Facility Location Games with Externalities*

Minming Li
Department of Computer Science,
City University of Hong Kong,
Hong Kong
City University of Hong Kong
Shenzhen Research Institute,
Shenzhen, China
minming.li@cityu.edu.hk

Lili Mei†
School of Computing and Information Sciences,
Caritas Institute of Higher Education,
Hong Kong
meilili@zju.edu.cn

Yi Xu
School of Economics & Management,
Xi'an University of Technology,
Xi'an, China
xuyi@xaut.edu.cn

Guochuan Zhang
College of Computer Science,
Zhejiang University,
Hangzhou, China
zgc@zju.edu.cn

Yinchao Zhao
School of Computing and Information Sciences,
Caritas Institute of Higher Education,
Hong Kong
zhaoyingchao@gmail.com

ABSTRACT

Facility location games study the scenario where a facility is to be placed based on the reported information from agents. In the society where there are relationships between agents, it is quite natural that one agent's gain will affect other agents' gain (either increase for a collaborator or decrease for a competitor). By using externality to represent this type of agent interaction, for the first time we introduce it into the facility location games in this paper. Namely, we study the extension where agents' utilities will be affected by other agents. We derive necessary and sufficient conditions for well known existing mechanisms and also prove strong lower bounds.

KEYWORDS

Mechanism Design; Facility Location Games; Externalities

ACM Reference Format:
Minming Li, Lili Mei, Yi Xu, Guochuan Zhang, and Yinchao Zhao. 2019. Facility Location Games with Externalities. In *Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), Montreal, Canada, May 13–17, 2019,* IFAAMAS, 9 pages.

1 INTRODUCTION

Facility location games have been extensively studied in the recent ten years after the seminal paper by Procaccia and Tennenholtz [31]. In the original setting, the social planner needs to locate a facility at a certain location based on the report from agents who are the potential customers of the facility, with an attempt to prevent each agent from lying, which will naturally happen if lying could increase the agent's gain. On the other hand, the social planner also wants to optimize a certain social objective. Researchers investigated the above mechanism design problem for various models since then along both the characterization and the design perspectives. However, to the best of our knowledge, all previous work on facility location games assumes that an agent's utility is only decided by the positions of the facilities and the position or preference of the agent herself. Other agents' information plays no role in the calculation of one agent's utility. However, in reality, one could often see the case of mutual influence among agents. For example, if an agent lives nearby a shopping mall and so does her friend, then this agent may have more incentive to go shopping (with the possible company of her friend) and therefore feels better, compared to the case that her friend lives far away and has low incentive in joining her in the shopping. Therefore, the valuation of an agent is not only decided by herself but also by her friend in this case. We call this type of influence *positive externalities*. On the other hand, suppose that the facility is a wholesale market and agents are retailers. In this case, the agents are competitors and other agents being closer to the facility definitely poses more threat to an agent since closer distance means faster turnaround time and being more competitive. We call this type of influence *negative externalities*. For the obnoxious facility scenario, we give examples of a facility which incurs interference to the WIFI connection. In the case of doing Skype chat, one would hope that the friends she talks to are not too close to the facility since that will cause the Skype chat quality to be downgraded. Inversely suppose the agents are participating in some competition via WIFI signal, then an agent would want her competitors to be close to the facility so that she could get advantage over her competitors. In this paper, we mainly focus on the scenarios where each agent has positive externalities.

In economics, externalities are defined as the benefits or costs that affect a party who did not choose to incur that cost or benefit [8]. In the context of multiagent systems, externalities can be interpreted as the interaction between different agents in terms of the relationships between them. For example, one agent may feel happier if her friend's utility increases. On the other hand, an agent

*This paper was partially sponsored by Project 11771365, 11531014 supported by National Natural Science Foundation of China and Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. UGC/FDS11/E03/16).
†Corresponding author

Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), N. Agmon, M. E. Taylor, E. Elkind, M. Veloso (eds.), May 13–17, 2019, Montreal, Canada. © 2019 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). All rights reserved.

may also feel worse if her enemy/competitor gains more utility. Although externalities are widely studied for the economics setting, they have never been studied in the facility location games. In this paper, when externalities exist in the context of facility location games, we consider that agents derive benefits or costs from others. Externalities make the valuations of agents for a certain location of the facility depend also on other agents' positions rather than only on their own location.

Our contributions

- We introduce externality into facility location games which could cover a wider range of real life applications. To a certain extent, the setting with externalities describes the relationships among persons, which the previous models of facility location games cannot demonstrate.
- For facility location games, we prove that any strategyproof mechanism cannot have a finite approximation ratio for all the social objectives we will consider in this paper if the externality coefficients are within $[0, 1)$, and characterize *sufficient and necessary conditions* for the externality coefficients so that existing well known mechanisms including the median mechanism are strategyproof.
- For obnoxious facility games, no strategyproof deterministic mechanisms achieve a finite approximation ratio for maximizing the total distance either. Again, we characterize *sufficient and necessary conditions* for externality coefficients so that the well known majority mechanism (a sufficient condition for an adapted majority mechanism) is strategyproof.

Related work

Facility location games have been receiving a considerable amount of focus in the recent literature. From the characterization perspective, the facility location game was first studied by Moulin [29]. He presented all characterization of strategyproof, Pareto efficient, and anonymous mechanisms on a line. Schummer and Vohra [33] characterized all the strategyproof mechanisms on other networks. The characterization of strategyproof mechanisms for the extended model of locating two facilities on a line was studied in [20, 28]. Dokow *et al.* [15] described the characterization of strategyproof deterministic mechanisms on discrete lines and cycles. Moreover, Barberà and Beviá [4] considered the scenario of locating multiple facilities.

Approximate mechanism design for facility location games was first explored by Procaccia and Tennenholtz [31]. In this setting, a facility on a line is to be built based on the reported locations. Each agent aims to minimize her cost which is the distance from herself to the facility. They studied two objective functions, minimizing the sum of the agents' costs and the maximum cost over all the agents. For the former objective function, they established an optimal mechanism which outputs the median position as the facility location. For the latter, a mechanism outputting the leftmost location is 2-approximation and best possible. They also considered the extended versions of locating two facilities and each agent having multiple locations, respectively. Subsequently, Alon *et al.* [1, 2] extended the original model to other networks. The results for the setting of each agent having multiple locations and two facilities to be located were improved in [25, 26].

Different variants on the cost function have been addressed as well. Mei *et al.* [27] defined a happiness factor to illustrate the satisfaction degree of each agent. Filos-Ratsikas *et al.* [18] proposed the model that each agent has two peaks. For the social objective function, Feldman and Wilf [17] discussed the model in which the social objective function is the sum of squares of agents' cost functions. Cai *et al.* [10] studied minimizing the maximum envy value which is the difference between the maximum cost and the minimum cost. In addition, some researchers [36, 37] introduced a property of mechanism called falsenameproofness, which prevents an agent from making many virtual agents in the internet.

For obnoxious facility location games, the first attempt belongs to Cheng *et al.* [12], where they focused on approximation mechanism design on a closed interval. Taking the social objective as maximizing the sum of all the agents' utilities, they designed a mechanism outputting one of the two endpoints which is preferred by more agents and showed that this majority mechanism is strategyproof and 3-approximation. Then, this model is further extended to tree and cycle networks in [13]. Ibara and Nagamochi [22] characterized strategyproof deterministic mechanisms, implying that the majority mechanism is best possible on tree networks.

Very recently, the setting of heterogeneous facility location games attracts much attention, which can be regarded as the mixture of facility location games and obnoxious facility location games. The setting where some agents want to stay close to the facility and the other agents want to stay far away from it was considered in [16, 40]. Serafino and Ventre discussed the model that each agent prefers to one of the two facilities or both of them in [34, 35]. In this model, the cost of each agent is the sum of the distances to the facilities she prefers and the social objective is to minimize the total cost over the agents. Yuan *et al.* [38] and Fong *et al.* [19] proposed a model that each agent has the optional preference for two heterogeneous facilities. And the cost (or utility) of each agent is the minimum (or maximum) distance to the facilities she prefers. Anastasiadis and Deligkas [3] analyzed the strategyproof mechanisms of heterogeneous facility location games. With the social objective of maximizing the minimum utility among the agents, they dealt with the cases where the numbers of agents are one and more, respectively.

In this paper we will take one step forward by studying the mechanism design of facility location games when externalities are considered. There is a large amount of work on externalities in other settings. Haghpanah *et al.* [21] studied the problem of designing auctions in social networks for goods that exhibit single-parameter submodular network externalities in which a bidder's value for an outcome is a fixed private type multiplied by a known submodular function of the allocation of her friends. There also exist another works investigating auction with positive externalities, such as [11, 24, 32]. Auction with negative externalities was studied in [5, 6, 14, 39]. The supply chain problem with positive and negative externalities was discussed in [9, 30]. Brânzei *et al.* [7] and Li *et al.* [23] introduced the externalities to the cake cutting problem.

2 PROBLEM STATEMENTS

In this section, we formally define the two location games we will tackle. In this paper, we mainly investigate the setting where the externalities are non-negative.

2.1 Facility location games

Let $N = \{1, 2, ..., n\}$ be a set of agents. We study the setting where all agents are located on a line. Each agent $i \in N$ has a location $x_i \in \mathbb{R}$, which may be different from the one she reports. We use $\mathbf{x} = (x_1, \ldots, x_n)$ to denote the *location profile*. The *externality* of agent $i \in N$ caused by agent $j \in N$ is denoted by $\alpha_{ij} \geq 0$. Let $\alpha_{ii} = 1$ $\forall i \in N$. we assume that each agent cares for any other agent not so much as herself, which means that $\alpha_{ij} < 1$ for $j \neq i$.

A *deterministic* mechanism in this setting is a function $f : \mathbb{R}^n \mapsto \mathbb{R}$, which maps a given location profile to a point in $\mathbb{R}$, i.e, a facility location. Let a, b be two points in $\mathbb{R}$. We will use $d(a, b)$ to denote the *distance* between a and b.

For a location profile $\mathbf{x} = (x_1, \ldots, x_n)$, if the facility is located at y, we define the *cost* of agent $i \in N$ to be

$$cost_i(y, \mathbf{x}) = \sum_{j=1}^{n} \alpha_{ij} d(y, x_j), \tag{1}$$

reflecting that an agent's cost is affected by the distances from other agents to the facility. As usual, each agent aims to minimize her cost.

A *randomized* mechanism is a function f from $\mathbb{R}^n$ to probability distributions over $\mathbb{R}$. If $f(\mathbf{x}) = P$, where P is a probability distribution on $\mathbb{R}$, the cost of agent $i \in N$ is defined as

$$cost_i(P, \mathbf{x}) = E_{y \sim P} cost_i(y, \mathbf{x}).$$

A mechanism f is *strategyproof* if an agent cannot benefit by reporting a false location, regardless of the strategies of the other agents. In other words, for all $\mathbf{x} \in \mathbb{R}^n$, $i \in N$ and $x'_i \in \mathbb{R}$, we have

$$cost_i(f(\mathbf{x}), \mathbf{x}) \leq cost_i(f(x'_i, \mathbf{x}_{-i}), \mathbf{x}),$$

where $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ is the location profile of all agents in $N \backslash \{i\}$.

We can view the costs with regard to externalities as the communication costs between agents (personal costs). The social goal is neutral with a bigger perspective. With this interpretation, we mainly consider social objectives which are independent of externalities, only on distances.

We consider minimizing *the sum of all the agents' distances to the facility location, i.e.*,

$$sd(y, \mathbf{x}) = \sum_{i=1}^{n} d(y, x_i)$$

and *the maximum agent's distance to the facility location*, denoted by

$$md(y, \mathbf{x}) = \max_{i \in N} d(y, x_i).$$

We call the above objectives *social distances*. Correspondingly, we have two social objective functions related to agent costs, namely *social cost*. One is minimizing the sum of all the agents' costs, formally, $sc(y, \mathbf{x}) = \sum_{i=1}^{n} cost_i(y, \mathbf{x})$; the other one is minimizing the maximum agent's cost $mc(y, \mathbf{x}) = \max_{i \in N} cost_i(y, \mathbf{x})$. Recall that in this paper, we mainly focus on social objectives on distances.

2.2 Obnoxious facility games

In obnoxious facility games, all agents are located on a closed interval, denoted by $I = [0, 1]$. Analogously, let $N = \{1, 2, ..., n\}$ be a set of agents. Again we can define $0 \leq \alpha_{ij} < 1$ and let $\alpha_{ii} = 1$. For a location profile $\mathbf{x} = (x_1, \ldots, x_n)$, if the facility is located at y, the utility of agent i is defined as

$$u_i(y, \mathbf{x}) = \sum_{j=1}^{n} \alpha_{ij} d(y, x_j). \tag{2}$$

Note that, in obnoxious facility games, each agent aims to maximize her utility.

In this game, strategyproofness of a mechanism means that for all $\mathbf{x} \in I^n$, $i \in N$, $x'_i \in I$, $u_i(f(\mathbf{x}), \mathbf{x}) \geq u_i(f(x'_i, \mathbf{x}_{-i}), \mathbf{x})$, where $\mathbf{x}_{-i}$ is the location profile of all agents except agent i.

The social objective is to *maximize the total distance* denoted by

$$st(y, \mathbf{x}) = \sum_{i=1}^{n} d(y, x_i).$$

In this paper, we mainly consider the above objective. There is another social objective which is to maximize the sum of all the agents' utilities. We call it social utility objective.

All α_{ij}'s are in general called externality coefficients, while for a specific i, we call α_{ij} ($j \neq i, 1 \leq j \leq n$) the externality coefficients of agent i. The relationships between agents can be studied by social networks. Thus, we assume that all the externality coefficients are common knowledge.

3 FACILITY LOCATION GAMES WITH EXTERNALITIES

Recall that the externality coefficients are non-negative and for $i \neq j$, $0 \leq \alpha_{ij} < 1 = \alpha_{ii}$. We first study deterministic mechanisms. It is a little bit disappointing that all the strategyproof deterministic mechanisms have unbounded approximation ratios for whatever objectives (social distances or social costs).

We first show the unbounded lower bound for minimizing the sum of distances objective. From the proof, it is easy to see that no deterministic mechanism can have finite lower bound for other objectives.

Theorem 3.1. *For $n \geq 4$, any deterministic strategyproof mechanism cannot have a finite approximation ratio for minimizing the sum of distances.*

Proof. We prove this theorem by contradiction. Assume that there exists a strategyproof deterministic mechanism f with a finite approximation ratio. We consider that the externality coefficients for each agent i ($j \neq i$) satisfies

$$\sum_{j=3}^{n} \alpha_{ij} > 1, \ i = 1, 2;$$

$$\sum_{j=1}^{2} \alpha_{ij} > 1, \ i = 3, \ldots, n;$$

$$\alpha_{ij} = 0, \ otherwise.$$

Namely, one can divide the agents into two groups. The first group consists of the first two agents $\{1, 2\}$, while the remaining $n - 2$

agents form the second group. Any agent in a group cares for the agents as a whole in the other group more than herself.

Let us first look at the following location profile $\mathbf{x}_0 = (0, 0, \dots, 0)$. Clearly, the best location is the point 0 with a cost of zero. Since f has a finite approximation ratio, it must hold that $f(\mathbf{x}_0) = 0$.

Next, we sequentially deal with the following location profiles, for $i = 1, 2, \dots, n$,

$$\mathbf{x}_i = (0, \dots, 0, \underbrace{1, \dots, 1}_{i})$$

Consider $\mathbf{x}_1$. To maintain the strategyproofness, we must ensure that

$$cost_n(f(\mathbf{x}_1), \mathbf{x}_1) \le cost_n(f(\mathbf{x}_0), \mathbf{x}_1) = 1,$$

since otherwise, agent n in location profile $\mathbf{x}_1$ can benefit by misreporting to 0. If $f(\mathbf{x}_1) \neq 0$, then

$$\begin{aligned} & cost_n(f(\mathbf{x}_1), \mathbf{x}_1) \\ = \quad & (\sum_{j=1}^{2} \alpha_{nj}) \cdot d(f(\mathbf{x}_1), 0) + d(f(\mathbf{x}_1), 1) \\ > \quad & d(f(\mathbf{x}_1), 0) + d(f(\mathbf{x}_1), 1) \ge 1, \end{aligned}$$

which causes a contradiction. Therefore $f(\mathbf{x}_1) = f(\mathbf{x}_0) = 0$. Along this way, we can show that $f(\mathbf{x}_{n-2}) = \cdots = f(\mathbf{x}_1) = 0$, until all agents in the second group stay at the point 1.

Then we turn to the location profile $\mathbf{x}_{n-1}$. Due to the strategyproofness of mechanism f, we need to guarantee the following inequalities,

$$cost_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-1}) \ge cost_2(f(\mathbf{x}_{n-1}), \mathbf{x}_{n-1}); \tag{3}$$

$$cost_2(f(\mathbf{x}_{n-1}), \mathbf{x}_{n-2}) \ge cost_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-2}). \tag{4}$$

Otherwise, agent 2 will lie in either of the two profiles $\mathbf{x}_{n-1}$ and $\mathbf{x}_{n-2}$.

Recall that $f(\mathbf{x}_{n-2}) = 0$. With inequality (3), we have $f(\mathbf{x}_{n-1}) \in [0, 2]$, while with inequality (4) we get $f(\mathbf{x}_{n-1}) \le 0$ or $f(\mathbf{x}_{n-1}) > 1$. Combining them together, the location $f(\mathbf{x}_{n-1})$ is either 0 or out of the interval $[0, 1]$.

Finally, let us focus on the location profile $\mathbf{x}_n$. Due to the strategyproofness, no matter where $f(\mathbf{x}_{n-1})$ is, we have

$$f(\mathbf{x}_n) \neq 1,$$

since otherwise, the first agent in location profile $\mathbf{x}_{n-1}$ can achieve her minimum cost by misreporting to location 1.

For location profile $\mathbf{x}_n$, where all agents are at the point 1, the optimal facility location is clearly at 1 with $sd(1, \mathbf{x}_n) = 0$. Unfortunately, however, the location of mechanism f is not at 1 which implies that $sd(f(\mathbf{x}_n), \mathbf{x}_n) > 0$. Hence, it cannot achieve any finite bound. The proof is completed. □

Remark: From the above theorem, it is easy to see that even if the externality coefficients are *symmetric*, i.e., $\alpha_{ij} = \alpha_{ji}$, the approximation ratios of any deterministic strategyproof mechanisms are unbounded either. It is natural to ask the following question: **can randomization help?** A little surprising, the answer remains negative.

Theorem 3.2. *For $n \ge 4$, no randomized strategyproof mechanisms have a bounded approximation ratio for minimizing the sum of distances.*

Proof. The same as the previous theorem, we give a proof by contradiction. In fact, we will use exactly the same profiles as those in the proof of Theorem 3.1.

The treatment for the profiles $\mathbf{x}_i$ for $i = 1, \dots, n-2$ is simple. No matter what probability distribution a randomized mechanism uses, one can show that $f(\mathbf{x}_i) = 0, i = 1, \dots, n-2$. If it is not true, agent $n-i+1$ at the location profile $\mathbf{x}_i$ will benefit by misreporting to 0 to get a fake location profile $\mathbf{x}_{i-1}$, where $i = 1, \dots, n-2$.

Now, we only need to take care of the last two profiles. Note that the cost of agent 2 in the location profile $\mathbf{x}_{n-2}$ is

$$cost_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-2}) = \sum_{j=3}^{n} \alpha_{2j}.$$

Then we consider the location profile $\mathbf{x}_{n-1}$. Let $f(\mathbf{x}_{n-1}) = P_{n-1}$. Since agent 2 in location profile $\mathbf{x}_{n-2}$ will not lie, we know that

$$cost_2(P_{n-1}, \mathbf{x}_{n-2}) \ge cost_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-2}) = \sum_{j=3}^{n} \alpha_{2j}. \tag{5}$$

Similarly, agent 2 in the location profile $\mathbf{x}_{n-1}$ will not lie, resulting in

$$\begin{aligned} cost_2(P_{n-1}, \mathbf{x}_{n-1}) \quad &= \quad (1 + \sum_{j=3}^{n} \alpha_{2j}) E_{y \sim P_{n-1}}[d(y, 1)] \\ &\le \quad cost_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-1}) \\ &= \quad 1 + \sum_{j=3}^{n} \alpha_{2j}, \end{aligned}$$

which implies that

$$E_{y \sim P_{n-1}}[d(y, 1)] \le 1. \tag{6}$$

Now we calculate the cost of agent 1 in the location profile $\mathbf{x}_{n-1}$.

$$\begin{aligned} & cost_1(P_{n-1}, \mathbf{x}_{n-1}) \\ = \quad & E_{y \sim P_{n-1}}[d(y, 0)] + \sum_{j=3}^{n} \alpha_{1j} E_{y \sim P_{n-1}}[d(y, 1)] \\ = \quad & cost_2(P_{n-1}, \mathbf{x}_{n-2}) + \big(\sum_{j=3}^{n} \alpha_{1j} - \sum_{j=3}^{n} \alpha_{2j} \big) E_{y \sim P_{n-1}}[d(y, 1)] \\ \ge \quad & \sum_{j=3}^{n} \alpha_{2j} (1 - E_{y \sim P_{n-1}}[d(y, 1)]) + \sum_{j=3}^{n} \alpha_{1j} \quad (7) \\ \ge \quad & \sum_{j=3}^{n} \alpha_{1j} \; . \quad (8) \end{aligned}$$

Inequality (7) comes from inequality (5). The last inequality holds due to inequality (6).

Finally, we move to the profile $\mathbf{x}_n$. Let $f(\mathbf{x}_n) = P_n$. The cost of agent 1 in location profile $\mathbf{x}_n$ is

$$\begin{aligned} cost_1(P_n, \mathbf{x}_n) &= E_{y\sim P_n}[d(y,1)] + \sum_{j=3}^{n} \alpha_{1j} E_{y\sim P_n}[d(y,1)] \\ &\geq E_{y\sim P_n}[d(y,0)] + \sum_{j=3}^{n} \alpha_{1j} E_{y\sim P_n}[d(y,1)] - 1 \\ &= cost_1(P_n, \mathbf{x}_{n-1}) - 1 \\ &\geq cost_1(P_{n-1}, \mathbf{x}_{n-1}) - 1 \quad (9) \\ &\geq \sum_{j=3}^{n} \alpha_{1j} - 1 \quad (10) \\ &> 0. \end{aligned}$$

Agent 1 cannot lie in profile $\mathbf{x}_{n-1}$ which makes inequality (9) hold. And inequality (10) holds due to inequality (8).

The above inequality implies that $E_{y\sim P'}[d(y,1)] > 0$. Thus, the social objective $sd(P', \mathbf{x}_n) > 0$. However, for location profile $\mathbf{x}_n$, the optimal solution is 0. Hence, the approximation ratio of mechanism f is unbounded. □

Remark: Similarly, we can show that for both social distance and social cost objectives, the lower bounds for any strategyproof randomized mechanisms are not finite either.

We now turn to characterize *sufficient and necessary conditions* for externality coefficients which could make certain mechanisms strategyproof. For any agent $i \in N$, we sort her externality coefficients α_{ij}, for $j \neq i$ in non-descending order. Let the sorted coefficients be $\beta_{ij}, j = 1, 2, \ldots, n-1$ such that $\beta_{i1} \leq \beta_{i2} \leq \cdots \leq \beta_{i(n-1)}$. Moreover, we have the following theorem.

Theorem 3.3. *Given any location profile $\mathbf{x} = (x_1, \ldots, x_n)$ sorted as $x_{j_1} \leq x_{j_2} \leq \cdots \leq x_{j_n}$, mechanism f which outputs $x_{j_k} (k = 1, \ldots, n)$ is strategyproof if and only if for each agent $i \in N$, her externality coefficients satisfy that*

$$\sum_{j=1}^{t} \beta_{ij} + \alpha_{ii} \geq \sum_{j=t+1}^{n-1} \beta_{ij}, \quad (11)$$

where $t = \min\{k-1, n-k\}$.

Proof. **Only if part.** We show this part by contradiction. We assume that there exists an agent $i \in N$ such that $\sum_{j=1}^{t} \beta_{ij} + \alpha_{ii} < \sum_{j=t+1}^{n-1} \beta_{ij}$.

We first deal with $k \leq \frac{n+1}{2}$, *i.e.*, $t = k-1$. We consider the following location profile $\mathbf{x}$ with n agents at locations $0, 1, \ldots, n-1$, respectively. In this location profile agent i is at 0 and the externality coefficient of agent i caused by the agent at location j is $\beta_{ij}, j = 1, \ldots, n-1$.

Now, the facility location of mechanism f is at $k-1$. The cost of agent i is

$$cost_i(k-1, \mathbf{x}) = \alpha_{ii}(k-1) + \sum_{j=1}^{k-1} \beta_{ij}(k-1-j) + \sum_{j=k}^{n-1} \beta_{ij}(j-k+1).$$

If agent i misreports to location n, then mechanism f outputs location k and the cost of agent i is

$$\begin{aligned} cost_i(k, \mathbf{x}) &= \alpha_{ii}k + \sum_{j=1}^{k-1} \beta_{ij}(k-j) + \sum_{j=k}^{n-1} \beta_{ij}(j-k) \\ &= cost_i(k-1, \mathbf{x}) + \alpha_{ii} + \sum_{j=1}^{k-1} \beta_{ij} - \sum_{j=k}^{n-1} \beta_{ij} \\ &< cost_i(k-1, \mathbf{x}), \end{aligned}$$

which implies that mechanism f is not strategyproof.

If $t = n-k$, *i.e.*, $k > \frac{n+1}{2}$, then consider a location profile $\mathbf{x}$ that n agents are at positions $-(n-1), -(n-2), \ldots, 0$, respectively, and agent i is at location 0. The externality coefficient of agent i caused by agent at location $-j$ is $\beta_{ij}, j = 1, 2, \ldots, n-1$.

Mechanism f outputs location $-(n-k)$. The cost of agent i is

$$cost_i(-(n-k), \mathbf{x}) = \sum\nolimits_{j=n-k+1}^{n-1} \beta_{ij}(j-(n-k)) + \sum\nolimits_{j=1}^{n-k} \beta_{ij}(n-k-j) + \alpha_{ii}(n-k).$$

If agent i misreports to location $-n$, then mechanism f returns location $-(n-k+1)$. The cost of agent i is

$$\begin{aligned} cost_i(-(n-k+1), \mathbf{x}) = &\sum_{j=n-k+1}^{n-1} \beta_{ij}(j-(n-k+1)) \\ &+ \sum_{j=1}^{n-k} \beta_{ij}((n-k+1)-j) + \alpha_{ii}(n-k+1) \\ = &\ cost_i(-(n-k), \mathbf{x}) - \sum_{j=n-k+1}^{n-1} \beta_{ij} + \sum_{j=1}^{n-k} \beta_{ij} + \alpha_{ii} \\ < &\ cost_i(-(n-k), \mathbf{x}). \end{aligned}$$

The above inequality implies that mechanism f is not strategyproof.

If part. For any location profile $\mathbf{x} = (x_1, \ldots, x_n)$, without loss of generality, we assume that $x_1 \leq x_2 \leq \cdots \leq x_n$. Hence, mechanism f outputs x_k. We consider the case that agent $i \leq k$ misreports to x_i'. The other case that agent $i > k$ misreports is analogous and the argument is omitted.

If agent i does not misreport, the cost of agent $i \leq k$ is

$$cost_i(x_k, \mathbf{x}) = \sum_{j=1}^{k} \alpha_{ij}(x_k - x_j) + \sum_{j=k+1}^{n} \alpha_{ij}(x_j - x_k).$$

We first deal with the case that agent $i \leq k$ misreports to $x_i' \leq x_k$. For agent $i < k$, in this case mechanism f outputs the same location. Hence, we only need to consider agent k misreports to $x_k' < x_k$. Let $\min\{d(x_{k-1}, x_k), d(x_k', x_k)\} = \delta$. We can see that the facility location of mechanism f is $x_k - \delta$. Moreover, the cost of agent k is

$$\begin{aligned}
cost_k(x_k - \delta, \mathbf{x}) &= \sum_{j=1}^{k-1} \alpha_{kj}(x_k - \delta - x_j) + \alpha_{kk}\delta \\
&\quad + \sum_{j=k+1}^{n} \alpha_{ik}(x_j - x_k + \delta) \\
&= cost_k(x_k, \mathbf{x}) + (\alpha_{kk} + \sum_{j=k+1}^{n} \alpha_{kj} - \sum_{j=1}^{k-1} \alpha_{kj})\delta \\
&\geq cost_k(x_k, \mathbf{x}) + (\alpha_{kk} + \sum_{j=1}^{n-k} \beta_{kj} - \sum_{j=n-k+1}^{n-1} \beta_{kj})\delta \\
&\geq cost_k(x_k, \mathbf{x}) + (\alpha_{kk} + \sum_{j=1}^{t} \beta_{kj} - \sum_{j=t+1}^{n-1} \beta_{kj})\delta \\
&\geq cost_k(x_k, \mathbf{x}).
\end{aligned}$$

Finally, we study the case that agent $i \leq k$ misreports to $x'_i > x_k$. Let $\min\{d(x_k, x_{k+1}), d(x_k, x'_i)\} = \Delta$. Then the facility location of mechanism f is $x_k + \Delta$. The cost of agent $i \leq k$ now is

$$\begin{aligned}
cost_i(x_k + \Delta, \mathbf{x}) &= \sum_{j=1}^{k} \alpha_{ij}(x_k + \Delta - x_j) \\
&\quad + \sum_{j=k+1}^{n} \alpha_{ij}(x_j - x_k - \Delta) \\
&= cost_i(x_k, \mathbf{x}) + (\sum_{j=1}^{k} \alpha_{ij} - \sum_{j=k+1}^{n} \alpha_{ij})\Delta \\
&\geq cost_i(x_k, \mathbf{x}) + (\alpha_{ii} + \sum_{j=1}^{k-1} \beta_{ij} - \sum_{j=k}^{n-1} \beta_{ij})\Delta \\
&\geq cost_i(x_k, \mathbf{x}) + (\alpha_{ii} + \sum_{j=1}^{t} \beta_{ij} - \sum_{j=t+1}^{n-1} \beta_{ij})\Delta \\
&\geq cost_i(x_k, \mathbf{x}).
\end{aligned}$$

□

We analyze the approximation ratios of the mechanism which outputs the k-th location. For minimizing the social distance objective, if $k < \frac{n+1}{2}$, the approximation ratio is $\frac{n-k}{k}$; for $k > \frac{n+1}{2}$, it is $\frac{k-1}{n-k+1}$; otherwise, the mechanism gives the optimal solution. For the maximum distance objective, the mechanism is 2-approximation, which is best possible even for the classic facility location game proposed in [31]. If $k = \lceil \frac{n}{2} \rceil$, we call the mechanism *median mechanism*.

Corollary 3.4. *If inequality (11) holds for some t, then it also holds for $t+1, t+2, \ldots, \lfloor \frac{n-1}{2} \rfloor$.*

Corollary 3.4 implies that if there exists some t which makes inequality (11) hold for each agent, then the median mechanism is strategyproof, which is best possible for both objectives.

Corollary 3.5. *If for each agent i, the total externality coefficients $\sum_{j \neq i} \alpha_{ij} \leq 1$, then inequality (11) holds for every t.*

Remark: Corollary 3.5 means that if every agent's total externality coefficients is less than 1 (everyone cares for all the others totally no more than herself), then there always exists strategyproof mechanisms.

4 OBNOXIOUS FACILITY GAMES WITH EXTERNALITIES

In obnoxious facility games, we also assume that $\alpha_{ij} \in [0, 1)$ for $i \neq j$. Using the similar profiles as in the proof of Theorem 3.1, we have the following theorem.

Theorem 4.1. *For $n \geq 4$, any deterministic strategyproof mechanism cannot achieve a finite approximation ratio for maximizing the total distance.*

Proof. We assume that there exists a strategyproof deterministic mechanism f with a finite approximation ratio. We use the same externality coefficients as those in the proof of Theorem 3.1 to construct profiles. Recall that

$$\sum_{j=3}^{n} \alpha_{ij} > 1, \; i = 1, 2;$$
$$\sum_{j=1}^{2} \alpha_{ij} > 1, \; i = 3, \ldots, n;$$
$$\alpha_{ij} = 0, \; otherwise.$$

Firstly, consider a location profile $\mathbf{x}_0 = (0, \ldots, 0)$. If $f(\mathbf{x}_0) = 0$, then the theorem is proved. Hence we only need to consider $f(\mathbf{x}_0) \neq 0$. Due to the strategyproofness , we can construct a location profile, which causes the approximation ratio of mechanism f to be unbounded.

Similar to the proof of Theorem 3.1, we consider the following location profiles sequentially for $i = 1, 2, \ldots, n$.

$$\mathbf{x}_i = (0, \ldots, 0, \underbrace{f(\mathbf{x}_0), \ldots, f(\mathbf{x}_0)}_{i})$$

Firstly, we consider location profile $\mathbf{x}_1$. We have

$$u_n(f(\mathbf{x}_0), \mathbf{x}_0) = (\sum_{j=1}^{2} \alpha_{nj} + 1) f(\mathbf{x}_0) \geq u_n(f(\mathbf{x}_1), \mathbf{x}_0);$$

$$u_n(f(\mathbf{x}_1), \mathbf{x}_1) \geq u_n(f(\mathbf{x}_0), \mathbf{x}_1) = \sum_{j=1}^{2} \alpha_{nj} f(\mathbf{x}_0).$$

The above inequalities hold since otherwise agent n can misreport either from 0 to $f(\mathbf{x}_0)$ or from $f(\mathbf{x}_0)$ to 0 and benefits.

According to the above two inequalities, we can see that $f(\mathbf{x}_1) = f(\mathbf{x}_0)$. Using the analogous analysis, we can show that

$$f(\mathbf{x}_{n-2}) = \cdots = f(\mathbf{x}_1) = f(\mathbf{x}_0).$$

In profile $\mathbf{x}_{n-2}$ (agent 1 and 2 are at 0, the remaining agents are at $f(\mathbf{x}_0)$), the utility of agent 2 is

$$u_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-2}) = f(\mathbf{x}_0).$$

Then we investigate location profile $\mathbf{x}_{n-1}$. Since agent 2 in location profile $\mathbf{x}_{n-2}$ cannot benefit by lying, we get that

$$u_2(f(\mathbf{x}_{n-1}), \mathbf{x}_{n-2}) \leq u_2(f(\mathbf{x}_{n-2}), \mathbf{x}_{n-2}) = f(\mathbf{x}_0),$$

which implies that $f(\mathbf{x}_{n-1}) = f(\mathbf{x}_0)$.

Finally, we deal with location profile $\mathbf{x}_n$. Similarly, we can verify that $f(\mathbf{x}_n) = f(\mathbf{x}_0)$. The social objective of mechanism f for profile $\mathbf{x}_n$ is 0 and the optimal solution is $\max\{nf(\mathbf{x}), n-nf(\mathbf{x})\} > 0$, which implies that the approximation ratio of mechanism f is unbounded. This completes the proof. □

Remark: Similarly, the proof of the above theorem also suggests that the lower bound is also unbounded for maximizing the social utility objective.

Similar to facility location games, we also try to characterize the *sufficient and necessary conditions* for the externality coefficients so that previous mechanisms in the literature are strategyproof. In classic obnoxious facility games, Cheng *et al.* [13] proposed the following mechanism, called *majority mechanism*. We show that the majority mechanism is strategyproof if and only if externalities do not exist, which implies that if externalities exist then the majority mechanism is not strategyproof.

Mechanism 1 (Majority mechanism). *Given a location profile $\mathbf{x} \in I^n$, let L denote the set of agents in $[0, \frac{1}{2}]$ and R be the set of agents in $(\frac{1}{2}, 1]$, respectively. Output $f(\mathbf{x}) = 0$ if $|L| \le |R|$ and otherwise output* 1.

Theorem 4.2. *For $n \ge 2$, the majority mechanism is strategyproof if and only if externalities do not exist.*

Proof. **Only if part**. We show this part by contradiction. Assume that there exists an agent $i \in N$ such that $\sum_{j \ne i} \alpha_{ij} > 0$. We sort the externality coefficients of agent i in non-descending order, denoted by $\beta_{ij}, j = 1, \dots, n-1$.

We consider a location profile $\mathbf{x}$ where $\lceil \frac{n}{2} \rceil$ agents are at location $\frac{1}{2}$ and $\lfloor \frac{n}{2} \rfloor$ agents are at location 1. In location profile $\mathbf{x}$, agent i is at location $\frac{1}{2}$. Moreover the externality coefficients of agent i caused by the agents at $\frac{1}{2}$ are $\beta_{ij}, j = 1, \dots, \lceil \frac{n}{2} \rceil - 1$. The remaining externality coefficients caused by agents at 1 are $\beta_{ij}, j = \lceil \frac{n}{2} \rceil, \dots, n-1$.

Let f be the majority mechanism. We can see that $f(\mathbf{x}) = 1$. The utility of agent i is

$$u_i(1, \mathbf{x}) = \frac{1}{2}(\sum_{j=1}^{\lceil \frac{n}{2} \rceil - 1} \beta_{ij} + 1).$$

If agent i misreports to 1, the majority mechanism outputs location 0. Now, the utility of agent i is

$$u_i(0, \mathbf{x}) = \frac{1}{2}(\sum_{j=1}^{\lceil \frac{n}{2} \rceil - 1} \beta_{ij} + 1) + \sum_{j=\lceil \frac{n}{2} \rceil}^{n-1} \beta_{ij}.$$

Since for agent i, $\sum_{j=1}^{n-1} \beta_{ij} > 0$ and $\beta_{i1} \le \dots \le \beta_{i(n-1)}$, we verify that

$$u_i(0, \mathbf{x}) > u_i(1, \mathbf{x}).$$

Hence, we show that the majority mechanism is not strategyproof.

If part. This part is to show that the majority mechanism is strategyproof for classic obnoxious facility games, which was proved in [13]. To be self-contained, we give a brief proof. Since for any agent $i \in N$, $\sum_{j \ne i} \alpha_{ij} = 0$ and $\alpha_{ij} \ge 0$, each agent gets the maximum utility at an endpoint which has larger distance to herself.

Without loss of generality, we assume that $|L| \le |R|$ ($|L| > |R|$ is similar), *i.e.*, the majority mechanism outputs 0. It is obvious that agents at $[\frac{1}{2}, 1]$ will not misreport.

Now we consider agent i at $[0, \frac{1}{2})$ misreports to $x_i' \in I$. Wherever x_i' is, the number of agents at R will not decrease, which implies that the majority mechanism will still output 1. □

On the other hand, we can re-interpret majority mechanism as outputting *the location the most agents prefer*, under which the classic model remains the same, while the model with externalities is totally different. Then we consider the adapted majority mechanism.

For a location profile $\mathbf{x}$, if $u_i(0, \mathbf{x}) \ge u_i(1, \mathbf{x})$ then we say agent i *prefers* 0 to 1 in location profile $\mathbf{x}$; otherwise we say agent i prefers 1 to 0. Let $N_0(\mathbf{x})$ be the set of agents who prefer 0 to 1 and $N_1(x)$ be the remaining agents.The adapted majority mechanism can be expressed as outputting 0 if $|N_0(\mathbf{x})| \ge |N_1(\mathbf{x})|$, otherwise outputting 1.

We give a sufficient condition for externality coefficients so that the adapted majority mechanism is not strategyproof.

Theorem 4.3. *If one agent has externalities and the other agents have no externalities, the adapted majority mechanism is not strategyproof.*

Proof. Suppose that agent $k \in N$ has externalities. Let $\beta_{k1}, \dots, \beta_{k(n-1)}$ be the externality coefficients of agent k with non-descending order.

We consider a location profile with $\frac{n-1}{2}$ agents at location $\frac{1}{2} - \epsilon$ and the remaining agents at $\frac{1}{2} + \epsilon$, where n is odd and $\epsilon > 0$ is an extremely small number. Agent k is at $\frac{1}{2} + \epsilon$. The externality coefficients of agent k caused by agents at $\frac{1}{2} - \epsilon$ are $\beta_{k(n-1)}, \beta_{k1}, \beta_{k2}, \dots, \beta_{k\frac{n-3}{2}}$, respectively. The remaining externality coefficients caused by agents at $\frac{1}{2} - \epsilon$ are $\beta_{k\frac{n-1}{2}}, \dots, \beta_{k(n-2)}$.

If the facility is at location 0, the utility of agent k is

$$\begin{aligned} u_k(0, \mathbf{x}) &= \frac{1}{2}(\sum_{j=1}^{n-1} \beta_{kj} + 1) \\ &+ \epsilon\Big(\sum_{j=\frac{n-1}{2}}^{n-2} \beta_{kj} - \sum_{j=1}^{\frac{n-3}{2}} \beta_{kj} + 1 - \beta_{k(n-1)}\Big). \end{aligned}$$

Correspondingly, if the facility is at 1, the utility of agent k is

$$\begin{aligned} u_k(1, \mathbf{x}) &= \frac{1}{2}(\sum_{j=1}^{n-1} \beta_{kj} + 1) \\ &- \epsilon\big(\sum_{j=\frac{n-1}{2}}^{n-2} \beta_{kj} - \sum_{j=1}^{\frac{n-3}{2}} \beta_{kj} + 1 - \beta_{k(n-1)}\big). \end{aligned}$$

It is easy to see that $u_k(0, \mathbf{x}) > u_k(1, \mathbf{x})$. Hence, $|N_0(\mathbf{x})| = \frac{n+1}{2}$, and the adapted majority mechanism outputs 0.

Then we construct a new location profile $\mathbf{x}'$ from location profile $\mathbf{x}$, where the agent with externality $\beta_{k(n-1)}$ moves from $\frac{1}{2} - \epsilon$ to 0 and other agents remain at the same locations. Denote the moved agent as agent i. If the facility is at 0, the utility of agent k in $\mathbf{x}'$ is

$$u_k(0, \mathbf{x}') = \frac{1}{2}\sum_{j=1}^{n-2}(\beta_{kj}+1) + \epsilon(\sum_{j=\frac{n-1}{2}}^{n-2}\beta_{kj} - \sum_{j=1}^{\frac{n-3}{2}}\beta_{kj}+1).$$

Similarly, if the facility is at 1, the utility is

$$\begin{aligned} u_k(1, \mathbf{x}') &= \frac{1}{2}\sum_{j=1}^{n-2}(\beta_{kj}+1) + \beta_{k(n-1)} \\ &- \epsilon(\sum_{j=\frac{n-1}{2}}^{n-2}\beta_{kj} - \sum_{j=1}^{\frac{n-3}{2}}\beta_{kj}+1). \end{aligned}$$

Set an appropriate $\epsilon > 0$ such that $u_k(1, \mathbf{x}') > u_k(0, \mathbf{x}')$. Thus, for profile $\mathbf{x}'$, $N_1(\mathbf{x}') = \frac{n+1}{2}$, which implies that the adapted majority mechanism outputs 1. It is easy to see that agent i can misreport from $\frac{1}{2} - \epsilon$ to 0 and benefits. □

Randomized mechanism. The optimal solution is either at 0 or at 1. It is natural to establish a randomized mechanism which only has positive probability outputting the two endpoints.

Consider a trivial randomized mechanism which outputs two endpoints with probability $\frac{1}{2}$, respectively. The mechanism does not take advantage of the agents' locations, therefore it is strategyproof. It is easy to see that this mechanism is 2-approximation.

If we consider randomized mechanisms with positive probabilities only at two endpoints, we can show that the above randomized mechanism is best possible.

THEOREM 4.4. *For $n \geq 4$, any randomized strategyproof mechanism which only has positive probabilities at two endpoints is at least 2-approximation for maximizing the total distance.*

PROOF. We prove this theorem by contradiction. Assume that a randomized mechanism f with positive probabilities only at two endpoints has approximation ratio less than 2. We use the same externality coefficients and location profiles in the proof of Theorem 3.1. Recall that the coefficients are

$$\begin{aligned} &\sum_{j=3}^{n}\alpha_{ij} > 1, \ i = 1, 2; \\ &\sum_{j=1}^{2}\alpha_{ij} > 1, \ i = 3, \ldots, n; \\ &\alpha_{ij} = 0, \ otherwise. \end{aligned}$$

And the location profiles are

$$\mathbf{x}_i = (0, \ldots, 0, \underbrace{1, \ldots, 1}_{i}), \ i = 0, 1, \ldots, n.$$

Let $P_i, \ i = 0, 1, \ldots, n$ be the probability distribution output by mechanism f for location profile $\mathbf{x}_i$. Formally, $f(\mathbf{x}_i) = P_i, i = 0, 1, \ldots, n$ which outputs location 0 with probability p_i and location 1 with probability $1 - p_i$.

Consider $\mathbf{x}_0$. Note that the approximation ratio is less than 2. The optimal solution is n while the mechanism solution is $st(P_0, \mathbf{x}_0) = (1 - p_0)n$, which implies that

$$p_0 < \frac{1}{2}.$$

We then consider location profile $\mathbf{x}_1$. Due to the strategyproofness of mechanism f, agent n will not lie in profile $\mathbf{x}_1$. Thus, we can get

$$u_n(P_1, \mathbf{x}_1) \geq u_n(P_0, \mathbf{x}_1).$$

and $u_n(P_i, \mathbf{x}_1) = \sum_{j=1}^{2}\alpha_{nj} + (1 - \sum_{j=1}^{2}\alpha_{nj})p_i, \ i = 0, 1.$

From the above, we can obtain that $p_1 \leq p_0 < \frac{1}{2}$.

Similarly, we can conclude that $p_{n-2} \leq \ldots \leq p_1 \leq p_0 < \frac{1}{2}$.

Next, we will show that $p_{n-1} \leq p_{n-2} < \frac{1}{2}$. Since agent 2 cannot lie in profile $\mathbf{x}_{n-1}$, we can establish the following inequality,

$$u_2(P_{n-1}, \mathbf{x}_{n-1}) \geq u_2(P_{n-2}, \mathbf{x}_{n-1}). \tag{12}$$

The utilities of agent 2 on probability P_{n-2} and P_{n-1} for location profile $\mathbf{x}_{n-1}$ are

$$u_2(P_i, \mathbf{x}_{n-1}) = (1 - p_i)(1 + \sum_{j=3}^{n}\alpha_{2j}), \ i = n-2, n-1. \tag{13}$$

With inequality (12) and (13), we can get that

$$p_{n-1} \leq p_{n-2}.$$

By analogous analysis, we can show that $p_n \leq p_{n-1}$. From all the above, we can conclude that

$$p_{n-1} \leq p_{n-2} \leq \ldots \leq p_1 \leq p_0 < \frac{1}{2}.$$

Observe that in location profile $\mathbf{x}_n$ all the agents are at location 1. The optimal solution is n, while the mechanism solution is $st(P_n, \mathbf{x}_n) = n \cdot p_n < \frac{n}{2}$, which implies the approximation ratio of mechanism f is bigger than 2.

□

Note that the proof with a little change can also be used to show a lower bound for maximizing social utility objective, with the restricted randomized mechanisms.

5 CONCLUSIONS

In this paper, we for the first time introduced externalities into facility location games. We focused on the case when externality coefficients are non-negative. On the negative side, we proved that any strategyproof mechanisms have unbounded approximation ratios if the externality coefficient $\alpha_{ij} \in (0, 1)$ for facility location games with both social distance and social cost objectives. For obnoxious facility games, we showed that any strategyproof deterministic mechanisms have unbounded approximation ratios for both maximizing the total distance and social utility objectives. On the positive side, we characterized *sufficient and necessary conditions* for externality coefficients so that well known existing mechanisms are still strategyproof, including the median mechanism and the majority mechanism. Exploring more results along both directions will be of interest as future work to enrich the study of externality for facility location games. We are also planning to study the implication of negative externalities.

REFERENCES

[1] Naga Alon, Michal Feldman, Ariel D. Procaccia, and Moshe Tennenholtz. 2010. Strategyproof approximation mechanisms for location on networks. (2010). arXiv preprint arXiv:0907.2049.

[2] Naga Alon, Michal Feldman, Ariel D. Procaccia, and Moshe Tennenholtz. 2010. Strategyproof approximation of the minimax on networks. *Mathematics of Operations Research* 35, 3 (2010), 513–526.

[3] Eleftherios Anastasiadis and Argyrios Deligkas. 2018. Heterogeneous facility location games. In *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. 623–631.

[4] Salvador Barberà and Carmen Beviá. 2006. Locating public facilities by majority: Stability, consistency and group formation. *Games and Economic Behavior* 56, 1 (2006), 185–200.

[5] Alexandre Belloni, Changrong Deng, and Saša Pekeč. 2017. Mechanism and network design with private negative externalities. *Operations Research* 65, 3 (2017), 577–594.

[6] Sayan Bhattacharya, Janardhan Kulkarni, Kamesh Munagala, and Xiaoming Xu. 2011. On allocations with negative externalities. In *Proceedings of the 7th International Workshop on Internet and Network Economics (WINE)*. 25–36.

[7] Simina Brânzei, Ariel D. Procaccia, and Jie Zhang. 2013. Externalities in cake cutting. In *Proceedings of the 23rd International Joint Conference on Artificial Intelligence (IJCAI)*. 55–61.

[8] James M. Buchanan and William C. Stubblebine. 1962. Externality. *Economica* 29, 116 (1962), 371–384.

[9] Gérard P. Cachon. 1999. Competitive supply chain inventory management. In *Quantitative Models for Supply Chain Management*. 111–146.

[10] Qingpeng Cai, Aris Filos-Ratsikas, and Pingzhong Tang. 2016. Facility location with minimax envy. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence (IJCAI)*. 137–143.

[11] Shuchi Chawla, Hu Fu, and Anna Karlin. 2014. Approximate revenue maximization in interdependent value settings. In *Proceedings of the 15th ACM Conference on Economics and Computation (EC)*. 277–294.

[12] Yukun Cheng, Wei Yu, and Guochuan Zhang. 2011. Mechanisms for obnoxious facility game on a path. In *Proceedings of the 5th International Conference on Combinatorial Optimization and Applications (COCOA)*. 262–271.

[13] Yukun Cheng, Wei Yu, and Guochuan Zhang. 2013. Strategy-proof approximation mechanisms for an obnoxious facility game on networks. *Theoretical Computer Science* 497 (2013), 154–163.

[14] Changrong Deng and Sasa Pekec. 2011. Money for nothing: exploiting negative externalities. In *Proceedings of the 12th ACM conference on Electronic Commerce (EC)*. 361–370.

[15] Elad Dokow, Michal Feldman, Reshef Meir, and Ilan Nehama. 2012. Mechanism design on discrete lines and cycles. In *Proceedings of the 13th ACM Conference on Electronic Commerce (EC)*. 423–440.

[16] Itai Feigenbaum and Jay Sethuraman. 2015. Strategyproof mechanisms for one-dimensional hybrid and obnoxious facility location models. In *Workshops at the 29th AAAI Conference on Artificial Intelligence*.

[17] Michal Feldman and Yoav Wilf. 2013. Strategyproof facility lacation and the least squares objective. In *Proceedings of the 14th ACM Conference on Electronic Commerce (EC)*. 873–890.

[18] Aris Filos-Ratsikas, Minming Li, Jie Zhang, and Qiang Zhang. 2017. Facility location with double-peaked preferences. *Autonomous Agents and Multi-Agent Systems* 31, 6 (2017), 1209–1235.

[19] Ken C.K. Fong, Minming Li, Pinyan Lu, Taiki Todo, and Makoto Yokoo. 2018. Facility location games with fractional preferences. In *Proceedings of the 32th AAAI Conference on Artificial Intelligence (AAAI)*. 1039–1046.

[20] Dimitris Fotakis and Christos Tzamos. 2014. On the power of deterministic mechanisms for facility location games. *ACM Transactions on Economics and Computation* 2, 4 (2014), 15:1–15:37.

[21] Nima Haghpanah, Nicole Immorlica, Vahab Mirrokni, and Kamesh Munagala. 2013. Optimal auctions with positive network externalities. *ACM Transactions on Economics and Computation* 1, 2 (2013), 13.

[22] Ken Ibara and Hiroshi Nagamochi. 2012. Characterizing mechanisms in obnoxious facility game. In *Proceedings of the 6th Annual International Conference on Combinatorial Optimization and Applications (COCOA)*. 301–311.

[23] Minming Li, Jialin Zhang, and Qiang Zhang. 2015. Truthful cake cutting mechanisms with externalities: Do not make them care for others too much!. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence (IJCAI)*. 589–595.

[24] Yunan Li. 2017. Approximation in mechanism design with interdependent values. *Games and Economic Behavior* 103 (2017), 225–253.

[25] Pinyan Lu, Xiaorui Sun, Yajun Wang, and Zeyuan Allen Zhu. 2010. Asymptotically optimal strategy-proof mechanisms for two-facility games. In *Proceedings of the 11th ACM Conference on Electronic Eommerce (EC)*. 315–324.

[26] Pinyan Lu, Yajun Wang, and Yuan Zhou. 2009. Tighter bounds for facility games. In *Proceedings of the 5th International Workshop on Internet and Network Economics (WINE)*. 137–148.

[27] Lili Mei, Minming Li, Deshi Ye, and Guochuan Zhang. 2016. Strategy-proof mechanism design for facility location games: Revisited (extended abstract). In *Proceedings of the 15th International Conference on Autonomous Agents and Multi-agent System (AAMAS)*. 1463–1464.

[28] Eiichi Miyagawa. 2001. Locating libraries on a street. *Social Choice and Welfare* 18, 3 (2001), 527–541.

[29] Hervé Moulin. 1980. On strategy-proofness and single peakedness. *Public Choice* 35, 4 (1980), 437–455.

[30] Serguei Netessine and Fuqiang Zhang. 2005. Positive vs. negative externalities in inventory management: Implications for supply chain design. *Manufacturing & Service Operations Management* 7, 1 (2005), 58–73.

[31] Ariel D. Procaccia and Moshe Tennenholtz. 2009. Approximate mechanism design without money. In *Proceedings of the 10th ACM Conference on Electronic Eommerce (EC)*. 177–186.

[32] Tim Roughgarden and Inbal Talgam-Cohen. 2013. Optimal and near-optimal mechanism design with interdependent values. In *Proceedings of the 14th ACM conference on Electronic Commerce (EC)*. 767–784.

[33] James Schummer and Rakesh V. Vohra. 2002. Strategy-proof location on a network. *Journal of Economic Theory* 104, 2 (2002), 405–428.

[34] Paolo Serafino and Carmine Ventre. 2014. Heterogeneous facility location without money on the line. In *Proceedings of the 21st European Conference on Artificial Intelligence (ECAI)*. 807–812.

[35] Paolo Serafino and Carmine Ventre. 2015. Truthful mechanisms without money for non-utilitarian heterogeneous facility location. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI)*. 1029–1035.

[36] Akihisa Sonoda, Taiki Todo, and Makoto Yokoo. 2016. False-name-proof locations of two facilities: Economic and algorithmic approaches. In *Proceedings of the 30th AAAI Conference on Artificial Intelligence (AAAI)*. 615–621.

[37] Taiki Todo, Atsushi Iwasaki, and Makoto Yokoo. 2011. False-name-proof mechanism design without money. In *Proceedings of the 10th International Conference on Autonomous Agents and Multi-agent System (AAMAS)*. 651–658.

[38] Hongning Yuan, Kai Wang, Ken C.K. Fong, Yong Zhang, and Minming Li. 2016. Facility location games with optional preference. In *Proceedings of the 23nd European Conference on Artificial Intelligence (ECAI)*. 1520–1527.

[39] Chaoli Zhang, Xiang Wang, Fan Wu, and Xiaohui Bei. 2018. Efficient auctions with identity-dependent negative externalities. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS)*. 2156–2158.

[40] Shaokun Zou and Minming Li. 2015. Facility location games with dual preference. In *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. 615–623.