

Forecast-Based Mechanisms for Demand Response

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ABSTRACT

We study mechanisms to incentivize demand response in smart energy systems. We assume agents that can respond (reduce their demand) with some probability if they prepare prior to the realization of the demand. Both preparation and response incur costs to agents. Previous work studies truthful mechanisms that select a minimal set of agents to prepare and respond such that a fixed demand reduction target is achieved with high probability. In this work we additionally consider the balancing responsibility of a retailer under a given demand forecast and imbalance price: the retailer is responsible to purchase additional reserve capacity at a high imbalance price to cover any excess in the demand. In this extended setting we study mechanisms that request only a subset of prepared agents to respond since the reduction target depends on the realization of the demand. We propose: (i) a sequential mechanism that in each round embeds a second-price auction and is truthful under some mild assumptions for the setting, and (ii) a truthful combinatorial mechanism that runs in polynomial time and uses VCG payments. We show that both mechanisms guarantee non-negative utility in expectation for both agents and the retailer (mechanism), and can further be used for simultaneous downward and upward flexibility. Last, we verify our theoretical findings in an empirical evaluation over a wide range of mechanism parameters.

KEYWORDS

mechanism design; demand response; demand forecast

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1 INTRODUCTION

There are two important electricity markets that facilitate commerce of electricity between energy producers and consumers: *day-ahead* and *balancing* markets [1]. Retailers (aggregators), based on demand forecasts, procure electricity in day-ahead markets to satisfy the demand of their consumers. Imbalances between the procured quantities and the actual (intra-day) demand are moderated in balancing markets, in which the reserve power of high-cost storage units and conventional fast-ramping generators, e.g., gas-turbines, is traded. As a result, imbalances result in increasing costs for retailers and they are further associated with excessive CO_2 emissions [5]. Maintaining balance between supply and demand is

therefore one of the main factors that determine the efficiency of both existing and envisioned *smart grid* systems [15].

To this end, *demand-side management* assumes that electricity users can alter their demand or generation behavior given economic incentives (e.g., smart tariffs, dynamic pricing), and therefore assist in reducing imbalances between supply and demand [15]. In practice, retailers currently agree upon long-term contracts to incentivize large-capacity users to reduce their demand if necessary [8]. The introduction of smaller-scale flexible users, such as intelligent home appliances, electric vehicles (EVs) and home batteries, can further alleviate retailers from costs related to balancing supply and demand [7]. However, the uncertain availability of such users requires more flexible contracts, such as short-term bilateral agreements that can be agreed upon and executed if necessary in a day-to-day manner.

We consider the following setting: a retailer based on a demand forecast procures electricity in the day-ahead market to satisfy the demand of its consumers. Since the demand is not certain, there is no guarantee that the actual demand (at the delivery time) is equal to the procured quantity. As a result, any imbalance between the procured quantity and the demand at the time of delivery should be adjusted in the balancing market with a much higher price than the procurement price. We consider agents that can reduce imbalances, after the realization of the demand (i.e., when demand is known but not finalized) and before the time of delivery, if requested by the retailer. Agents decide whether to prepare with some cost before the realization of the demand; prepared agents are able (with some probability) to respond if requested after the realization of the demand. Agents' responses can be observed and incur extra costs to agents. The following example illustrates an instantiation of the model of agents used in our setting:

Example 1.1. Consider a neighborhood with multiple EVs that are parked and plugged into charging stations. Some of the vehicles may be fully charged, while others may be charging. In case of excess demand fully charged vehicles can be utilized to provide the extra needed electricity out of their battery, while vehicles that undergo charging can pause their charging. Each vehicle has a preparation cost, which is the opportunity/planning cost caused by extending its stay in a charging station. The probability of response refers to the uncertain availability of a vehicle to reduce its demand upon request. Last, the response cost is associated to the operating cost of response, such as the cost of battery degradation.

In this setting we design mechanisms to incentivize uncertain demand response, i.e., agents that their availability to reduce demand is not certain (see Example 1.1) while additionally considering the balancing responsibility of a retailer under a given demand forecast and imbalance price. More specifically, given the demand forecast,

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the imbalance price and the characteristics of agents (i.e., preparation cost, response probability and response cost), we design mechanisms that: (i) elicit truthful information with regards to the characteristics of agents, (ii) select a subset of agents to prepare, (iii) do not require all prepared agents to respond but only upon request (i.e., until imbalance is resolved), (iv) determine the order that prepared agents are asked to respond, (v) compute rewards and penalties for selected agents in order to incentivize them to prepare and respond (if able) if requested, and (vi) reduce the expected balancing cost of the retailer and overall increase social welfare.

The main contributions of this work can be summarized as follows:

- We study implications that arise by dependencies between agents that are requested to respond sequentially.
- We propose a sequential mechanism that in each round embeds a second-price auction and is truthful under some assumptions for the setting, and a truthful combinatorial mechanism that runs in polynomial time and uses VCG payments.
- We show that both mechanisms guarantee non-negative utility in expectation for both agents and the mechanism, and can further be used for simultaneous downward and upward flexibility.
- We empirically evaluate the proposed mechanisms over a wide range of parameters and find that they achieve up to 16% reduction in the balancing cost of the retailer and up to 14% increase in social welfare compared to the case when no demand response is used. Last, we provide an evaluation of related work [12] in our extended setting.

To the best of our knowledge, this work is the first to propose mechanisms that connect incentives for uncertain demand response with the balancing responsibility of the retailer, given the demand forecast of the consumers and the imbalance price for the retailer.

2 RELATED WORK

In the closest state-of-the-art work, Ma et al. [12] propose mechanisms to incentivize reliable demand reduction in electricity grids. The proposed mechanisms, that are based on greedy allocation and critical value payments [9], achieve a fixed demand reduction target with some reliability while minimizing the number of selected agents (see Sec. 5 for a detailed description). Our work differentiates from the aforementioned work in the following ways: (i) Ma et al. [12] use a fixed demand reduction target. On the contrary, we propose mechanisms to allocate demand response when the demand reduction target is not known and only the demand forecast (distribution) is used to select agents. (ii) Ma et al. [12] propose a two-stage setting: in the first stage, a set of agents is selected, and in the second stage, all selected agents are asked to respond. In contrast, we consider mechanisms that request prepared agents to respond until an imbalance between supply and demand is resolved (or can not be reduced further), and thus the probability of requesting selected agents to respond is less or equal to one. (iii) Ma et al. [12] do not consider the actual need for demand response (fixed demand reduction target). In this paper, however, we consider that agents are requested to respond based on the realization of the demand, which can prevent excessive costs for demand response. Last, (iv) Ma et al. [12] evaluate their proposed mechanisms with

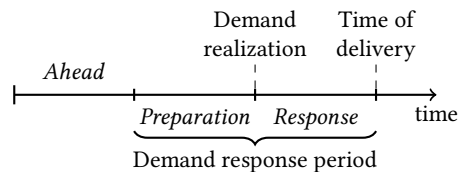


Figure 1: Timeline of the model formulation.

regards to the resulting payments to agents. On the contrary, we consider both the expected balancing cost for the retailer and the social cost of demand response, and thus the overall social welfare.

The aforementioned work by Ma et al. [12] considers unit responses by agents (i.e., each agent can reduce one unit of demand), while a later extension of this work generalizes to multi-unit responses, uncertainties in preparation costs of agents, as well as a multi-effort probability of response [11]. Other related work studies demand response contracts under uncertainty, where the reserve cost for the retailer (i.e., cost for not reaching the reduction target) is considered [4, 17]. However, no prior work has studied mechanisms to incentivize uncertain demand response given the additional information of the demand forecast, as we do in this paper.

Aside from mechanisms that encourage demand reduction in settings of the smart grid, a related line of research studies the design of electricity tariffs that incentivize uncertainty reduction in the demand-side by alleviating the risk of imbalances from the retailers using risk-sharing [13], or prediction-of-use tariffs [20]. The formation of virtual power plants using scoring rules is yet another solution concept for matching volatile supply and demand [18]. However, none of these works models demand response considering its explicit costs and uncertain availability.

3 PROBLEM FORMULATION

In this section we outline our problem setting: Section 3.1 formulates the balancing responsibility of the retailer, Section 3.2 introduces demand response agents and Section 3.3 illustrates how demand response is used by the retailer.

3.1 Retailer’s Balancing Responsibility

We consider a single retailer of electricity that is the balancing responsible party. The demand of the retailer’s portfolio of consumers is described by the discrete random variable X . The demand forecast $f_X(x) = \mathbb{P}_X(X = x)$ is the probability mass function (PMF) of X . We denote with $x \in \mathbb{Z}_0^D$ the realization of the demand, where $x \sim X$ and D is the upper bound of the support of X .

Consider the timeline in Figure 1. Similarly to previous work [13], we consider that the retailer procures the quantity $b \in \mathbb{Z}^+$ at unit price $p \in \mathbb{R}^+$ ahead of the realization of the demand, during the *ahead* period, in the day-ahead market. When no demand response is used, the retailer pays any positive imbalance between the demand realization x and the procured quantity b at unit price $p' \in \mathbb{R}^+$ (*imbalance price*) at the time of delivery in the balancing market. We assume that $p' > p$, and that the prices p and p' are determined by an exogenous process and can not be influenced by the retailer (*price-taker*). We further assume that the procurement quantity b is predetermined (see Assumption 1 in Sec. 4) and we focus on the

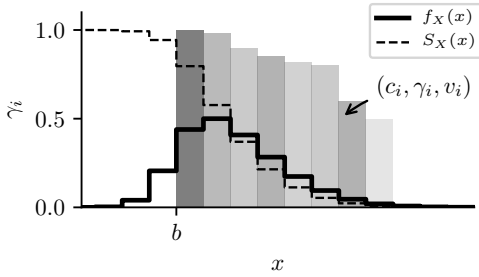


Figure 2: Demand response model: bars represent prepared agents prior to demand realization. For demand realization $x > b$, the retailer requests prepared agents to respond sequentially (from left to right) until the positive imbalance $(x - b)$ is resolved. The ordering of the agents is an example ordering with regards to the response probability.

expected balancing cost of the retailer:

$$C_{-DR} = p' \mathbb{E}_X[x - b | x > b], \quad (1)$$

where only positive imbalance from the procured quantity incurs a balancing cost to the retailer.

In practice, both demand excess (positive imbalance) and shortages (negative imbalance) result in balancing costs for retailers. However, to align our model with related work [12], we first consider the expected positive imbalance (see Eq. 1). In Section 4.4 we generalize to the case where both positive and negative imbalances incur balancing cost to the retailer.

REMARK 1. Our choice for a discrete model, where both the demand x and the procurement b are discrete variables, is motivated by markets that have trading volumes that are multiples of a unit quantity (as it is usual in day-ahead and balancing electricity markets) or markets with discrete items.¹

3.2 Demand Response Agents

We consider agents that are flexible and can reduce their demand and consequently any positive imbalance between the procured quantity and the realized demand, after demand realization and before the time of delivery, during the *response* period (see Fig.1).²

Let $A = \{0, 1, \dots, n-1\}$ denote the finite set of demand response agents. Let also $d_i \in \{-1, +1\}$, $\forall i \in A$ denote the flexibility of agent i , i.e., for $d_i = -1$ agent i can reduce its demand by one unit, while for $d_i = +1$ agent i can increase its demand by one unit. We assume unit downward flexibility, i.e., $d_i = -1, \forall i \in A$. Later in this paper (see Section 4.4) we also consider the case of both downward and upward unit flexibility where $d_i \in \{-1, +1\}$.

We follow previous work to define the timing of demand response and additional characteristics of agents with regards to

¹Our model can be generalized to continuous variables if we neglect the need for decimal reduction, or if there is a min. price to participate in the balancing market.

²Demand response retail market programs take place in short time periods (e.g., 15-min) and are based on time-ahead “realization” of the demand [22], i.e., when demand is very close to the real value. Real-time imbalances are not handled by demand response agents since this requires time (notification and response), but instead by automatic generation control or spinning reserves.

their ability to respond [12]. The type of agent i , θ_i , is the triplet (c_i, γ_i, v_i) . Prior to demand realization and during the *preparation* period (see Fig.1), agent i decides whether to prepare with preparation cost $c_i \geq 0$. After demand realization and during the *response* period, if agent i is prepared, it is able with probability $\gamma_i \in (0, 1]$ to respond. If agent i is able to respond, it can decide either to respond with response cost $v_i \geq 0$, or not without any cost. The decision of agent i to prepare and the ability to respond can not be observed, while the response can be observed.

3.3 Demand Response Model

We proceed to discuss how demand response agents can be used by the retailer. Consider a set of agents that decide to prepare prior to demand realization, and demand realization $x > b$, i.e., positive imbalance. During the response period (see Fig.1), the retailer requests prepared agents to respond (reduce their demand) in some order; each agent that responds reduces the imbalance quantity $(x - b)$ by one unit. The response of an agent can be observed before the next agent is asked to respond. If the imbalance is resolved (enough agents have responded), the retailer stops requesting agents to respond. Otherwise, if imbalance is not resolved (there are no more prepared agents to respond), the retailer pays the remaining imbalance quantity with price p' at the time of delivery.

Example 3.1. Figure 2 presents the demand forecast $f_X(x)$, the dashed curve illustrates the survival function $S_X(x) = \mathbb{P}_X(X > x)$. Bars of different color intensity (darker means higher total cost for demand response) represent prepared (prior to demand realization) agents starting from the procured quantity of the retailer b . The height of the bars show the probability that agent i is able to respond, γ_i , and bars’ width show the quantity that each agent can reduce its demand. For demand realization $x > b$, the retailer requests sequentially (from left to right) prepared agents to respond until imbalance $(x - b)$ is zero or no more agents can be requested.

In contrast to related work that does not consider the realization of the demand [11, 12], in the following sections we design mechanisms that request agents to respond only if there is a positive imbalance; therefore, the probability that a selected agent is requested to respond is not equal to one but it is influenced by: (i) the demand forecast $f_X(x)$, (ii) the order in which is asked to respond, and (iii) the response probabilities of preceding agents.

4 DEMAND RESPONSE MECHANISM M

In this section we first define the general mechanism M in which selected agents are asked to respond sequentially to reduce a positive imbalance. In Section 4.1 we compute the probability that agents are requested to respond, the expected utility of both agents and the retailer (mechanism), and we analyze dependencies between agents that arise in our setting. Sections 4.2 and 4.3 outline our proposed mechanisms. Last, Section 4.4 generalizes our proposed mechanisms to the case where both positive and negative imbalances incur balancing cost to the retailer.

Recall that p' is the imbalance price, X is the random variable of the demand, b is the procurement quantity, and θ_i is the type of agent i . We define the general mechanism $M(X, b, p', \hat{\theta}) \rightarrow (s_i, o_i, r_i, t_i), \forall i \in A$, in which all available agents report their types

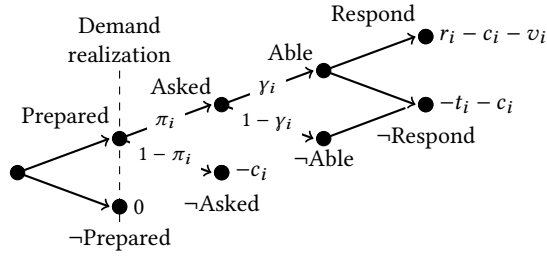


Figure 3: Schematic sequence of agent’s i decisions (preparation and response), stochastic transitions (the probability of which is placed midway in arrows), and costs (illustrated on the right side of end nodes). Reward r_i is transferred from the mechanism to agent i in case of response, penalty t_i is paid to the mechanism otherwise.

$\hat{\theta} = \{\hat{\theta}_0, \hat{\theta}_1, \dots, \hat{\theta}_{n-1}\}$ ($\hat{\theta}_i$ is the reported type of agent i) to the mechanism M during the preparation period (see Fig.1).

ASSUMPTION 1. *The retailer does not have access to the available flexibility (reports $\hat{\theta}$) during the ahead period, and thus the procurement quantity b is already determined before the preparation period.*

The quadruplet (s_i, o_i, r_i, t_i) is the resulting allocation for agent i , where $s_i \in \{0, 1\}$ denotes the selection of agent i , $o_i \in \mathbb{Z}_0^{n-1}$ the order in which agent i may be requested to respond by the mechanism, $r_i \geq 0$ the reward that is transferred from the mechanism to agent i in case agent i responds after is requested by the mechanism, and $t_i \geq 0$ the penalty that is paid to the mechanism if agent i does not respond after request. Payments from and towards the mechanism take place after the realization of the demand and after observing the response of agents if requested by the mechanism (*contingent payments*).

Consider agent i that is selected by the mechanism ($s_i = 1, o_i \in \mathbb{Z}_0^{n-1}$) with reward r_i and penalty t_i . Figure 3 illustrates the general mechanism M , where π_i denotes the probability that agent i is requested to respond by the mechanism (see Sec. 4.1.1). With knowledge of r_i and t_i , agent i decides whether to prepare prior to the realization of the demand (preparation period). During the response period, the mechanism asks agent i to respond with probability π_i . If agent i is able to respond (with probability γ_i), it decides whether to respond during the response period.

4.1 Request Probability & Interdependencies

In this section we compute the probability π_i with which mechanism M requests agent i to respond. We further compute the utilities of both agents and the retailer (mechanism) under mechanism M . Last, we study implications that arise by dependencies between agents that are requested to respond sequentially.

4.1.1 Probability of Response Request. To compute the probability π_i we consider that agents report their true types θ . Consider agent i with unit flexibility (recall that $d_i = -1, \forall i \in A$) and type $\theta_i = (c_i, \gamma_i, v_i)$. We assume w.l.o.g. that $s_i = 1, o_i = i, \forall i \in A$, i.e., all agents are selected and the order that are requested to respond follows the indexing of agents. We further assume that all agents prepare and respond if requested (agent i is able to respond

with probability γ_i). Let $a_i \in \{0, 1\}$ denote the observed action of agent i after request, it is equal to one in case of response and zero otherwise.

$$\pi_i = \begin{cases} S_X(b), & i = 0 \\ S_X(b+i) + \sum_{k=0}^{i-1} f_X(b+k+1) \mathbb{P}\left(\sum_{j=0}^{i-1} a_j \leq k\right), & i > 0 \end{cases} \quad (2)$$

is the probability of response request from the mechanism to agent i , where $\mathbb{P}(\sum_j a_j \leq k)$ is the probability that less than or equal to k agents respond from the agents preceding i , i.e., $\forall j < i$. For $i = 0$, π_i is equal to the probability that demand is larger than b , i.e., the first agent in the order is always asked to respond if there is positive imbalance. For $i > 0$, Eq. (2) further accounts for failures (inability to respond) of agents preceding i in case $x < (b+i+1)$.

The quantity $\sum_j a_j$ in Eq. (2) is the sum of independent Bernoulli variables and follows a Poisson binomial distribution [23]. The probability $\mathbb{P}(\sum_j a_j \leq k)$ is the cumulative distribution function of a Poisson binomial distribution for k successes.

$$\mathbb{P}\left(\sum_j a_j \leq k\right) = \sum_{l=0}^k \sum_{L \in F_l} \prod_{q \in L} \gamma_q \prod_{m \in L^c} (1 - \gamma_m), \quad (3)$$

where for each number of successes $l \in [0, k]$, F_l contains all sets of size l in the powerset of A and L^c is the complementary set of L , i.e., $L \cup L^c = A$. For experiments presented later in this paper (see Sec. 5), we compute the probability in Eq. (3) using a closed-form expression based on the discrete Fourier transform of the characteristic function of the distribution [6].

4.1.2 Utilities under Mechanism M . Let C_{DR} denote the expected cost for the mechanism M (expected balancing cost plus the cost for demand response) under allocation $(s_i, o_i, r_i, t_i), \forall i \in A$. To derive the cost C_{DR} , we assume that agents report their true types θ and w.l.o.g. that $s_i = 1, o_i = i, \forall i \in A$. We further assume that all selected agents prepare and respond if requested.

$$C_{DR} = \sum_{i \in A} \pi_i \gamma_i r_i - \pi_i (1 - \gamma_i) t_i + p' \sum_{x=b+1}^D f_X(x) \sum_{k=0}^{\min\{x-b-1, n\}} \mathbb{P}\left(\sum_{i \in A} a_i = k\right) (x - b - k), \quad (4)$$

where the first term is equal to the expected payments towards and from the agents. The expected payment to agent i is equal to: $\pi_i \gamma_i r_i$, and the expected income from agent i : $\pi_i (1 - \gamma_i) t_i$. The second term of Eq. (4) computes the expected balancing cost for $x \in [b+1, D]$ and $k \in [0, \min\{x-b-1, n\}]$ (k is the number of responses) and it considers only the cases in which there is a remaining imbalance (thus all agents have been requested to respond). Note that, there is no imbalance for $k \geq (x-b)$, and k can not be greater than the number of agents ($k \leq n$).

We define the (retailer’s) expected utility of the mechanism U_M as the difference between the expected balancing cost when no demand response is used (see Eq. 1) and the expected cost for mechanism M (see Eq. 4).

$$U_M = C_{\neg DR} - C_{DR}. \quad (5)$$

where the utility of the mechanism depends on the cost C_{DR} that is influenced by the allocation. Note that C_{-DR} depends only on the procurement quantity (see Eq. 1 and Assumption 1).

Similarly, given reward r_i and penalty t_i the expected utility of agent i is:

$$u_i = \pi_i \gamma_i (r_i - v_i) - \pi_i (1 - \gamma_i) t_i - c_i, \quad (6)$$

where π_i is the probability of response request in Eq. (2). Agent i pays c_i to prepare. If agent i is asked and able to respond, pays v_i and gets reward r_i . Otherwise, if agent i is asked and can not respond, agent i pays t_i to the mechanism.

Definition 4.1. A mechanism is called *dominant-strategy incentive compatible* (DSIC), or truthful, if no agent can increase its utility by misreporting its type to the mechanism, and *individually rational* (IR) if agents get non-negative utility in expectation (i.e., agents are willing to participate). Furthermore, a mechanism is called individual rational for the center (CR) if the center's (mechanism) expected utility is non-negative [16].³

4.1.3 Interdependent Tasks with Uncertain Executions. As discussed earlier in this paper, the allocation is determined by both the selection of agent i and the order that the mechanism requests agent i to respond. We showed that the latter influences the probability of response request to agent i (see Sec. 4.1.1) and therefore tasks (each order in the allocation) are interdependent. The valuation of agent i for a given allocation depends on the probabilities that preceding agents are able to respond. However, there exist no efficient mechanism (in the class of Groves mechanisms) that satisfies DSIC, IR and CR when there are interdependencies between tasks with uncertain executions [2, 16, 24].

Given the above impossibility result, in Sections 4.2 and 4.3 we design mechanisms that select agents to perform demand response by removing dependencies between tasks and satisfy all DSIC, IR and CR properties.

4.2 Sequential-Task Mechanism

In this section we define the sequential mechanism Seq_M , which selects agents for each order in the allocation sequentially.

4.2.1 Minimum Acceptable Reward. Given the expected utility of agent i in Eq. (6), we define the *minimum acceptable reward* for agent i , \hat{r}_i : the minimum reward for which it is rational for agent i to prepare prior to demand realization (during the preparation period) and respond if it is able with probability γ_i during the response period. The minimum acceptable reward \hat{r}_i (based on report $\hat{\theta}_i$) that yields positive expected utility for agent i is:

$$r_i \geq \frac{\pi_i (1 - \hat{\gamma}_i) t_i + \hat{c}_i}{\pi_i \hat{\gamma}_i} + \hat{v}_i \triangleq \hat{r}_i, \quad (7)$$

where \hat{r}_i is the lower bound of the reward r_i . We further set an upper bound for the reward r_i , it should not be larger or equal to the imbalance price p' that the retailer pays in case agent i does not respond (and no other agent responds), $r_i < p'$.

³We adopt the notion of center's rationality (CR) considering the utility of the mechanism for the retailer; equivalently, we can say that mechanism M satisfies CR if it is IR for the retailer to adopt mechanism M .

4.2.2 Mechanism Seq_M . We define the mechanism Seq_M as follows: $Seq_M(X, b, p', \hat{\theta}, T) \rightarrow (s_i, o_i, r_i, t_i = T), \forall i \in A$, where $T \geq 0$ is a fixed penalty. Let A' denote the set of available agents (i.e., agents that are not yet selected) and κ the order in which the mechanism requests an agent to respond. Initially, $A' = A$, $\kappa = 0$, and $s_i = 0, \forall i \in A$. We detail the steps of Seq_M below:

- (1) Collect reports from all available agents, $\hat{\theta}_i, \forall i \in A'$.
- (2) Compute $\hat{r}_i, \forall i \in A'$ for $o_i = \kappa$ as in Eq. (7); π_i is computed with regards to previously selected agents as in Eq. (2).
- (3) Consider in order κ agent $w = \arg \min_{i \in A'} \hat{r}_i$ (lowest \hat{r}_i), and reward $r_w = \min_{i \neq w \in A'} \hat{r}_i$ (second lowest \hat{r}_i).
- (4) For $r_w < p'$, select agent w , i.e., ($s_w = 1, o_w = \kappa, r_w = \min_{i \neq w \in A'} \hat{r}_i, t_w = T$), remove agent w from the set of available agents, $A' = A' - w$. Then, go to step (1) and increase the order κ by one, $\kappa = \kappa + 1$. If $r_w \geq p'$, stop without selecting agent w ($s_w = 0$).

We consider that Seq_M takes place during the preparation period (see Fig. 1). At each round, the computed reward and the fixed penalty is communicated to the selected agent that decides whether to prepare before the demand realization and respond if it is requested. After demand realization and in case of positive imbalance, Seq_M requests agents to respond sequentially according to the order that they are selected until imbalance is zero, or there are no more agents to respond. If agent i is asked to respond, Seq_M pays r_i to the agent in case of response, and receives penalty t_i otherwise.

4.2.3 Incentives and Truthfulness of Seq_M . We proceed to discuss agents' incentives to report truthfully and participate in Seq_M , and further show that Seq_M can only benefit by selecting agents to perform demand response.

ASSUMPTION 2. Agents do not have access to: (i) the reports of other agents $\hat{\theta}_{-i}$, the number of participating agents, and the distribution of agents' types (no communication), (ii) the reward that is communicated to the selected agent after each round of the mechanism (no price discovery), and (iii) the demand forecast of the retailer.

THEOREM 4.2. Given Assumption 2, Seq_M is DSIC and IR.

PROOF. We first show that the mechanism is IR for the agents. For report $\hat{\theta}_i$, the minimum acceptable reward \hat{r}_i as it is computed in Eq. (7) yields zero expected utility for agent i , $u_i = 0$. Any reward $r_i \geq \hat{r}_i$ yields positive utility $u_i \geq 0$. Let \hat{r}_j be the second lowest minimum acceptable reward. It holds by definition that $\hat{r}_j \geq \hat{r}_i$ and consequently $u_i \geq 0$. Therefore, it is rational for selected agent i to prepare and respond if requested.

We proceed to show that a selected agent can not improve its utility by misreporting to the mechanism. Given Assumption 2 and the definition of Seq_M , each round of the mechanism is an isolated Vickrey (second-price) auction [19], since there is no information propagating to the next rounds, i.e., *no externalities*.⁴ In each round, agents deterministically choose to report truthfully to the mechanism since any round can be the last round in which they can obtain positive expected utility with reward lower than p' . \square

Note that, given Theorem 4.2, π_i is computed in step (2) of Seq_M (see Sec. 4.2.2) based on truthful reports.

⁴See [10] for sequential mechanisms with externalities.

PROPOSITION 4.3. For any fixed penalty $T \geq 0$, Seq_M is CR.

PROOF. Note that, rewards are lower than p' and response is requested only if there is an imbalance that otherwise has to be paid with price p' . It follows that the utility of Seq_M (see Sec. 4.1.2) is always non-negative. \square

4.3 Independent-Task Mechanism

Recall that the probability of response request to agent i , π_i , depends on the response probabilities of preceding agents in the allocation. In this section we design a truthful combinatorial mechanism by removing dependencies between selected agents.

4.3.1 *Mechanism Ind_M* . We define the mechanism Ind_M , which requests agents to respond as follows:

Definition 4.4. For demand realization $x > b$, Ind_M requests all agents up to order $\lambda = x - b - 1$ to respond.

Intuitively, the mechanism Ind_M asks agents up to a specific order, that corresponds to the imbalance quantity, to respond. It follows that, the probability that agent i is asked to respond in Eq. (2) is independent of preceding agents ($o_j < o_i$) and $\pi_i = S_X(b + o_i)$.

Similarly to Seq_M (see Sec. 4.2.2), Ind_M fixes a penalty, $t_i = T, \forall i \in A$. In addition, Ind_M fixes a reward $R, r_i = R, \forall i \in A$, which is the reward that the mechanism pays agent i after response. Mechanism Ind_M can be written as: $Ind_M(X, b, p', \hat{\theta}, R, T) \rightarrow (s_i, o_i, r_i = R, t_i = T, z_i), \forall i \in A$, where z_i is a payment from agent i to the mechanism upon allocation. Each selected agent i gets reward $r_i = R$ if it is requested and responds, and pays penalty $t_i = T$ in case of no response.

Unlike Seq_M that may request all selected agents to respond to resolve some imbalance quantity, failing to respond under Ind_M yields balancing cost p' to the mechanism, since all selected agents have to respond to resolve any positive imbalance. This means that under Ind_M , fixing $T = p'$ "transfers" the balancing responsibility to selected agents.

4.3.2 *Optimal Allocation & VCG Payments*. Given reward R , penalty T , and considering $z_i = 0$, the expected utility of agent i for order o_i and based on the report $\hat{\theta}_i$ is:

$$\hat{u}_i(o_i) = \pi_i \hat{\gamma}_i (R - \hat{v}_i) - \pi_i (1 - \hat{\gamma}_i) T - \hat{c}_i, \quad (8)$$

where $\pi_i = S_X(b + o_i)$ (see Def. 4.4). We define the optimal allocation o^{opt} (assignment of each agent to an order) such that:

$$o^{opt} = \arg \max_{o \in O} \sum_{i \in A} \max(0, \hat{u}_i(o_i^{opt})), \quad (9)$$

where O is the set of all permutations. The term $\max(0, \hat{u}_i(o_i^{opt}))$ ensures that no agent is selected with negative expected utility, therefore $s_i = 0$, if $\hat{u}_i(o_i^{opt}) \leq 0$.

LEMMA 4.5. The problem of finding the optimal allocation o^{opt} in Eq. (9) can be solved in polynomial time $O(n^3)$.

PROOF. When the probability of response request π_i is independent of previously allocated agents, the optimal allocation problem can be formulated as the *linear assignment problem* (LAP), where n agents are assigned n tasks. In our setting, each task stands for an order $o_i \in \mathbb{Z}_0^{n-1}$. It can be solved optimally in polynomial time,

$O(n^3)$, by the *Hungarian method* [21], while there exist implementations that can speed up further the computation of the optimal assignment in LAPs [3]. \square

To simplify notation in the remainder of this section, we define $u'_i(o_i^{opt}) \triangleq \max(0, \hat{u}_i(o_i^{opt}))$. We use the VCG payment rule to compute the payment of agent i to the mechanism upon allocation:

$$z_i = \left(\sum_{j \in A \setminus i} u'_j(o_j^{opt-i}) - \sum_{j \in A \setminus i} u'_j(o_j^{opt}) \right), \quad (10)$$

where o^{opt-i} is the optimal allocation without agent i present [14]. Intuitively, z_i is equal to the difference between the sum of utilities that agents other than i get without agent i present, and the sum of utilities they get under its presence (marginal loss). Note that, agents that are not selected cause zero marginal loss to other agents.

We consider that Ind_M takes place during the preparation period (see Fig. 1), where each selected agent i pays z_i upon allocation. After demand realization, Ind_M requests selected agents up to the order λ . Selected agents in the set $\{j : o_j \leq \lambda\}$ are asked to respond, Seq_M pays R to agents that respond, and receives penalty T otherwise.

THEOREM 4.6. The mechanism Ind_M is DSIC and IR.

PROOF. First, it follows from Lemma 4.5 that o^{opt} maximizes the sum of agents' utilities. By definition of the VCG mechanism [14], Ind_M is DSIC and IR. Agent i maximizes its utility by reporting truthfully, $\hat{\theta}_i = \theta_i$, and it is rational for agent i to prepare and respond if requested under reward R and penalty T . \square

PROPOSITION 4.7. For any $R \leq p'$ and $T \geq 0$, Ind_M is CR.

PROOF. For any $R \leq p'$ and $T \geq 0$, no allocation can yield losses for the mechanism (see proof of Proposition 4.3). \square

4.4 General Flexibility Mechanisms

In previous sections we have considered a setting where only positive imbalance from the procured quantity b results in balancing cost for the retailer (see Sec. 3.1). In this section we show that mechanisms Seq_M and Ind_M can generalize in settings where both positive and negative imbalances result in balancing costs.

In addition to the set of downward flexibility agents A (also denoted by A^- in the remainder of this paper), we consider the set of upward flexibility agents A^+ , with reports $\hat{\theta}^+ = \{(\hat{c}_i, \hat{\gamma}_i, \hat{v}_i) : \forall i \in A^+\}$ and $d_i = +1, \forall i \in A^+$. Both Seq_M and Ind_M can be applied on the set A^+ , independently from the set A^- , under the same imbalance price p' or a different price for negative imbalances, with only a small adjustment of the request probability π_i . For Seq_M , the probability that agent i is asked to respond π_i for $i > 0$ in Eq. (2) becomes:

$$\pi_i = F_X(b - i - 1) + \sum_{k=0}^{i-1} f_X(b - k - 1) \mathbb{P}\left(\sum_{j=0}^{i-1} a_j \leq k\right), \quad (11)$$

where $F_X(x) = 1 - S_X(x) = \mathbb{P}_X(X \leq x)$. For $i = 0$, Eq. (2) becomes $\pi_i = F_X(b - 1)$. Similarly, for Ind_M , $\pi_i = F_X(b - o_i - 1)$.

Both Seq_M and Ind_M hold their properties (DSIC, IR, and CR) since they are applied independently on different sets of agents.

5 EXPERIMENTAL EVALUATION

In this section we empirically evaluate the performance of the proposed mechanisms Seq_M and Ind_M . We also provide an evaluation of the mechanisms proposed by Ma et al. [12] (in our extended setting), which we detail below:

Fixed-Reward/Penalty Ma et al. Mechanisms. Fixed-reward Ma et al. mechanism fixes a reward R , a reduction target Z and a reliability target τ . For agent reports $\hat{\theta}$ (reports are the same as in Sec. 4), it computes the maximum penalty that each agent is willing to pay such that the agent retains non-negative utility (similarly to Eq. (7) that computes the minimum acceptable reward). Then, the mechanism sorts agents in a decreasing order with regards to their maximum willingness to pay, and selects the minimum number of agents such that: $\mathbb{P}(\sum a \geq Z) \geq \tau$, i.e., the probability of reaching the reduction target Z is higher than the reliability target τ . The mechanism sets penalty (in case of unsuccessful response) for each selected agent i that is equal to the smallest willingness to pay from the set of agents that would have been selected without agent i present. Intuitively, that is the lowest willingness to pay for agent i to get selected. After allocation, the mechanism asks all selected agents to respond. Similarly, fixed-penalty Ma et al. mechanism fixes penalty T and computes the minimum acceptable reward (as in Eq. (7) for $\pi_i = 1$) [11]. Both fixed-reward and fixed-penalty mechanisms make a *deep market* assumption, i.e., there are enough agents in the market (economy) to fulfill the requirements with regards to the reduction target and the reliability.

Fixed-reward and fixed-penalty Ma et al. mechanisms do not consider the balancing responsibility of the retailer after the realization of the demand. Hence, the following sections do not serve as a direct comparison of the mechanisms proposed in this paper and the mechanisms proposed by Ma et al. [12]; instead, they focus on the added value of information considered by our mechanisms (i.e., demand forecast, imbalance price), and the advantage of only requesting agents to respond after the realization of the demand.

Experimental Setup. We consider a market with $n = 200$ agents. Each agent has preparation cost $c_i \sim \mathcal{U}[0, p']$, response probability $\gamma_i \sim \mathcal{U}[0.5, 1]$, and response cost $v_i \sim \mathcal{U}[0, p' - c_i]$, where $\mathcal{U}[\alpha, \beta]$ denotes a uniform distribution from α to β . Note that $c_i + v_i \leq p'$, i.e., the sum of the costs for preparation and response is lower or equal to the imbalance price p' , which is a relevant assumption for the setting. For the demand distribution we use a discretized skew normal distribution $\mathcal{N}(\mu_X, \sigma_X, \alpha_X)$, where $\mu_X = 500$, $\sigma_X = 100$, $\alpha_X = 10$ (e.g., see Fig. 2). The procurement quantity is set to $b = \mathbb{E}_X[x]$, and $p' = 0.6$. Our results are averaged over 200 independent runs where the demand distribution is fixed.

Mechanism Parameters. For Seq_M , we use fixed penalty $T \in \{0.1, 0.2, \dots, 1.0\} \times p'$. For Ind_M , we use $R \in \{0.1, 0.2, \dots, 0.9\} \times p'$ and $T \in \{0, 0.5, 1\} \times p'$. Furthermore, for fixed-reward Ma et al. mechanism, we use $R \in \{0.4, 0.5, \dots, 2.0\} \times p'$ (for $R < 0.4 \times p'$, negative penalties are induced to selected agents by the mechanism). For fixed-penalty Ma et al. mechanism, we use $T \in \{0.0, 0.1, \dots, 2.0\} \times p'$. Last, for both fixed-reward and fixed-penalty Ma et al. mechanisms, we use reduction target $Z \in \{0.1, 0.2, \dots, 1.0\} \times \mathbb{E}_X[x - b | x > b]$ and reliability $\tau = 0.95$.

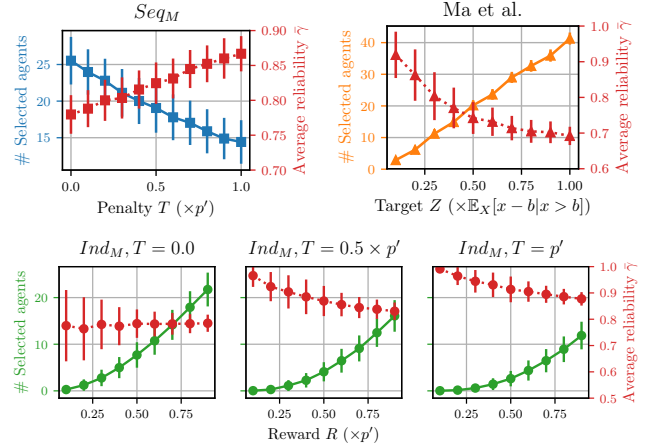


Figure 4: Mean and standard deviation of the number and the average reliability of selected agents. Continuous lines correspond to values on left vertical axes and dotted lines to right vertical axes. Seq_M mechanism (top left), Ind_M mechanism (bottom row), and Ma et al. mechanisms (top right).

Number of Selected Agents & Average Reliability. Figure 4 shows the number and the average reliability $\bar{\gamma}$ of selected agents under Seq_M , Ind_M , and Ma et al. mechanisms. The number of selected agents in both Ma et al. mechanisms is influenced by the reduction target Z and the reliability target τ , and not by the fixed-reward R and fixed-penalty T . For Seq_M and Ind_M , reward R and penalty T affect the number of selected agents. For $T = 0$, Seq_M selects approximately 25 agents, this corresponds to a reduction target $Z = 0.6 \times \mathbb{E}_X[x - b | x > b]$ for Ma et al. mechanisms. For $T = p'$, Seq_M selects on average 15 agents. The number of selected agents for Ind_M is lower than for Seq_M since Ind_M asks agents up to the quantity of the imbalance to respond (see Definition 4.4); Ind_M does not count for possible failures of agents that otherwise would increase the probabilities of response requests and consequently select more agents. As anticipated, the average reliability $\bar{\gamma}$ is influenced by the reward R and penalty T parameters. For higher penalty T , fewer agents with higher reliability are selected by our proposed mechanisms.

Social Welfare & Balancing Cost. Figure 5 illustrates the utility space of the mechanism (retailer) U_M and the agents $U_A = \sum_i u_i$, on the horizontal and the vertical axis respectively. The star marker shows the case when no demand response is used, and thus the mechanism pays positive imbalances with price p' . For every drawn set of agents out of 200 independent runs, we compute the analytical expected utility under each mechanism based on Eq. (4). For all mechanisms, the solid color marker shows the point where the utility of agents (U_A) is maximum, and the solid marker with black colored borders shows the point where social welfare ($U_M + U_A$) is maximum. The parameters used for the mechanisms are shown in parentheses, where target Z is multiplied with the expected positive imbalance $\mathbb{E}_X[x - b | x > b]$ and R, T with the imbalance price p' . Transparent markers show points in the utility space for parameters that are not shown in the figure.

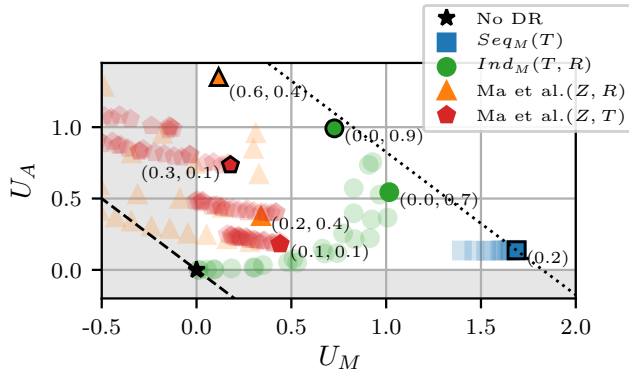


Figure 5: Expected utilities of the mechanism U_M and the agents U_A under all mechanisms for a wide range of parameters. For Ma et al. mechanisms, the reduction target Z is shown with regards to the expected imbalances $\mathbb{E}_X[x - b|x > b]$, and for all mechanisms reward R and penalty T with regards to the imbalance price p' .

The shaded area illustrates the utility space where either the mechanism, the agents, or both have negative utility in expectation (when compared to the case of no demand response). In comparison to Ma et al. mechanisms that only consider incentives for the agents (satisfy IR for participating agents), both Seq_M and Ind_M guarantee non-negative expected utility for both agents and the mechanism (both satisfy IR and CR) since they consider both the demand forecast and the balancing cost of the mechanism.

Next, we evaluate all mechanisms with regards to the utility of the mechanism U_M and the social welfare ($U_M + U_A$). Parallel lines in Figure 5 illustrate points of equal social welfare, the dashed line for the case of no demand response, and the dotted line for the maximum social welfare under both our proposed mechanisms (almost equal): $Seq_M(T = 0.2)$ and $Ind_M(T = 0, R = 0.9p')$. When compared to the case of no demand response, the expected social welfare increases by 14% for $Seq_M(T = 0.2p')$, 13% for $Ind_M(T = 0, R = 0.9p')$, 11% for fixed-reward Ma et al. ($Z = 0.6 \times \mathbb{E}_X[x - b|x > b], R = 0.4p'$), and last, 6% for Ma et al. with fixed penalty ($Z = 0.3 \times \mathbb{E}_X[x - b|x > b], T = 0.1p'$). The utility of the mechanism increases (i.e., expected balancing cost decreases) by: 13% for $Seq_M(T = 0)$, 7% for $Ind_M(T = 0, R = 0.7p')$ and 2 ~ 3% for both fixed-reward and fixed-penalty Ma et al. mechanisms. Compared to Ma et al. mechanisms in this extended setting, Seq_M and Ind_M improve both social welfare and the utility of the mechanism since they request agents to respond only if there is positive imbalance.

Simultaneous Upward & Downward Flexibility. Last, we show that both Seq_M and Ind_M reduce balancing costs substantially for the retailer in the case where both positive and negative imbalances from the procured quantity incur balancing cost to the retailer.

We consider that any absolute deviation from the procurement quantity ($b = \mathbb{E}_X[x]$) is balanced with price p' . As described in Sec. 4.4, Seq_M and Ind_M can be used to allocate both upward and downward flexibility agents. We draw equal number of both types of agents, $|A^-| = |A^+| = 200$. For Seq_M , we use $T = 0$. For Ind_M ,

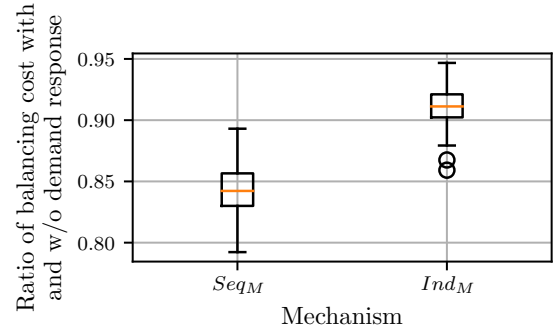


Figure 6: Ratio of expected balancing cost (absolute imbalance case) for the mechanism with and without the use of demand response (lower is better).

$R = 0.6p'$ and $T = 0$. We keep the distribution of agent types and the demand distribution same as those of earlier experiments. Figure 6 presents the ratio of the expected balancing cost for Seq_M and Ind_M with and without demand response (C_{DR}/C_{-DR}). On average, $Seq_M(T = 0)$ mechanism achieves a 16% reduction in the balancing cost of the mechanism, while $Ind_M(R = 0.6p', T = 0)$ yields 9% reduction.

6 CONCLUSIONS

In this paper we studied a highly relevant problem in energy systems: how to incentivize uncertain demand response under a given demand forecast and imbalance price. We proposed two mechanisms: a sequential mechanism (Seq_M) that is truthful under some mild assumptions (see Th. 4.2), and a truthful combinatorial mechanism (Ind_M) that runs in polynomial time and uses VCG payments (see Th. 4.6). Both mechanisms require only a subset of selected agents to respond, while they guarantee non-negative utility in expectation for both agents and the retailer (mechanism). The proposed mechanisms can further be used in settings where both positive and negative imbalances result in balancing cost for the retailer. Last, we verified the theoretical properties of both mechanisms in an empirical evaluation over different parameters. Our proposed mechanisms achieved up to 16% reduction in the balancing cost of the retailer and 14% increase in social welfare compared to when no demand response is used.

With regards to future extensions of this work, it is of interest to study how reports for demand response can influence the procurement decision of the retailer (e.g., by relaxing Assumption 1). Last, future work may further consider agents with continuous responses as a challenging generalization of our proposed setting.

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