehicle Aggregators: A Coalitional Approach. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), Stockholm, Sweden, July 10–15, 2018,* IFAAMAS, 9 pages.

1 INTRODUCTION

To date, there exists a world-wide fleet of more than two million electric vehicles (EVs), combining purely electrical and hybrid [15]. Furthermore, EV sales are growing exponentially in most countries and there are targets to achieve 50 to 200 million of EVs at a global scale in the next decade [14]. These high penetration targets aim to reduce the use of fossil fuels and improve environmental conditions. However, the transition from conventional to electric vehicles is not without challenges [24]. Specifically, compared to traditional fuel powered vehicles, EVs present a novel and heavy strain to existing electricity networks, which will need to accommodate a new type of consumer with high demand. Careless managing of a fleet of EVs can cause great demand peaks and network congestion, which can compromise the good functioning of the electricity grid and require the use of expensive and polluting generation methods. On the positive side, in contrast with conventional electricity consumption such as heating or lighting, EVs offer a high degree of flexibility: on average, a given EV is idle up to 95% of the time [34].

In order to deal with these challenges and to exploit the flexibility inherent to EVs, the last decade has seen the introduction of the concept of the EV aggregator [16]: an intermediary between a fleet of EVs and the electricity grid and markets. Potential examples of EV aggregators are EV charging enabled car parks, micro-grids, *etc.* The aggregator is able to control the charging (and potentially discharging) of its fleet, and this way informed collective decisions can be made. In contrast with individual EV operation, the much higher degree of coordination possible when a fleet is centrally managed by an aggregator offers great benefits. For example, electricity consumption to charge the fleet's batteries can be spread over time, avoiding expensive and polluting demand peaks. In particular, in this work we focus on EV aggregators participating in day-ahead markets, in order to purchase the electricity needed to meet their clients' energy requirements. In more detail, day-ahead markets match electricity supply and demand on an hourly basis (see Section 2), and are the main source of whole-sale electricity. Here, increased electricity demand means increased prices, resulting in the so-called *price impact*, and hence it is in every market participant's interest to avoid unnecessary demand peaks.

The participation of an EV aggregator in this type of market has been extensively studied in the literature in recent years, both under price-taker (where no price impact is considered) and price-maker (where price impact is considered) approaches (see [12, 21] for reviews). All these works consider a single aggregator participating in the day-ahead market. However, given the fast growing numbers of EVs and the very large targets established for the near future, we envision an scenario where different EV aggregators co-exist in the same day-ahead market. These aggregators may vary in nature and size, but it is reasonable to assume that they are self-interested. Indeed, reduced electricity costs translate in more profit for the aggregator and/or more benefits for their EV fleet. In this scenario, reduced overall costs can be achieved by inter-aggregator coordination, producing more informed and optimised bidding. However, this coordination is challenging, as an aggregator may choose to cheat the system if greater personal benefit is perceived.

Against this background, there are some works considering a multi-aggregator setting [19, 23, 27, 32, 33], but do not consider self-interested aggregators. More in line with this paper, a first step towards inter-aggregator cooperation in our setup can be found in [21]. They propose a day-ahead bidding coordination mechanism which uses techniques from the field of mechanism design. By employing a third-party coordinator, which collects the EV

Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018), M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.), July 10–15, 2018, Stockholm, Sweden.

aggregator's electricity requirements and performs joint bidding, reduced price impact and lower costs are achieved. Truthful cooperation is encouraged by employing payment systems based on a Vickrey-Clarke-Groves (VCG) mechanism. Their results show that significant cost reductions arise from utilising the coordination mechanism when compared to individually optimal bidding. However, they do not study the stability of the coordination mechanism, *i.e.* what would happen if different coordinators are present, and the aggregators choose whether to form smaller cooperative groups.

In order to address this issue, in this paper we study the coordination approach introduced in [21] from a different perspective, by using tools from cooperative game theory [6]. Cooperative game theory studies games in which the participating agents can form *coalitions* in order to improve their performance, and has been successfully applied in smart grid related studies. Specifically, cooperation among small power producers is studied in [5, 7]. [7] considers small distributed energy producers which are able to form coalitions to make joint offers in electricity markets. This is shown to be beneficial to gain market visibility, and to reduce the uncertainty related to their bids, resulting in increased profit. [5] considers a similar scenario, and focuses on profit distribution among the coalition members. Similarly, cooperation among wind producers is studied in [3], their results show that cooperation increases their profit. Moreover, cooperative game theory has been employed to aggregate demand response providers, in order to improve their performance and grid stability [9, 20]. Lastly, cooperation among independent households with distributed generation and storage capabilities is studied in [1]. Their results show that battery degradation can be greatly reduced, while obtaining significant better energy efficiency, by cooperation. However, none of these works studies EV charging, which presents several distinctive challenges, as detailed below.

Against this background, we study the formation of EV aggregator coalitions in order to coordinate day-ahead bidding. Specifically, we focus on finding payoff allocations (*i.e.* payment mechanisms) which result in fair and stable coalitions. Moreover, our scenario presents several challenges: complex hourly interdependence (see Section 3.1), a complex underlying optimisation problem (see Section 3.2), and the presence of externalities in the coalitional game (see Section 4.2). In more detail, this paper makes the following contributions to the state of the art:

- We present the first application of cooperative game theory to study the cooperation of EV aggregators participating in day-ahead markets.
- We propose a coalitional game and prove that it has a non-empty core. Moreover, we propose a payment mechanism that lies in the core, specifically, the least-core.
- We compare this payment mechanism against the well-known Shapley value, whose computational complexity is softened by employing a sampling approximation with bounded error.
- We present a realistic empirical evaluation that uses real market and driver data to compare the least-core and the Shapley value payment mechanisms.

The rest of the paper is structured as follows. Section 2 introduces the considered day-ahead market and the mathematical formalism to quantify price impact. Section 3 presents the considered EV aggregator and day-ahead bidding models. We are then ready to present our cooperation model and study its theoretical properties in Section 4. An empirical evaluation is presented in Section 5. Finally, we conclude in Section 6.

2 THE DAY-AHEAD MARKET

This section details the day-ahead market structure considered in this paper and present in most countries. Moreover, we discuss how to quantify the price impact of buy orders (electricity demand), which is an important aspect of our work.

Day-ahead markets divide each day into 24 hourly slots, each running a separate uniform-priced double-sided auction. Before closure time (usually noon) on day D, bids and offers for each hourly slot of day $D+1$ must be submitted to the market. Then, a matching algorithm determines the accepted bids and offers, and establishes an hourly uniform price using marginal pricing, this is, the price of the intersection between supply and demand.

Bids (buy orders) and offers (sell orders) for each hourly slot are quantity-price pairs. For bids (offers), the price represents the highest (lowest) price the participant is willing to pay (sell for). As is common in most markets, we define a minimum price $p_{\min} = 0$ and some maximum price, $p_{\max}$. After closure time, the auctioneer aggregates all buy and sell orders, by high-price and low-price priorities, respectively. This generates the aggregated demand and supply curves, and their intersection determines the accepted orders and the resulting uniform price, as depicted in Fig. 1a.

Clearly, the arrival of a new buy order pushes the clearing price up if it gets accepted (*i.e.* if it lies towards the left-hand side of the intersection). Fig. 1b illustrates the effect of a new buy order with quantity E placed at price $p_{\max}$. The price increase (price impact) depends on the new order's price and quantity, and on the supply and demand curves. Price impact is an essential market characteristic associated with large market participants, and careful managing is required to avoid pushing prices up unnecessarily. Price impact has been studied in the electricity markets literature by employing residual curves [13], which are detailed below.

Employing standard notation, for any given hour t, let $D_t(p)$ and $S_t(p)$ be the aggregated demand and supply curves respectively, as a function of price, p. The residual supply curve is defined as $R_t(p) = S_t(p) - D_t(p) = E$, and represents the amount of energy, E, an agent could bid for while maintaining a clearing price p. Conversely, the clearing price when bidding a quantity E is given by $p = R_t^{-1}(E)$. Introducing the notation $\mathcal{P}_t(E) = R_t^{-1}(E)$, the clearing price when the new agent bids an amount E is $p = \mathcal{P}_t(E)$, and the price impact Δp of this order is given by $\Delta p = \mathcal{P}_t(E) - \mathcal{P}_t(0)$, where $\mathcal{P}_t(0)$ represents the *base price* at hour t, i.e. the price without the agent's new bid. This formalism is depicted in Figs. 1b and 1c.

We are now ready to introduce the EV aggregator model and the day-ahead bidding algorithm.

3 EV AGGREGATOR PARTICIPATION IN DAY-AHEAD MARKETS

As discussed in Section 1, an EV aggregator is responsible for the charging of a fleet of EVs and, to this end, purchases the required electricity from the day-ahead market (see Section 2) [4]. We will start by describing the aggregator structure and operation. Then, we will describe the bidding algorithm, which is from [21]. Finally, we will show how two or more independent aggregators can coordinate their bidding.

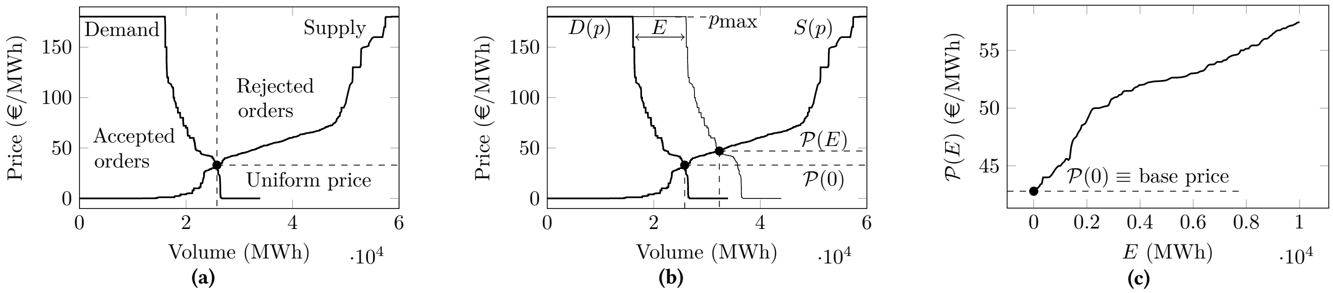


Figure 1: (a) Aggregated supply and demand curves, and market clearing mechanism. (b) Price impact of a buy order with volume E and maximum price $p_{\max}$. (c) Final price function $\mathcal{P}(E)$. Source: OMIE, 01/11/2016, 11th hour. [21]

3.1 EV Aggregator Model

In our model, EVs arrive and depart dynamically over time. When an EV arrives to the charging point, it communicates the desired departure time, t_d^i, and desired state of charge at departure, SoC_d^i, to the aggregator. We assume that arrival time and state of charge, t_d^0 and SoC_0^i can be automatically inferred by the aggregator. Each EV has a maximum charging speed, $P_{\max}^i$ in kW, which depends on two factors: the available physical infrastructure, and the EV's battery. The charging schedule of the EV is then left at the aggregator's discretion, which can choose when to perform the charging while guaranteeing the desired state of charge by departure time. This flexibility allows charging the battery in an informed way, rather than randomly, or at arrival, providing cheaper electricity costs.

Due to the nature of the day-ahead market, electricity bids need to be placed between 12 and 36 hours before delivery time (assuming market closure at noon, see Section 2). This requires the market participants to forecast their electricity needs and to bid accordingly.

Following [4, 21], we model the requirements of an EV by employing two vectors with 24 entries each, $\mathbf{r}^{\min,i}$ and $\mathbf{r}^{\max,i}$. Specifically, $r_t^{\min,i}$ is the amount of energy needed at hour t assuming charging has been left for the last possible moment and that the charging requirements need to be fulfilled. Conversely, $r_t^{\max,i}$ is the amount of energy needed at hour t assuming charging starts as soon as possible. For example, consider an EV arriving at 3pm, stating 9pm departure time and 8kWh charging needs with $P_{\max} = 3$kW. Then, $\mathbf{r}^{\min,i}$ would be as specified in Table 1. Specifically, if 6pm is reached with no charging done, at least 2kW of energy needs to be charged between 6-7pm in order to fulfil the EV driver requirements. The same applies with 3kW between 7-8pm and 8-9pm. Similarly, for the same scenario, the requirement vector $\mathbf{r}^{\max,i}$ would be as specified in Table 2.

$r_3^{\min,i}$	$r_4^{\min,i}$	$r_5^{\min,i}$	$r_6^{\min,i}$	$r_7^{\min,i}$	$r_8^{\min,i}$	$r_9^{\min,i}$
0	0	0	2	3	3	0

Table 1: Example of requirement vector $\mathbf{r}^{\min,i}$

$r_3^{\max,i}$	$r_4^{\max,i}$	$r_5^{\max,i}$	$r_6^{\max,i}$	$r_7^{\max,i}$	$r_8^{\max,i}$	$r_9^{\max,i}$
3	3	2	0	0	0	0

Table 2: Example of requirement vector $\mathbf{r}^{\max,i}$

Then, two global energy requirement vectors, $\mathbf{R}^{\min}$ and $\mathbf{R}^{\max}$, can be obtained by summing the hourly requirements of all the EVs associated to the particular aggregator, *i.e.* $R_t^{\min} = \sum_{i=1}^N r_t^{\min,i}$ and $R_t^{\max} = \sum_{i=1}^N r_t^{\max,i}$.

In order to make informed bids in the day-ahead market, several quantities need to be forecasted by the aggregator (denoted by a hat): hourly energy requirements, $\hat{R}_t^{\min}$ and $\hat{R}_t^{\max}$, hourly number of available EVs, $\hat{N}_t$, and hourly price impact functions, $\hat{\mathcal{P}}_t$.

Considering advanced forecasting approaches is outside of the scope of this paper and simple forecasting is employed in the simulation experiments shown in Section 5. Specifically, data from the previous day is the forecast for the day after [4, 12, 21]. However, we note that all theoretical results presented in this paper are independent of the forecasting approach used.

3.2 Day-Ahead Bidding Algorithm

Now that the day-ahead and EV aggregator models have been detailed, we are ready to present the day-ahead bidding algorithm. The algorithm is from [21] and reproduced here for convenience. The mathematical problem is defined as follows: given an EV aggregator's forecasted requirements and price impact functions, find the optimal distribution of energy quantities to bid across the 24 hourly slots of the next day, $(E_0, \ldots, E_{23})$, in order to satisfy its clients' charging needs while minimising the total cost of the purchased energy. We assume that the agent's bids are set at maximum price, $p_{\max}$, in order to guarantee execution. Hence only bidding hours and quantities need to be decided.

As discussed in [21], and in order to avoid a complex minimisation landscape with multiple minima, the forecasted hourly price impact functions $\hat{\mathcal{P}}_t$ (see Sections 2 and 3.1) are approximated by quadratic convex functions. Specifically, they are given by $\hat{\mathcal{P}}_t^{\text{convex}} = a_t E_t^2 + b_t E_t + \hat{\mathcal{P}}_t(0)$, where all the coefficients a_t and b_t are restricted to be positive. Formally, the optimisation algorithm is given by Eqs. (1), (2a), (2b), (2c). In more detail, the objective function (1) minimizes the total cost of the purchased energy. The constraints guarantee that the amount of purchased energy is enough to satisfy the forecasted demand (2a), that it is not purchased before the forecasted arrival of the EVs (2b) and that the energy purchased at each hour is not greater than the amount that the aggregator is able to charge at the given hour, based on the forecasted number of available vehicles (the aggregator cannot store energy). It is worth noting that the number of constraints is always 72, independent on the fleet size. Also, given the convexity of the problem, there exists a unique global minimum, which we are guaranteed to find.

$$\min_{\{E_t\}} \sum_t \hat{\mathcal{P}}_t^{\text{convex}}(E_t) \cdot E_t \tag{1}$$

$$\sum_{j=0}^{t} E_j \geq \sum_{j=0}^{t} \hat{R}_j^{\min}, \ \forall t = 0, \ldots, 23 \tag{2a}$$

$$\sum_{j=0}^{t} E_j \leq \sum_{j=0}^{t} \hat{R}_j^{\max}, \ \forall t = 0, \ldots, 23 \tag{2b}$$

$$E_t/\Delta t \leq \hat{N}_t P_{\max}, \ \forall t = 0, \ldots, 23 \tag{2c}$$

3.3 Joint Bidding

The bidding algorithm detailed in the previous section for a single aggregator can be extended to perform joint bidding, where a number of independent aggregators join their requirements and apply the optimisation algorithm globally. In more detail, let C be a set of EV aggregators. Then, following [21], let $\hat{R}_t^{\min, i}$ and $\hat{R}_t^{\max, i}$ be aggregator i's forecasted energy requirements for hour t , and $\hat{N}_t^i$ the number of available EVs from aggregator i, as specified in Section 3.2. The combined requirements of all the aggregators in C are then:

$$\hat{R}_t^{\min} = \sum_{i \in C} \hat{R}_t^{\min, i} \quad (3) \qquad \hat{R}_t^{\max} = \sum_{i \in C} \hat{R}_t^{\max, i} \quad (4) \qquad \hat{N}_t = \sum_{i \in C} \hat{N}_t^i \quad (5)$$

To find the optimal global energy bids, the bidding optimisation algorithm given by Eqs. (1), (2a), (2b), (2c) can be applied with constraints given by the combined requirements (3), (4) and (5). This will result in obtaining a global day-ahead energy volume E_t^{global} for each hour t, which can be then distributed among the aggregators in C. The redistribution mechanism is defined in [21], and allocates an hourly energy schedule to each participating aggregator.

4 COORDINATION AMONG AGGREGATORS

In this section, we present our novel coalitional analysis of inter-aggregator cooperation in day-ahead markets. By employing techniques from cooperative game theory, we model the scenario as a coalitional game, and discuss and prove several desirable theoretical properties. In more detail, we focus on finding payment mechanisms which incentivise the aggregators to form coalitions and cooperate, rather than strategically manipulate the system.

Cooperation is achieved by employing a coordination system, proposed in [21], which consists of three stages. Firstly, the cooperating aggregators report their forecasted energy requirements to the coordinator. The coordinator is then able to perform coordinated bidding, as shown in Section 3.3. Secondly, the purchased energy is distributed among the participating aggregators according to their reported preferences. Thirdly, the coordinator must compute suitable payments for each of the participating aggregators. This step is key to ensure that cooperation rather than strategic manipulation is encouraged.

The rest of the section is structured as follows. We first describe the proposed EV aggregator coalitional game. Then, the presence of externalities is discussed. The main theoretical properties of the game are then described, including superadditivity and balancedness. Lastly, the Shapley value is considered, together with the nucleolus and the least-core imputations.

4.1 Defining the Aggregator Coalitional Game

We start by presenting the basic concepts of cooperative game theory [6], before proceeding to describe our proposed game. Consider a set of players $N = \{1, \ldots, n\}$, *i.e.* the set of EV aggregators participating in the day-ahead market.

Definition 4.1 (Coalition). A *coalition* is any subset of players $C \subseteq N$. The number of players in the coalition C is given by its cardinality $|C|$. All possible coalitions are denoted by the power set of N, 2^N. The *grand coalition* is the set of all players, N.

Definition 4.2 (Coalition structure). A *coalition structure* over N is a collection of non-empty subsets $CS = \{C_1, \ldots, C_k\}$ such that $\cup_{j=1}^k C_j = N$ and $C_i \cap C_j = \emptyset \ \forall i \neq j$.

Definition 4.3 (Characteristic function game). A *characteristic function game* G is given by a pair (N, v), where N is a finite and non-empty set of players, and $v : 2^N \longrightarrow \mathbb{R}$ is a *characteristic function.* The value $v(C)$ is usually referred to as the *value* of the coalition C.

Note that the characteristic function assigns a value to the whole coalition, not to its individual members. Games in which a coalition value, $v(C)$, can be divided in any way among its members are called *transferable utility* (TU) games.

Focusing on our scenario, consider a realisation of the market with hourly prices $\mathbf{p} = (p_0, \ldots, p_{23})$. Then, the aggregators purchasing an energy schedule given by $\mathbf{E} = (E_0, \ldots, E_{23})$ will incur a total electricity cost given by:

$$\text{cost}(\mathbf{p}, \mathbf{E}) = \sum_{t=0}^{23} p_t \cdot E_t \tag{6}$$

This provides a natural way to define the value function of our coalitional game. In more detail, for a coalition C, $v(C)$ must represent the electricity costs paid by the members of C when they perform coordinated bidding. However, the price impact present in our market model introduces an extra layer of complexity, as any market participant affects the resulting prices with their bids. More specifically, the cost paid by a coalition C depends not only on the members of the coalition itself, but on all the other aggregators as well. This situation is treated in detail in the next section.

4.2 Value Function with Externalities

The first thing to note is that our setting deviates from traditional characteristic function games. This is due to the presence of *externalities* [6, Ch 5.2]. Specifically, in classical game theory, the value of a coalition C, $v(C)$, only depends on the coalition itself. However, in our market structure with price impact (see Section 2), a given coalition C is also affected by the aggregators not in the coalition. In more detail, any market participant will affect the resulting market prices, hence affecting every other participant's costs. Formally, the resulting prices depend on the whole coalition structure, $\mathbf{p} = \mathbf{p}(CS)$, and thus so does the value function of our game: $v(C, CS)$. Games with such value functions are called *partition function games* [31].

A coalitional game with externalities can be studied in partition function form. However, the resulting game has poor theoretical properties and does not yield useful results, as we show in Section 4.2.1. Another usual procedure when dealing with a coalitional game with externalities is to introduce a *conjecture* on the behaviour of the outsider agents [2]. In more detail, when considering a coalition C, the behaviour of the outsider agents, $N \setminus C$ is assumed to be deterministic, and to follow the chosen conjecture, hence recovering the classical theory where the value of the coalition only depends on the coalition itself. The earliest proposed conjecture is the so-called *α-conjecture* [2], which assumes that the outsider players act as to minimise the payoff of the deviated coalition. However, it does not seem appropriate in our setting, as an aggregator trying to minimise a coalition's payoff through price impact would automatically harm itself as well. More recent conjectures proposed in the literature include the *γ-conjecture* and the *outsider coalition conjecture*. Both are reasonable in our setting and are further explored in the next two subsections.

4.2.1 The outsider coalition (oc) conjecture. Introduced in [10], it assumes that, when a coalition C deviates from the grand coalition, all the outsiders join together and form a counter coalition $N \setminus C$. Hence, the resulting coalition structure is $C^{\text{oc}} = \{C, N \setminus C\}$. Formally, the resulting prices depend on the coalition structure, and we can write $\mathbf{p} = \mathbf{p}(C^{\text{oc}})$. Similarly, the amount of energy purchased by the members of coalition C depends on C itself and on the coalition structure C^{oc}, $\mathbf{E} = \mathbf{E}_C(C^{\text{oc}})$. Therefore, the value function can be defined as:

$$v_{\text{oc}}(C) := -\text{cost}(\mathbf{p}(C^{\text{oc}}), \mathbf{E}_C(C^{\text{oc}}))$$

Even though this conjecture seems reasonable in our scenario, the resulting coalitional game (N, v_{oc}) has poor stability properties. To see this, we first introduce the following definition.

Definition 4.4 (Superadditive game). A coalitional game (N, v) is *superadditive* if for every pair of disjoint coalitions $C_1, C_2 \subset N$ such that $C_1 \cap C_2 = \emptyset$, we have $v(C_1) + v(C_2) \leq v(C_1 \cup C_2)$.

In other words, in a superadditive game, the grand coalition has the incentive to form, as the agents can earn at least as much profit by working together. Unfortunately, the considered game is not superadditive, as proven below.

Theorem 4.5. *The coalitional game (N, v_{oc}) is not superadditive.*

Proof. We present a counter-example which employs a simplified market structure with three hours and synthetic prices. In more detail, consider hourly prices given by: $\hat{\mathcal{P}}_1(E_1) = 10 + E_1$, $\hat{\mathcal{P}}_2(E_2) = 5 + E_2/2$, $\hat{\mathcal{P}}_3(E_3) = 10 + E_3$. Consider nine identical EV aggregators, $N = \{1, \ldots, 9\}$, with the following individual energy requirements: $R^{\max} = (1, 0, 0)$, $R^{\min} = (0, 0, 1)$ and a maximum charging speed $P_{\max} = 1$. Considering the following pair of coalitions, $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, it holds: $v_{\text{oc}}(C_1 \cup C_2) < v_{\text{oc}}(C_1) + v_{\text{oc}}(C_2)$. Hence the coalitional game (N, v_{oc}) is not superadditive. □

Thus, the grand coalition does not necessarily form, in which case full coordination is not achieved. This counter-example also applies to the partition function game described in Section 4.2, which is not superadditive either.

4.2.2 The γ-conjecture. Another common choice is the so-called *γ-conjecture* [8], in which the outsider agents select their individual best strategies. Hence, the resulting coalition structure is $C^\gamma = \{C\} \cup \{\{i\} | i \in N,\ i \notin C\}$. Formally, we can write $\mathbf{p} = \mathbf{p}(C^\gamma)$ and $\mathbf{E} = \mathbf{E}_C(C^\gamma)$. Then, the value function can be defined as:

$$v_\gamma(C) := -\text{cost}(\mathbf{p}(C^\gamma), \mathbf{E}_C(C^\gamma)) \tag{7}$$

As we will show in Section 4.3, this conjecture has nice theoretical properties. As a result, we will adopt it throughout the rest of the paper. For convenience, we will drop the subscript γ and write v henceforth.

4.3 Properties of the Coalitional Game

The coalitional game proposed in the previous section, (N, v), has several desirable properties. Specifically, we will show that it is superadditive and balanced, hence it has a non-empty core. Thus, all the EV aggregators are incentivised to cooperate together (grand coalition), and a payment mechanism can be implemented which results in a stable grand coalition, with no sub-coalition having an incentive to deviate. We will now detail and prove these properties.

For convenience, and extending the notation presented in the previous subsection, let $\text{cost}\big(\mathbf{p}(C_2^\gamma), \mathbf{E}_{C_1}(C_2^\gamma)\big)$ be the total electricity cost paid by the members of $C_1 \subseteq N$ when coalition $C_2 \subseteq N$ performs coordinated bidding (see Section 3.3), and all other participants perform individual bidding (see Section 3.2).

We are now ready to show that the game is superadditive.

Lemma 4.6. *For all coalitions $C_1, C_2 \subseteq N$ such that $C_1 \subset C_2$, it holds that:*

$$cost\Big(\mathbf{p}((C_1 \cup C_2)^\gamma), \mathbf{E}_{C_1}((C_1 \cup C_2)^\gamma)\Big) \leq cost\Big(\mathbf{p}(C_1^\gamma), \mathbf{E}_{C_1}(C_1^\gamma)\Big)$$

Proof. This lemma trivially follows from the fact that coordinated bidding with more participants can only decrease the total costs. Hence the price paid by members of coalition C_1 when coordination happens inside $C_1 \cup C_2$ can only be lower, or equal, than when coordination happens only inside C_1. The equality case happens only when the members of C_1 and C_2 have non-overlapping energy requirements, or when the price impact of their combined bids is not high enough. □

Theorem 4.7. *The coalitional game (N, v) is superadditive.*

Proof. Consider any two disjoint coalitions, $C_1, C_2 \subseteq N$. Then,

$$\begin{aligned} v(C_1 \cup C_2) \geq v(C_1) + v(C_2) \Leftrightarrow\ & \text{cost}\Big(\mathbf{p}((C_1 \cup C_2)^\gamma), \mathbf{E}_{C_1 \cup C_2}((C_1 \cup C_2)^\gamma)\Big) \\ & \leq \text{cost}\Big(\mathbf{p}(C_1^\gamma), \mathbf{E}_{C_1}(C_1^\gamma)\Big) + \text{cost}\Big(\mathbf{p}(C_2^\gamma), \mathbf{E}_{C_2}(C_2^\gamma)\Big) \end{aligned}$$

Given the following identity:

$$\begin{aligned} &\text{cost}\big(\mathbf{p}((C_1 \cup C_2)^\gamma),\ \mathbf{E}_{C_1 \cup C_2}((C_1 \cup C_2)^\gamma)\big) \\ &= \text{cost}\big(\mathbf{p}((C_1 \cup C_2)^\gamma),\ \mathbf{E}_{C_1}((C_1 \cup C_2)^\gamma)\big) \\ &+ \text{cost}\big(\mathbf{p}((C_1 \cup C_2)^\gamma),\ \mathbf{E}_{C_2}((C_1 \cup C_2)^\gamma)\big) \end{aligned}$$

the expression above reads:

$$\begin{aligned} &\text{cost}\Big(\mathbf{p}((C_1 \cup C_2)^\gamma), \mathbf{E}_{C_1}((C_1 \cup C_2)^\gamma)\Big) \\ &+ \text{cost}\Big(\mathbf{p}((C_1 \cup C_2)^\gamma), \mathbf{E}_{C_2}((C_1 \cup C_2)^\gamma)\Big) \\ &\leq \text{cost}\Big(\mathbf{p}(C_1^\gamma), \mathbf{E}_{C_1}(C_1^\gamma)\Big) + \text{cost}\Big(\mathbf{p}(C_2^\gamma), \mathbf{E}_{C_2}(C_2^\gamma)\Big) \end{aligned}$$

which is always true, applying Lemma 4.6. □

This result shows that overall costs are minimised when the grand coalition forms. The main issue is now how to distribute the value of the grand coalition, *i.e.* the resulting costs, among its members, the so-called *payoff allocation*.

Definition 4.8 (Payoff allocation). A vector $\mathbf{x} = (x_1, \ldots, x_n)$ is a payoff allocation vector for a coalition structure $CS = \{C_1, \ldots, C_k\}$ over N if $x_i \geq 0$ for all $i \in N$, and $\sum_{i \in C_j} x_i \leq v(C_j)$ for any $j = 1 \ldots, k$.

(1) (*Efficiency*) An allocation $\mathbf{x}$ is *efficient* if $\sum_{i \in C_j} x_i = v(C_j)$ for any $j = 1 \ldots, k$.

(2) (*Individually rational*) An allocation is *individually rational* if $x_i \geq v(\{i\})$ for all $i \in N$.

This value distribution can be done in an arbitrary way. However, certain allocations have special properties, such as efficiency and individual rationality, which make them desirable, as defined below.

Definition 4.9 (Imputation). A payoff allocation for the grand coalition N is said to be an *imputation* if it is both efficient and individually rational.

Definition 4.10 (The Core). Given a TU characteristic function game (N, v), the *core*, C, is defined as the set of imputations such that no sub-coalition can obtain a payoff which is better than the sum of the members current payoffs. Explicitly:

$$C := \left\{ \mathbf{x} \in \mathbb{R}^N \middle| \sum\nolimits_{i \in N} x_i = v(N),\ \sum\nolimits_{i \in S} x_i \geq v(S),\ \forall S \subseteq N \right\}$$

In other words, payoff allocations lying in the core provide stability, as no sub-coalition has an incentive to deviate from the grand coalition in order to increase its profit. However, the core of coalitional game can be empty [6]. A classical result guarantees the non-emptiness of the core for certain games, the so-called *balanced* games. After the pertinent definitions, we prove that our game is balanced, and thus has a non-empty core.

Definition 4.11 (Balanced function). A function $\alpha : 2^N \longrightarrow \mathbb{R}$ is said to be *balanced* if for all $i \in N$, we have $\sum_{C \in 2^N} \alpha(C)\ \mathbf{1}\{i \in C\} = 1$, where $\mathbf{1}$ is the indicator function.

Definition 4.12 (Balanced game). A coalitional game (N, v) is balanced if for any balanced function α, we have $\sum_{C \in 2^N} \alpha(C) v(C) \leq v(N)$.

Theorem 4.13 ([28]). *A coalitional game has a non-empty core if and only if it is balanced.*

Theorem 4.14. *Our proposed coalitional game (N, v) is balanced. As a result, it has a non-empty core.*

Proof. Let $\alpha : 2^N \longrightarrow \mathbb{R}$ be an arbitrary balanced function. Balancedness of the coalitional game follows from Lemma 4.6:

$$\sum_{C \in 2^N} \alpha(C) v(C) = -\sum_{C \in 2^N} \alpha(C)\ \mathrm{cost}(\mathbf{p}(C^Y), \mathbf{E}_C(C^Y))$$
$$\leq -\sum_{C \in 2^N} \alpha(C)\ \mathrm{cost}(\mathbf{p}(N^Y), \mathbf{E}_C(N^Y))$$
$$= -\sum_{C \in 2^N} \sum_{i \in N} \alpha(C) \mathbf{1}_{\{i \in C\}} \mathrm{cost}\Big(\mathbf{p}(N^Y), \mathbf{E}_{\{i\}}(N^Y)\Big)$$
$$= -\sum_{i \in N} \mathrm{cost}\Big(\mathbf{p}(N^Y, \mathbf{E}_{\{i\}}(N^Y)\Big) = -\mathrm{cost}(\mathbf{p}(N^Y, \mathbf{E}_N(N^Y)) = v(N)$$

Thus the coalitional game is balanced and, by the Bondareva-Shapley Theorem, it has a non-empty core. □

The non-emptiness of the core of our proposed coalitional game guarantees the existence of at least one payoff allocation which stabilises the grand coalition.

4.4 Fair Payoff Allocation

After proving that the proposed game has a non-empty core, we now seek a payoff allocation lying in it. One of the most widely used allocations is the *Shapley value*, defined as:

Definition 4.15 (Shapley value [6]). Given a coalitional game (N, v), the *Shapley value* assigns a payoff SV_i to each player $i \in N$ given by:

$$SV_i(v) = \sum_{C \subseteq N \setminus \{i\}} \frac{|S|!(N - |S| - 1)!}{N!} [v(C \cup \{i\} - v(C)]$$

The Shapley value is the only payment allocation satisfying the *efficiency*, *symmetry*, *dummy action* and *additive* axioms [6, Ch 2.2.1]. Hence, it is traditionally considered to present a fair payoff distribution. Moreover, there exist the following positive results which guarantee that the Shapley value lies in the core of a particular type of coalitional games, *convex* games.

Definition 4.16 (Convex game). A coalitional game (N, v) is *convex* if it has a supermodular value function:

$$v(C_1) + v(C_2) \leq v(C_1 \cup C_2) + v(C_1 \cap C_2)\ , \forall C_1, C_2 \subseteq N$$

Theorem 4.17 ([29]). *If (N, v) is a convex game, then the Shapley value is the barycentre of its core.*

However, it turns out that our proposed coalitional game is not convex, as proven by the counter-example in the following result.

Theorem 4.18. *The coalitional game (N, v) is not convex.*

Proof. We present a counter-example which employs a simplified market structure with three hours and synthetic prices. In more detail, consider hourly prices given by: $\hat{\mathcal{P}}_1(E_1) = 10 + 20E_1$, $\hat{\mathcal{P}}_2(E_2) = 0.1 + 20E_2$, $\hat{\mathcal{P}}_3(E_3) = 10 + 20E_3$. Consider three identical EV aggregators, $N = \{1, 2, 3\}$, with the following individual energy requirements: $R^{\max} = (1, 0, 0)$, $R^{\min} = (0, 0, 1)$ and a maximum charging speed $P_{\max} = 1$. Considering the following pair of coalitions, $C_1 = \{1, 2\}$ and $C_2 = \{2, 3\}$, it holds: $v(C_1 \cup C_2) + v(C_1 \cap C_2) < v(C_1) + v(C_2)$. Hence the coalitional game is not convex. □

As a result, the Shapley value is not guaranteed to be in the core of our proposed coalitional game (N, v). Therefore, we seek a different payoff allocation which is guaranteed to be in the core.

4.5 Imputations in the Core

Another widely used imputation is the *nucleolus*. It employs a different approach than the Shapley value, trying to minimise player dissatisfaction, which is defined next.

Definition 4.19 (Excess). Given a coalitional game (N, v) and a payment allocation $\mathbf{x} \in \mathbb{R}^N$, the dissatisfaction of coalition C is measured by the *excess* defined as: $e(\mathbf{x}, C) = v(C) - \sum_{i \in C} x_i$.

Any payoff vector $\mathbf{x}$ generates an excess vector, $\mathbf{e}(\mathbf{x}) = (e(\mathbf{x}, C_1), \ldots, e(\mathbf{x}, C_{2^N})) \in \mathbb{R}^{2^N}$, where $C_1, \ldots, C_{2^N}$ is the list of subsets of N ordered in non-increasing order by their excess under $\mathbf{x}$. Then, two deficit vectors can be compared lexicographically. Given two payoff vectors $\mathbf{x}, \mathbf{y}$, we have $\mathbf{e}(\mathbf{x}) \leq_{\text{lex}} \mathbf{e}(\mathbf{y})$ if there exists $k \in \mathbb{R}$ such that for all $i < k$, $e_i(\mathbf{x}) = e_i(\mathbf{y})$, and $e_k(\mathbf{x}) \leq e_k(\mathbf{y})$.

Definition 4.20 (Nucleolus). Given a coalitional game (N, v), its *nucleolus* is given by the lexicographically minimal imputation.

The nucleolus is in the core of a coalitional game with non-empty core, as the core is the set of imputations with negative excess [30]. Hence, it is in the core of our proposed coalitional game. However, computing the nucleolus for a game with n players requires solving 2^n linear programs [26], which is prohibitive for all but the smallest coalitional games. In order to maintain computational tractability and develop a scalable system, we will employ an approximation to the nucleolus which lies in the core and presents better scaling properties. Specifically, we propose utilising the *least-core* [18]. This imputation only minimises the *worst-case excess* for all coalitions, instead of finding the imputation that lexicographically minimises the vector of excesses. As a result, the least-core is much less computationally expensive than computing the nucleolus. Moreover, if the core is non-empty, the least-core belongs to it [30].

In more detail, following the exposition in [3], the least-core can be defined as the payment allocation $\mathbf{x}$ which solves the following minimisation problem:

$$e^* = \min_{\mathbf{x}\in\mathbb{R}^n, e\in\mathbb{R}} e, \text{ s.t.} \begin{cases} v(C) - \sum_{i\in C} x_i - e \leq 0, \forall C \subset N \\ v(N) - \sum_{i\in N} x_i = 0 \end{cases} \tag{8}$$

This is a linear program with $n+1$ variables and 2^n+1 constraints, presenting reduced computational complexity than the nucleolus.

We have described two stabilising payoff allocations for the grand coalition. Given the fact that we want our system to be able to scale to large scenarios, computational tractability is an important characteristic. The analysis of runtimes of the two payoff allocations is presented in Section 5.3.

5 EXPERIMENTAL EVALUATION

In this section, we study the performance of the coalitional game proposed in Section 4, employing real market and driver behaviour data from the Iberian Peninsula. The purpose of this empirical study is two-fold. First, we show the global welfare benefits of coordinated bidding against uncoordinated independent bidding. Secondly, we compare two different payment allocations: the Shapley value (see Section 4.4) and the least-core (see Section 4.5). In order to maintain computational tractability, an error-bounded approximation of the Shapley value is employed, as described in Section 5.2.

5.1 Experimental Setup

This case study considers a night-time residential scenario in which EVs arrive in the evening and need to be charged by the next morning. We consider medium-sized electric vehicles with battery capacities of 24kWh. Charging speed is considered to be the same for all EVs and set to $P_{\max} = 3.7$kW. Charging efficiency is considered to be 90%, meaning that 10% of the consumed electricity is lost and does not contribute to the charging of the battery. This parameter choices are consistent with the literature [11].

We employ real market data from the OMIE day-ahead market[1]. Specifically, we utilise weekday data from November 2016. Detailed order data is available online, allowing us to build the aggregated hourly supply and demand curves, and compute price impacts employing residual supply curves as shown in Section 2.

Similarly, we utilise real driver data from a Spanish driving behaviour survey [25]. Specifically, it determines the distribution of times for the first and last trip from and to home. These distributions are given in Table 3. To account for driver mobility, each EV will make use of its aggregator's services with 80% probability every day.

With respect to energy requirements, the state of charge of an EV at arrival and departure times is drawn from a uniform distribution as follows: $\text{SoC}_0 \in [\text{SoC}_{\text{total}}/4, \text{SoC}_{\text{total}}/2]$ and $\text{SoC}_f \in [2 \cdot \text{SoC}_{\text{total}}/3, \text{SoC}_{\text{total}}]$. Consequently, the EV charging requirements range between a large percentage of the battery (up to 75%), to a small percentage (down to 16%), accounting for long and short trips home.

t_0	Time	19h	20h	21h	22h	23h
	Probability	0.16	0.25	0.32	0.12	0.15
t_d	Time	6h	7h	8h	9h	10h
	Probability	0.04	0.02	0.34	0.5	0.1

Table 3: Possible arrival (t_0) and departure (t_d) times rounded to the nearest hour, with their respective probabilities.

5.2 Approximating the Shapley value

The Shapley value (see Section 4.4) is known to be computationally expensive. Specifically, its computational complexity is $2^n \cdot O(v)$, where $O(v)$ is the complexity of the value function. In order to improve its computational tractability, often an approximation is employed [17]. In this paper, we apply the state-of-the-art error-bound approximation proposed in [17, Ch 4.1] for superadditive games.

In more detail, let $SV_i(v)$ denote the Shapley value for a given agent $i \in N$, and $\widetilde{SV}_i(v)$ the approximated Shapley value for the same agent. Instead of considering the $v(C \cup \{i\} - v(C))$ contributions from *all* subsets of $N \setminus \{i\}$, the approximation randomly samples a number of them, $m_{\epsilon,\delta}$, which depends on the desired level of precision:

$$\mathbb{P}\left(\left|SV_i(v) - \widetilde{SV}_i(v)\right| \geq \epsilon\right) \leq \delta$$

The specific formula for $m_{\epsilon,\delta}$ is detailed in [17, Ch 4.1]. Throughout our simulations, we employ $\epsilon, \delta = 5\%$, in order to balance precision and computational tractability.

5.3 Experimental Results

We present the results from three different simulations. Firstly, we study the global cost reductions provided by coordination, when the grand coalition performs joint bidding (see Section 3.3), in comparison to individual bidding (see Section 3.2). Results indicate that significant cost reductions can be achieved by coordination, as shown in Fig. 2. Now that the efficacy of coordinated inter-aggregator bidding has been shown, we turn our attention to the two proposed payment mechanisms, the least-core and the approximated Shapley value.

In more detail, we present the results from two different simulations. Firstly, we consider an scenario where four different EV aggregators of different sizes, namely 10, 20, 30 and 40k EVs participate in the market. Apart from size, they share the same characteristics, as described in Section 5.1. The purpose of this simulation is to study how the proposed payment mechanisms capture the greater electricity costs of larger market participants. Results are shown in Figs. 4a and 4b. Specifically, Fig. 4a presents the daily prices assigned to each aggregator by the least-core payment mechanism. We can

[1] http://www.omie.es/en/inicio

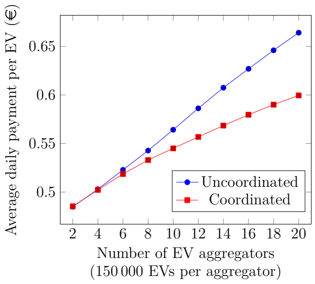


Figure 2: Global cost comparison: uncoordinated versus coordinated bidding.

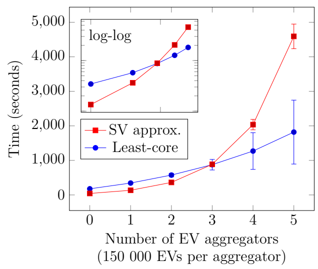
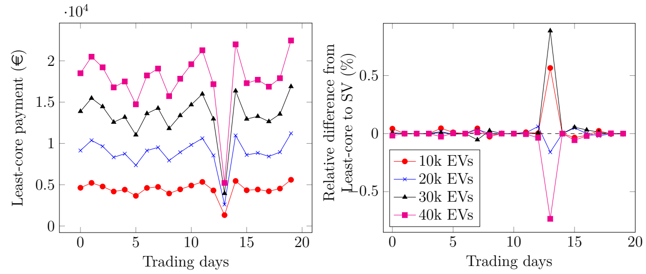
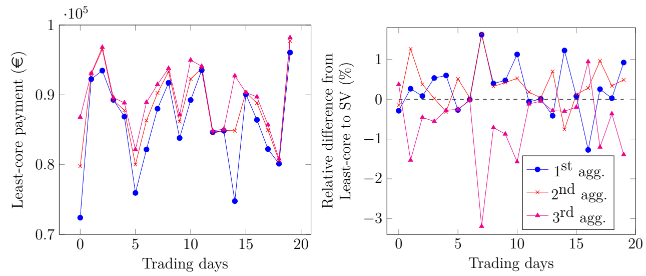


Figure 3: Runtimes for the approx. SV and least-core payment mechanisms.



(a) **(b)**

Figure 4: (a) Daily least-core payment allocations for four aggregators with different sizes. (b) Daily difference between the least-core and approx. Shapley value payments.



(a) **(b)**

Figure 5: (a) Daily least-core payment allocations for three aggregators of the same size (150k EVs) but with different flexibilities. (b) Daily difference between the least-core and approx. Shapley value payments.

see that it correctly assigns larger payments to larger aggregators. Moreover, Fig. 4b presents the daily difference between least-core and approximated Shapley value payments for each aggregator. We can see that the differences are of very small magnitude, concluding that both payment mechanisms are both very close in all cases.

Lastly, we consider a similar scenario, where three different aggregators participate in the market. In this case, they all have the same size, 150k EVs each, but have different charging flexibilities. In more detail, instead of employing the real arrival and departure data provided in Table 3, we employ synthetic data in order to capture the effects of charging flexibility on allocated prices. Specifically, we consider an evening charging scenario where all EVs depart at midnight. The first aggregator receives EVs at 14h, the second at 16h and the third at 18h. Thus, the first aggregator has the highest flexibility and the third is the most constrained. This choice of arrival and departure times is motivated by the typical hourly prices found in the OMIE market, where electricity is cheaper around 15-17h, and more expensive afterwards. Results are presented in Figs. 5a and 5b. We can see that first aggregator, the most flexible, is able to obtain energy at cheaper hours, hence being allocated cheaper payments. Conversely, the third aggregator which is forced to operate at expensive hours, incurs larger payments. The second aggregator lies in between. Similarly to the previous scenario, we can see that least-core and approximated Shapley value payments are very close together for all aggregators and every trading days.

As just described, the least-core and Shapley value payments are very close in all considered scenarios. This suggests that both payment mechanisms present good stability and fairness properties. Moreover, given their computational complexity (see Fig. 3), the approximated Shapley value presents much better scaling properties, and is more suitable for the application of this framework to large scenarios. We note that the presented results do not depend on the particular trading days shown here. Longer simulations have been run utilising different trading months, and the results obtained follow the same trend.

6 CONCLUSION AND FUTURE WORK

In this paper, we present the first study of coperative game theory applied to EV charging. We define a cooperative game in which self-interested EV aggregators can cooperate with each other in order to effectively bid in the day-ahead market. We propose employing a γ-conjecture in order to obtain a game in characteristic function form. Next, we show that the resulting game is superadditive, hence the larger the formed coalition, the larger electricity cost reductions for the EV aggregators. Moreover, we prove that our game is balanced and thus has a non-empty core. The payment mechanism given by the least-core is proposed in order to distribute payments among the grand coalition of aggregators. As the least-core belongs to the core of our game, this payment mechanism stabilises the grand coalition. Lastly, we present numerical simulations which employ real market and driver behaviour data, in order to show the efficacy of coordinated bidding. The least-core payments are compared with a more computationally tractable approximation to the Shapley value. The good agreement between the two suggests that they both present good stability and fairness properties.

Future work will focus on the extension of the proposed coalition framework to a vehicle-to-grid (V2G) setting [34]. In more detail, V2G involves selling electricity stored in the batteries of EVs back to the grid, in order to obtain a profit and/or support grid functioning in times of scarcity. Plenty of works in the literature have shown the benefits of V2G in a variety of scenarios. However, the scenario considered in this paper has not been studied.

ACKNOWLEDGEMENTS

This work was supported by an EPSRC Doctoral Training Centre grant (EP/L015382/1). All the data from the simulations described in the experimental evaluation (Section 5) is publicly available in [22].

REFERENCES

[1] Muddasser Alam, Sarvapali D. Ramchurn, and Alex Rogers. 2013. Cooperative Energy Exchange for the Efficient Use of Energy and Resources in Remote Communities. *Proceedings of the 12th International Conference on Autonomous Agents in Multiagent Systems (AAMAS 2013)* (2013), 731–738. http://eprints.soton.ac.uk/346637/

[2] Robert J Aumann. 1961. The Core of a Cooperative Game Without Side Payments. *Trans. Amer. Math. Soc.* 98, 3 (1961), 539–552.

[3] E. Baeyens, E. Y. Bitar, P. P. Khargonekar, and K. Poolla. 2013. Cooperative Aggregation of Wind Power. *IEEE Transactions on Power Systems* 28, 4 (2013), 1–11.

[4] Ricardo J. Bessa, Manuel A. Matos, Filipe Joel Soares, and João A Peças Lopes. 2012. Optimized bidding of a EV aggregation agent in the electricity market. *IEEE Transactions on Smart Grid* 3, 1 (2012), 443–452. https://doi.org/10.1109/TSG.2011.2159632

[5] Jörg Bremer and Michael Sonnenschein. 2013. Estimating shapley values for fair profit distribution in power planning smart grid coalitions. *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)* 8076 LNAI (2013), 208–221. https://doi.org/10.1007/978-3-642-40776-5-19

[6] Georgios Chalkiadakis, Edith Elkind, and Michael Wooldridge. 2012. *Computational Aspects of Cooperative Game Theory*. 170 pages. https://doi.org/10.2200/S00355ED1V01Y201107AIM016

[7] Georgios Chalkiadakis, Valentin Robu, Ramachandra Kota, Alex Rogers, and Nick Jennings. 2011. Cooperatives of Distributed Energy Resources for Efficient Virtual Power Plants. *The Tenth International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2011)* Aamas (2011), 787–794. http://eprints.soton.ac.uk/271950/

[8] Parkash Chander and Henry Tulkens. 1997. The Core of an Economy with Multilateral Environmental Externalities. *International Journal of Game Theory* 26, 3 (1997), 379–401. https://doi.org/10.1007/BF01263279

[9] Archie C Chapman, Sleiman Mhanna, and Gregor Verbič. 2017. Cooperative Game Theory for Non-linear Pricing of Load-side Distribution Network Support. In *The 3rd IJCAI Algorithmic Game Theory Workshop*.

[10] Yukihiko Funaki and Takehiko Yamato. 1999. The core of an economy with a common pool resource: A partition function form approach. *International Journal of Game Theory* 28, 2 (1999), 157–171. https://doi.org/10.1007/s001820050010

[11] Enrico H. Gerding, Sebastian Stein, Sofia Ceppi, and Valentin Robu. 2016. Online mechanism design for vehicle-to-grid car parks. *IJCAI International Joint Conference on Artificial Intelligence* 2016-Janua (2016), 286–293.

[12] Marina Gonzalez Vaya and Goran Andersson. 2015. Optimal Bidding Strategy of a Plug-In Electric Vehicle Aggregator in Day-Ahead Electricity Markets under Uncertainty. *IEEE Transactions on Power Systems* 30, 5 (2015), 2375–2385. https://doi.org/10.1109/TPWRS.2014.2363159

[13] Rocio Herranz, Antonio Muñoz San Roque, José Villar, and Fco Alberto Campos. 2012. Optimal demand-side bidding strategies in electricity spot markets. *IEEE Transactions on Power Systems* 27, 3 (2012), 1204–1213. https://doi.org/10.1109/TPWRS.2012.2185960

[14] International Energy Agency. 2016. *Global EV Outlook 2016: Beyond one million electric cars*. Technical Report. http://www.eia.gov/forecasts/aeo/pdf/0383(2016).pdf

[15] International Energy Agency. 2017. *Global EV Outlook 2017: Two million and counting*. Technical Report. https://doi.org/10.1787/9789264278882-en

[16] Willett Kempton, Jasna Tomic, Steven Letendre, Alec Brooks, and Timothy Lipman. 2001. Vehicle-to-Grid Power: Battery, Hybrid, and Fuel Cell Vehicles as Resources for Distributed Electric Power in California. *Fuel Cell* IUCD-ITS-R, June (2001), 95. http://escholarship.org/uc/item/0qp6s4mb.pdf

[17] Sasan Maleki. 2015. *Addressing The Computational Issues of the Shapley Value With Applications in The Smart Grid*. Ph.D. Dissertation. University of Southampton.

[18] M Maschler, Bezalel Peleg, and Lloyd S. Shapley. 1979. Geometric Properties of the Kernel, Nucleolus, and Related Solution Concepts. *Mathematics of Operations Research* 4, 4 (1979), 303–338.

[19] Joy Chandra Mukherjee and Arobinda Gupta. 2017. Distributed Charge Scheduling of Plug-In Electric Vehicles Using Inter-Aggregator Collaboration. *IEEE Transactions on Smart Grid* 8, 1 (2017), 331–341. https://doi.org/10.1109/TSG.2016.2515849

[20] Geaorid O'Brien, Abbas El Gamal, and Ram Rajagopal. 2015. Shapley Value Estimation for Compensation of Participants in Demand Response Programs. *IEEE Transactions on Smart Grid* 6, 6 (2015), 2837–2844. https://doi.org/10.1109/TSG.2015.2402194

[21] Alvaro Perez-Diaz, Enrico H. Gerding, and Frank McGroarty. 2018. Coordination and payment mechanisms for electric vehicle aggregators. *Applied Energy* 212 (2018), 185–195. https://doi.org/10.1016/j.apenergy.2017.12.036

[22] Alvaro Perez-Diaz, Enrico H. Gerding, and Frank McGroarty. 2018. Dataset for "Coordination of Electric Vehicle Aggregators: A Coalitional Approach". (2018). https://doi.org/10.5258/SOTON/D0413

[23] Wei Qi, Zhiwei Xu, Zuo Jun Max Shen, Zechun Hu, and Yonghua Song. 2014. Hierarchical coordinated control of plug-in electric vehicles charging in multifamily dwellings. *IEEE Transactions on Smart Grid* 5, 3 (2014), 1465–1474. https://doi.org/10.1109/TSG.2014.2308217

[24] Emmanouil S. Rigas, Sarvapali D. Ramchurn, and Nick Bassiliades. 2015. Managing Electric Vehicles in the Smart Grid Using Artificial Intelligence : A Survey. *Ieee Transactions on Intelligent Transportation Systems* 16, 4 (2015), 1619–1635.

[25] Juan Manuel Ropero Ortega. 2014. Encuesta Movilidad Cotidiana. *Ministerio de Fomento.* 1 (2014), 1–5. https://doi.org/10.1007/s13398-014-0173-7.2

[26] J.K. Sankaran. 1991. On finding the nucleolus of a n-person cooperative game. *International Journal of Game Theory* 19 (1991), 329–338.

[27] Chengcheng Shao, Xifan Wang, Xiuli Wang, Chao Du, and Biyang Wang. 2016. Hierarchical Charge Control of Large Populations of EVs. *IEEE Transactions on Smart Grid* 7, 2 (2016), 1147–1155. https://doi.org/10.1109/TSG.2015.2396952

[28] Lloyd S. Shapley. 1967. On balanced sets and cores. *Naval Research Logistics Quarterly* 14, 4 (1967), 453–460. https://doi.org/10.1002/nav.3800140404

[29] Lloyd S. Shapley. 1971. Cores of complex games. *International Journal of Game Theory* 1, 1 (1971), 11–26.

[30] Tamas Solymosi. 2014. The kernel is in the least core for permutation games. *Central European Journal of Operations Research* (2014), 795–809. https://doi.org/10.1007/s10100-014-0342-y

[31] R. M. Thrall and W. F. Lucas. 1963. n-Person Games in Partition Function Form. *Naval Research Logistics (NRL)* 10 (1963), 281–298.

[32] Hongyu Wu, Mohammad Shahidehpour, Ahmed S Alabdulwahab, and Abdullah Abusorrah. 2016. A Game Theoretic Approach to Risk-Based Optimal Bidding Strategies for Electric Vehicle Aggregators in Electricity Markets With Variable Wind Energy Resources. *IEEE Transactions on Sustainable Energy* 7, January (2016), 118. https://doi.org/10.1109/TSTE.2015.2498200

[33] James J. Q. Yu, Junhao Lin, Albert Y. S. Lam, and Victor O. K. Li. 2016. Maximizing Aggregator Profit through Energy Trading by Coordinated Electric Vehicle Charging. In *IEEE International Conference on Smart Grid Communications (SmartGridComm)*. arXiv:1508.00663 http://arxiv.org/abs/1508.00663

[34] Chengke Zhou, Kejun Qian, Malcolm Allan, and Wenjun Zhou. 2011. Modeling of the cost of EV battery wear due to V2G application in power systems. *IEEE Transactions on Energy Conversion* 26, 4 (2011), 1041–1050. https://doi.org/10.1109/TEC.2011.2159977