Multi-unit Budget Feasible Mechanisms for Cellular Traffic Offloading*

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ABSTRACT

Cellular traffic offloading is nowadays an important problem in mobile networking. Since the offloading resource owners (agents) are self-interested and have private costs, it is highly challenging to design procurement mechanisms that motivate agents to reveal their true costs and achieve guaranteed performance under the constraint of a strict budget. In this paper, we model cellular traffic offloading as a multi-unit budget feasible procurement auction design problem with diminishing return valuations. We design a novel greedy-based randomized mechanism, and prove it is budget-feasible, truthful, individually rational and a $(3 + 2 \ln N)$-approximation, where N is the total number of available resource units. We also propose a deterministic mechanism which achieves $(2 + \ln N + \sqrt{2 + 3 \ln N + \ln^2 N})$ - approximation. We prove no budget-feasible and truthful mechanism can do better than $\ln N$-approximation in our setting, thus our mechanism approaches the optimal to a constant factor. In addition to solving the cellular traffic offloading problem, our work successfully extends solvable valuation class of greedy-based multi-unit budget-feasible mechanism with performance guarantees from the concave-additive valuations to more general local diminishing return valuations.

KEYWORDS

Auctions; Algorithmic mechanism design; Budget feasible mechanisms; Multi-unit procurement auctions

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1 INTRODUCTION

With the proliferation of smart mobile devices and applications, the global mobile data traffic keeps rapidly growing. According to a white paper recently updated by Cisco [8], global mobile data traffic grew 63% in 2016, and it will continue to increase another 7-fold between 2016 and 2021. Such a massive amount of mobile traffic would certainly deteriorate existing cellular networks' service quality if the traffic is not handled properly.

Compared with updating the cellular network's infrastructure or building more towers, offloading part of cellular traffic through existing alternative wireless networks such as femtocells, WiFi, etc. eases the burden of cellular networks and enhance users' experience in a timely and economical manner [3, 10, 20, 21], and has already been widely applied in practice. According to Cisco [8], 60% of total mobile data traffic was offloaded onto the fixed network through Wi-Fi or femtocell in 2016. In total, 10.7 exabytes of mobile data traffic were offloaded onto the fixed network each month. Therefore, it is of great application value to study how to optimally offload the cellular traffic.

Unlike the macrocells which are owned by cellular service providers (CSPs), femtocell devices or WiFi hotspots are often owned by third-party entities (or be called *agents*) such as schools, restaurants, residences, etc. Shifting cellular traffic to these agents requires to consume their own resources (e.g., bandwidth, data quota, electricity, etc.), thus the agents need to be well motivated, and their costs, which are privately known by themselves, should be well compensated [21]. Under such circumstances, finding a good offloading solution is no longer a conventional optimization problem. Instead, what we need is a well-designed procurement auction mechanism, that generates an optimized procurement solution which approximates the optimal one in an ideal omniscient scenario.

In this paper, we consider a general setting that the macrocell of a CSP is divided into several non-overlapping small regions. Each agent is located inside one region and provides offloading resources for this region. The CSP, given a fixed budget, aims to purchase resource units from the agents to optimally mitigate the overloading issue of the macrocell base station, which can be formalized as optimizing a special *diminishing return demand valuation function* [6], which captures a fundamental principle of economics [17] and is widely adopted in modeling economic systems. Clearly, the required mechanism belongs to the class of budget feasible mechanisms [2, 4–7, 18], and can be further characterized with the following features:

1) *Multi-unit procurement auction.* Bidders can sell multiple units of a homogeneous item with privately known costs;
2) *Multi-submarket and a shared budget.* The market is divided into several submarkets, and a global budget is shared between all the submarkets;

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3) *Symmetric submodular demand valuation function for each submarkets.* For each region, the marginal value of a purchased item is non-increasing.

As far as we know, although there is some closely related work, our problem in the setting above has not been solved yet. We emphasize that our problem is highly challenging since solving it requires to effectively correlate all sub-markets, strictly bound the total payment under a fixed budget, and achieve a guaranteed performance lower bound compared with the optimal solution in the ideal case at the same time. We successfully devise a novel approach that copes well with these challenges and achieves good performance in the multi-region multi-unit procurement auction. The main contributions of this paper can be summarized as follows:

1) We model the valuation of offloading resources purchased among the regions as a local diminishing return (LDR) function, which intuitively captures optimality in each region and fairness between the regions.
2) We propose a greedy-based randomized mechanism, and prove it is budget-feasible, truthful, individually rational and $(3 + 2 \ln N)$ - approximation (N equals the total number of resource units in the market).
3) We also propose a deterministic mechanism, which is proved to be budget-feasible, truthful, individual rational and $(2 + \ln N + \sqrt{2 + 3 \ln N + \ln^2 N})$ - approximation.
4) We prove no budget-feasible and truthful mechanism can do better than $\ln N$-approximation in our setting, thus our mechanism approaches the optimal to a constant factor.
5) We extend solvable valuation class of greedy-based multi-unit budget-feasible mechanism with performance guarantees from the concave-additive valuations to more general local diminishing return valuations.

2 RELATED WORK

Most of the pioneering studies on designing incentive mechanisms for cellular traffic offloading, e.g., [10, 16, 22, 23], focus on the settings where the underlying traffic or offloading demands are known to the CSP or can be estimated precisely and efficiently, and the optimization objective is social welfare maximization. Basically, for these settings the celebrated Vickrey-Clarke-Groves (VCG) mechanism [9, 13, 19] is a perfect solution. The problem of maximizing CSP's *capacity gain* under the constraint of a strict budget was firstly studied by [21]. In this work the offloading resources owned by each agent can an be continuously divided and sold. The objective of maximizing the weighted sum of the capacity gain of all the regions was studied in depth, and a $\frac{(\alpha-1)w_{min}}{4\alpha}$-approximation mechanism, where α is the global bidder dominance and w_{min} is the minimum of all region's weights, was proposed. Intuitively, this mechanism is based on properly dividing the budget among the regions, and then run a sub-mechanism independently in each region. The sub-mechanisms was designed based on the *random sampling and profit extraction* framework proposed in [1].

Our setting has a lot in common with that studied in [21]. However, there are two major differences:

1) The offloading resources for sale in our setting are *divided into fixed-sized atomic units* instead of *being continuously dividable;*
2) The valuation in each region is *symmetric submodular* instead of being *additive*.

For our setting, we adopt the *proportional share allocation rule* [18], which is the core of budget-feasible mechanisms for maximizing submodular demand valuation functions, to design a greedy-based mechanism that selects units from different regions with a global view, instead of performing random sampling auctions independently in each submarkert.

Budget feasible mechanism design [2, 4–7, 18] is an important branch of nowadays prior-free optimal mechanism design research, which aims to optimize *payment related objectives* in auctions without the availability of the prior knowledge of the agents' type distribution. *Multi-unit* budget feasible mechanism design is a relatively new setting in this area that has been investigated by [6]. They studied a series of nested valuation classes:

bounded knapsack $\subseteq$ concave additivity $\subseteq$ diminishing return $\subseteq$ sub-modularity $\subseteq$ sub-additivity

Their results demonstrate the existence of a greedy-based randomized $4(1 + \ln n)$-approximation mechanism for concave additive valuations. However, the existence of greedy algorithms to construct budget-feasible mechanism for larger classes of valuations is left open. Therefore, they turned to a different approach, random sampling[4, 12], to propose a $O(\frac{\log^2 n}{\log \log n})$-approximation mechanism for sub-additive valuations. As will be shown in our paper, the valuation function in our setting, which we called local diminishing return (LDR), is actually a class located between concave additivity and diminishing return. We can demonstrate that, the scope of valuation functions that can be handled by greedy-based mechanisms can be at least extended to include the LDR class. Moreover, it is necessary to notice that, instead of to find mechanism that are budget-feasibly in expectation (as in [6]), we aim to find mechanism that is strict budget-feasible.

3 PRELIMINARIES

3.1 Cellular traffic offloading

3.1.1 Agents, units and regions. In cellular traffic offloading the CSP gains extra network capacity by purchasing offloading resources (or service) from n agents in the macrocell sector, denoted as $[n] = \{1, ..., n\}$. We assume each agent $i \in [n]$ has

- a *capacity* $\sigma_i \in \mathbb{N}$, representing the maximal number of resource units she can provide, and
- a *unit cost* $c_i \in \mathbb{R}^+$, representing her cost of providing each resource unit.

The resource units owned by each agent $i \in [n]$ can be denoted as $(i, 1), (i, 2), ..., (i, \sigma_i)$, where (i, j) refers to agent

i's j-th unit. Hence, the total number resource units in the system is $N = \sum_{i=1}^{n} \sigma_i$. Note that, femtocells have a much smaller coverage compared with macrocell base stations, offloading resources purchased from one femtocell is not able to handle all traffic from the entire macrocell. To deal with this practical issue, we can divide the entire sector into m non-overlapping *regions*, denoted as $[m] = \{1, ..., m\}$, each of which is fully covered by the femtocells that reside in it. So from a region's point of view, there is no difference between the units purchased from different agents inside it.

We denote an *allocation* as $\mathcal{A} = (a_1, ..., a_n)$ where each $a_i \in \{0, 1, ..., \sigma_i\}$ is the number of units obtained from agent i. We can see $\mathcal{A}$ as the set $\{(i, j) : i \in [n], j \leq a_i\}$, that is, intuitively we assume each agent's units will always be considered in their naming order. Specially, we use e^i to denote the allocation where $a_i = 1$ and $a_j = 0$ for all $j \notin i$.

Definition 3.1 (capacity gain). The *capacity gain* of a region $j \in [m]$, denoted as $s_j(\mathcal{A})$, equals to the total units purchased within this region.

We can define a function $r : [n] \to [m]$, and let $r(i)$ denote the region agent i belongs to, and then the agents in region j can be denoted as $R_j = \{i \in [n] : r(i) = j\}$. Hence, we have

$$s_j(\mathcal{A}) = \sum_{i \in R_j} a_i. \tag{1}$$

3.1.2 Local diminishing return (LDR) valuation. The aim of cellular traffic offloading is to mitigate the overloading issue of the base station which may be unevenly distributed in the macrocell. For each region j, we can specify a weight $w_j \in R^+$, which intuitively represents the *value* obtained by using a resource unit in this region to offload some data traffic. So, as the number of units obtained in a region increases, the marginal value of the ith unit is $w_j \cdot \Pr(d_j \geq i)$, where d_j is a random variable representing the resource demand in region j, and $\Pr(d_j \geq i)$ is the probability that the ith unit will be used. We denote $\Pr(d_j \geq i)$ as δ_i^j. Therefore, given an allocation $\mathcal{A}$, the obtained value in region j is

$$v_j(\mathcal{A}) = \sum_{i=1}^{s_j(\mathcal{A})} w_j \delta_i^j = w_j \cdot \sum_{i=1}^{s_j(\mathcal{A})} \delta_i^j \tag{2}$$

where $1 \geq \delta_1^j \geq \delta_2^j \geq \delta_3^j \geq \cdots \geq 0$ is a sequence of nonnegative real numbers upper bounded by 1 (we assume they are given in advance). $\sum_{i=1}^{s_j(\mathcal{A})} \delta_i^j$ can be understood as the *expected offloading (amount)* in region j.

Definition 3.2 (allocation valuation function). The value of allocation $\mathcal{A}$, denoted as $v(\mathcal{A})$, is the sum of the values obtained in all regions, i.e.,

$$v(\mathcal{A}) = \sum_{j=1}^{m} \sum_{i=1}^{s_j(\mathcal{A})} w_j \delta_i^j = \sum_{j=1}^{m} w_j \sum_{i=1}^{s_j(\mathcal{A})} \delta_i^j \tag{3}$$

Given an allocation $\mathcal{A} = (a_1, a_2, ..., a_i, ..., a_n)$, adding one extra resource unit (i, j) of agent i will result in a new allocation $\mathcal{A} + e_i = (a_1, a_2, ..., a_i + 1, ..., a_n)$, in which the expected offloading in the system increases by $m_{\mathcal{A}}(i, j) = \delta^{r(i)}_{s_{r(i)}(\mathcal{A})+1}$ in region $r(i)$. We call $m_{\mathcal{A}}(i, j)$ as unit (i, j)'s *marginal expected offloading*. Moreover, in this case the value will increase by $v(\mathcal{A} + e_i) - v(\mathcal{A}) = m_{\mathcal{A}}(i, j) \cdot w_{r(i)}$, which we refer to as unit (i, j)'s *marginal value*.

Definition 3.3 (diminishing return [6]). A allocation valuation function v is called a diminishing return function, if for any $\mathcal{A}$ and $\mathcal{A}'$ such that $a_i \leq a_i'$ for each i, and for any agent j, we have $v(\mathcal{A} + e_j) - v(\mathcal{A}) \geq v(\mathcal{A}' + e_j) - v(\mathcal{A}')$.

It is easy to see that, for any $\mathcal{A}$ and $\mathcal{A}'$ such that $a_i \leq a_i'$ for each i, and for any agent j,

$$\begin{aligned} v(\mathcal{A} + e_j) - v(\mathcal{A}) &= \delta^{r(j)}_{s_{r(j)}(\mathcal{A})+1} \cdot w_{r(j)} \geq \delta^{r(j)}_{s_{r(j)}(\mathcal{A}')+1} \cdot w_{r(j)} \\ &= v(\mathcal{A}' + e_j) - v(\mathcal{A}') \end{aligned} \tag{4}$$

According to the above definition, $v(\cdot)$ belongs to the *diminishing return* class. It actually forms a proper subset of the *diminishing return* class, since each unit's marginal value is only locally depended on the allocation in her region. Moreover, when there is at most one agent in each region it collapse to the class of *concave additivity*. We call this valuation class as *local diminishing return* (LDR) functions and formally defined it as follows.

Definition 3.4 (local diminishing return, LDR). A local diminishing return function v is a diminishing return function where the agents are partitioned into disjointed sets (let $r(i)$ denote the set agent i belongs to), and for any allocation $\mathcal{A}$ and $\mathcal{A}'$, and any agent j, we have $v(\mathcal{A} + e_j) - v(\mathcal{A}) = v(\mathcal{A}' + e_j) - v(\mathcal{A}')$ if $a_i = a_i'$ for all i such that $r(i) = r(j)$.

3.2 Economic model

3.2.1 The optimization problem. We assume the agents are *rational in the sense of game theory*, that is, they always try to maximize their utilities and need to be well-motivated to provide their offloading resources. We also assume the actual value of the unit cost c_i are each agent i's *private information*. As usual, we also refer to c_i as agent i's type. The type space of agent i, $\Theta_i = \mathbb{R}^+$, denoting agent i's all possible types, is public information. The CSP is given a fixed *budget* $B \in \mathbb{R}^+$, which represents the maximum amount of money she can spend. The problem of the CSP is to decide how many units should be purchased from each agent, i.e., an allocation $\mathcal{A}$, to maximize the valuation $v(\mathcal{A})$. As the type of each agent is only known by herself, this problem is not an optimization problem in the conventional sense.

3.2.2 Formalize it as an auction design problem. Fortunately, methodologies from algorithmic mechanism design can be adopted for this setting: The CSP can run a procurement auction with all the agents. As the seller, each agent submits a bid $b_i \in \Theta_i$ to the buyer. Based on all agents' bids, the buyer determines the auction's outcome using a predefined auction mechanism $\mathcal{M} = (\boldsymbol{x}, \boldsymbol{p})$ where

- $\boldsymbol{x}$ is an *allocation function*. For each bid vector $\boldsymbol{b} \in \Theta_1 \times \cdots \times \Theta_n$, $\boldsymbol{x}(\boldsymbol{b}) = (a_1, ..., a_n)$ is an allocation;
- $\boldsymbol{p}$ is a *payment function*. For each bid vector $\boldsymbol{b} \in \Theta_1 \times \cdots \times \Theta_n$, $\boldsymbol{p}(\boldsymbol{b})$ is the payment vector to the agents.

We use $x_i(\boldsymbol{b})$ and $p_i(\boldsymbol{b})$ to denote agent i's allocation and payment respectively. Obviously, we have $x_i(\boldsymbol{b}) \in \{0, ..., \sigma_i\}$ and $p_i(\boldsymbol{b}) \in \{0\} \cup \mathbb{R}^+$ for all $i \in [n]$. We call agent i a *winner* if it sells some units to the buyer (i.e., $x_i(\boldsymbol{b}) > 0$) and receives a positive payment (i.e., $p_i(\boldsymbol{b}) > 0$) according to the *outcome* $(\boldsymbol{x}(\boldsymbol{b}), \boldsymbol{p}(\boldsymbol{b}))$.

Definition 3.5 (utility). The *utility* of agent i for bid vector $\boldsymbol{b}$, denoted as $u_i(\boldsymbol{b})$, is defined as

$$u_i(\boldsymbol{b}) = p_i(\boldsymbol{b}) - c_i x_i(\boldsymbol{b}) \tag{5}$$

Intuitively, the utility of each agent i is defined as the difference between agent i's received payment and agent i's cost. Note that, we have assume that the agents always try to maximize their utilities.

Finally, the required properties of our aiming mechanism can be formalized as follows:

- *Budget feasible (*BF*)*, i.e., the total payment of the mechanism is upper bounded by a given budget:
$$\sum_{i=1}^{n} p_i(\boldsymbol{b}) \leq B \text{ for all } \boldsymbol{b} \in \Theta;$$
- *Truthful*, i.e., each agent maximizes her utility by bidding her true type, regardless of the types bid by others:
$$u_i(c_i, \boldsymbol{b}_{-i}) \geq u_i(b_i, \boldsymbol{b}_{-i}) \text{ for all } (b_i, \boldsymbol{b}_{-i}) \in \Theta;$$
A mechanism that is a randomization over truthful mechanisms is called *universally truthful.*
Note that, we seek deterministic mechanisms that are truthful, or randomized mechanisms that are universally truthful.
- *Individually rational (*IR*)*, i.e., each agent i's utility is non-negative. If truthfulness is satisfied, it only requires:
$$u_i(\boldsymbol{c}) \geq 0;$$
- *Tractable and performance guaranteed*, i.e., the computation of the allocation and payment function is tractable, and there is some theoretical performance lower bound.

4 A RANDOMIZED MECHANISM

By generalizing Myerson's famous characterization for truthful single-parameter auctions [14], the following result for procurement auctions has been proved.

LEMMA 1. *[11] A single-parameter procurement auction $(\boldsymbol{x}, \boldsymbol{p})$ is truthful, if and only if for any agent i and bids of other agents b_{-i} fixed,*

1) *$x_i(b_i)$ is monotone non-increasing; and*
2) *$p_i(b_i) = x_i(b_i) \cdot b_i + \int_{b_i}^{+\infty} x_i(z)dz$.*

For the multi-unit case considered in our setting, the following result can be further derived.

COROLLARY 2. *A single-parameter multi-unit procurement auction $(\boldsymbol{x}, \boldsymbol{p})$ is truthful, if and only if for any agent i and bids of other agents b_{-i} fixed,*

1) *$x_i(b_i)$ is monotone non-increasing; and*
2) *$p_i(b_i) = \sum_{j=1}^{x_i(b_i)} t_j$, where $t_j = \inf\{b_i : x_i(b_i) < j\}$, i.e., the threshold bid of agent i for unit (i, j) being selected by the mechanism.*

PROOF. $x_i(b_i)$ being monotone non-increasing is directly followed by lemma 1. Let $k = x_i(b_i)$ and we have

$$\begin{aligned} p_i(b_i) &= kb_i + \int_{b_i}^{t_k} kdz + \int_{t_k}^{t_{k-1}} k - 1dz + \cdots + \int_{t_2}^{t_1} 1dz \\ &= t_k + t_{k-1} + \cdots + t_1 \end{aligned} \tag{6}$$

Therefore, $p_i(b_i)$ is the sum of agent i's threshold bid of each of her sold unit. □

Intuitively, for the multi-unit procurement auction setting considered in this paper, to design a truthful mechanism, we can focus on finding out a monotone allocation, by which the number of units purchased from each agent i decreases as b_i increases. Then, the payment to agent i is just the sum of the threshold bid of each unit she sold out.

4.1 A monotone allocation function

Our mechanism is based on sorting all the N units (i, j) according to *marginal value per cost* (MVPC) decreasingly, with ties broken lexicographically, first by i and then by j. Notice that, we will use this tie breaking rule as default. The ordered list can be denoted as $\mathcal{L} = \langle (i_1, j_1), ..., (i_N, j_N) \rangle$, where for each $\ell \in [N]$, (i_ℓ, j_ℓ) is the unit that selected in stage ℓ. Let $\mathcal{A}_{\ell-1}$ be the set $\{(i_1, j_1), ..., (i_{\ell-1}, j_{\ell-1})\}$, then

$$(i_\ell, j_\ell) \in \arg \max_{(i,j) \notin \mathcal{A}_{\ell-1}} \frac{m_{\mathcal{A}_{\ell-1}}(i, j) w_{r(i)}}{b_i} \tag{7}$$

For each (i_ℓ, j_ℓ) in the list, we use m_ℓ^+, w_ℓ^+ and b_ℓ^+ as abbreviations for $m_{\mathcal{A}_{\ell-1}}(i_\ell, j_\ell)$, $w_{r(i_\ell)}$ and b_{i_ℓ} respectively.

Now, we can specify a greedy-based allocation function as follows. Basically, this function generalizes the *proportional share allocation rule* [18] to the multi-unit setting considered in this paper: firstly, it reorders all the units by decreasing marginal value per cost, then it picks up all the units with cost bid no higher than their marginal value proportional share of the given budget γB. Note that, $0 \leq \gamma \leq 1$ is a constant which will be used in bounding the total payment within the given budget.

GREEDY-LDR(S, B, γ)

1. Order all the units of the agents in S according to MVPC decreasingly, to the order list $\mathcal{L} = \langle (i_1, j_1), ..., (i_N, j_N) \rangle$;
2. Let k be the last position in $\mathcal{L}$ satisfying
$$\frac{b_k^+}{m_k^+ w_k^+} \leq \frac{\gamma B}{\sum_{\ell \leq k} m_\ell^+ w_\ell^+};$$
3. Pick up the first k units in $\mathcal{L}$, that is, output $\mathcal{A} = (a_1, ..., a_n)$ where $a_i = |\{\ell : \ell \leq k \text{ and } i_\ell = i\}|$.

For any two units (i, h) and (j, k), we denote as $(i, h) \prec (j, k)$, if (i, h) is ranked ahead of (j, k) (in $\mathcal{L}$) . Then, we can obtain the following result which depicts the relative position of the units of any two agents in the same region.

PROPOSITION 3. *For any 2 agents i, j in the same region,*

1) *If $b_i < b_j$ then $(i, h) \prec (j, k)$ for any h, k.*
2) *If $b_i = b_j$ then $(i, h) \prec (j, k)$ for any h, k iff $i < j$.*

PROOF. *1)* Assume $b_i < b_j$ and there are $h \in [\sigma_i]$, $k \in [\sigma_j]$ satisfying $(j, k) \prec (i, h)$. So,

$$\frac{m_{\mathcal{A}}(j,k) w_{r(j)}}{b_j} \geq \frac{m_{\mathcal{A}}(i,h) w_{r(i)}}{b_i}, \tag{8}$$

where $\mathcal{A}$ are the units ordered before (j, k). Since $r(i) = r(j)$, we have $m_{\mathcal{A}}(j, k) = m_{\mathcal{A}}(i, h)$ and therefore $b_i \geq b_j$, and this is obviously a contradiction!

2) Trivial due to the tie breaking rule. □

The above result implies the units of different agents from the same region never intersect with each other in $\mathcal{L}$. They are scattered along $\mathcal{L}$ from left to right in the order of increasing cost bid and break ties by increasing naming order.

PROPOSITION 4. *The marginal value of a unit (i, j) depends only on agent i's local rank on cost bid.*

PROOF. Given a region $h \in [m]$, we sort the agents i in this region as $i^h_1, ..., i^h_{|R_h|}$ in the order of increasing cost bid, breaking ties by i. According to proposition 3, an arbitrary unit (i^h_k, j) has $\Delta = \sum_{\ell=1}^{k-1} \sigma_{i^h_\ell} + j - 1$ units from the same region ranked before it in $\mathcal{L}$. Therefore its marginal value is $\delta^{r(i^h_k)}_{\Delta+1} w_{r(i^h_k)}$. Clearly, it only depends on the local rank k. □

Afterward, based on the above 2 results, we can prove the monotonous of the allocation.

LEMMA 5. GREEDY-LDR(S, B, γ) *is monotone.*

PROOF. For any unit (i_s, j_s) in the winning set, we can divide the rest units in the system into 4 sets:

1) S^-_1, the units in region $r(i_s)$ ranked before (i_s, j_s);
2) S^+_1, the units in region $r(i_s)$ ranked after (i_s, j_s);
3) S^-_0, the units outside region $r(i_s)$ ranked before (i_s, j_s);
4) S^+_0, the units outside region $r(i_s)$ ranked after (i_s, j_s).

Now we suppose agent i_s bids a cost $b' \leq b^+_s$, obtains a marginal expected offloading m' and the above 4 sets becomes $\hat{S}^-_1, \hat{S}^+_1, \hat{S}^-_0$ and $\hat{S}^+_0$ respectively. It is easy to show that no unit (i_ℓ, j_ℓ) from S^+_1 can be moved to the front of (i_s, j_s):

- If $i_\ell = i_s$, i.e., they belong to the same agent, we still have $(i_s, j_s) \prec (i_\ell, j_\ell)$ because of the tie breaking rule,
- otherwise, we have $b^+_\ell \geq b^+_s > b'$, and by proposition 3, we have $(i_s, j_s) \prec (i_\ell, j_\ell)$.

That is, we have $\hat{S}^-_1 \subseteq S^-_1$, and therefore $m' \leq m^+_s$. Since the marginal value of all the units in $S^-_0 \cup S^+_0$ are trivially invariant, no unit (i_h, j_h) in S^+_0 can be moved to the front of (i_s, j_s) , since the weighted marginal value rate of (i_h, j_h) is invariant, and

$$\frac{m^+_h w^+_h}{b^+_h} \leq \frac{m^+_s w^+_s}{b^+_s} < \frac{m' w^+_s}{b'} \tag{9}$$

Thus we also have $\hat{S}^-_0 \subseteq S^-_0$, and moreover

$$\begin{aligned} \frac{b'}{m' w^+_s} < \frac{b^+_s}{m^+_s w^+_s} &\leq \frac{\gamma B}{\sum_{\ell \leq s} m^+_\ell w^+_\ell} \\ &= \frac{\gamma B}{\sum_{(i_\ell, j_\ell) \in S^-_1 \cup S^-_0 \cup \{(i_s, j_s)\}} m^+_\ell w^+_\ell} \\ = & \frac{\gamma B}{w^+_s \cdot \sum_{j=1}^{|S^-_1|+1} \delta^{r(i_s)}_j + \sum_{(i_\ell, j_\ell) \in S^-_0} m^+_\ell w^+_\ell} \\ \leq & \frac{\gamma B}{w^+_s \cdot \sum_{j=1}^{|\hat{S}^-_1|+1} \delta^{r(i_s)}_j + \sum_{(i_\ell, j_\ell) \in \hat{S}^-_0} m^+_\ell w^+_\ell} \end{aligned} \tag{10}$$

Therefore, according the specification of function GREEDY-LDR(S, B, γ), each unit will still be allocated after biding a lower cost, and thus monotonous hold. □

4.2 Finding the threshold payment

The algorithm for computing the threshold payment of unit (i, j), which will be used as a key building block of our payment determination algorithm, can be specified as follows:

TP(i, j)

1. Order the agents in region $h = r(i)$ as $(i^h_1, i^h_2, ..., i^h_{|R_h|})$ according to increasing cost bid, and let ℓ satisfy $i^h_\ell = i$.
2. Order all the units except that from agent i, according to decreasing MVPC, as $\mathcal{L}' = \langle (i'_1, j'_1), ..., (i'_{n-\sigma_i}, j'_{n-\sigma_i}) \rangle$
3. for each $j \in [|R_h|] \setminus \{\ell\}$, find out the first l_j satisfying $i'_{l_j} = i^h_j$ and the last r_j satisfying $i'_{r_j} = i^h_j$;
4. Find out the last position k' s.t. $\frac{c'_{k'}}{w'_{k'} \cdot m'_{k'}} \leq \frac{\gamma B}{\sum_{\ell \leq k'} m'_\ell w'_\ell}$
Let $r_0 = 0$; $k = r_{\ell-1} + 1$; $\tau = 1$; $S = \emptyset$;
while $k \leq k' + 1$ **do**
– **if** $k == l_{\ell+\tau} + 1$ **do** $k = r_{\ell+\tau} + 1; \tau + +$;
– **else**
$\Delta = \sum_{j=1}^{\ell-1} \sigma_{i^h_j} + \sum_{j=2}^{\tau} \sigma_{i^h_{\ell+j-1}} + j$;
$t = min\{\delta^h_\Delta \frac{c'_{k'}}{m'_{k'}}, \gamma B \delta^h_\Delta \cdot \frac{1}{\sum_{i \leq k-1} m'_k + r^h_{\Delta-j+1} + \cdots + r^h_\Delta}\}$;
$S = S \cup \{t\}$; $k + +$;
– **endif**
endwhile
5. return the max value in S.

PROPOSITION 6. TP(i, j) *correctly returns the threshold payment of unit (i, j).*

Proof. TP(i, j) firstly deletes all the units from agent i and sorts the remaining units to sequence S' according to decreasing weighted marginal value rate (steps 1∼ 2). Afterward, for each agent (ranked as j by increasing cost) in region $r(i)$, it marks the first and the last of its units in S' by l_j and r_j respectively (step 3). According to proposition 3, we have

$$l_1 \leq r_1 < l_2 \leq r_2 < \cdots < l_{\ell-1} \leq r_{\ell-1}$$
$$< l_{\ell+1} \leq r_{\ell+1} < \cdots < l_{|J_h|} \leq r_{|J_h|} \tag{11}$$

Therefore, we can search in the intervals

$$(r_{\ell-1}, \cdots, l_{\ell+1}], (r_{\ell+1}, \cdots, l_{\ell+2}], (r_{\ell+2}, \cdots, l_{\ell+3}], ...$$

and try to find out the highest value agent i can bid to let unit (i, j) be placed next to one of the units in these intervals and be picked up (steps 4 ∼ 5). Note that, by agent i's current bid, unit (i, j) will be insert into the interval $(r_{\ell-1}, \cdots, l_{\ell+1}]$ and be picked up, and being insert into an interval in front requires agent i to bid lower (by proposition 4). So, the search can start from interval $(r_{\ell-1}, \cdots, l_{\ell+1}]$. □

4.3 Bounding the total payment

First of all, we can try to establish an upper bound for each winning unit's threshold bid. Note that, for a winning unit (i, j), we use s_{ij} to refer to (i, j)'s rank of marginal value in the winning set.

Proposition 7. *If (i, j) wins, then $b_i \leq \frac{\gamma B}{s_{ij}}$.*

Proof. Given a bid vector $\boldsymbol{b} = \{b_1, ..., b_n\}$, for which the mechanism picks up the first k pairs, i.e., $(i_1, j_1), .., (i_k, j_k)$, in $\mathcal{L}$. We then reorder these k pairs according to marginal values decreasingly, as $(i^\sharp_1, j^\sharp_1), .., (i^\sharp_k, j^\sharp_k)$. We also write $(i^\sharp_\ell, j^\sharp_\ell)$'s marginal expected offloading, $b_{i^\sharp_\ell}$ and $w_{r(i^\sharp_\ell)}$ as $m^\sharp_\ell$, $b^\sharp_\ell$ and $w^\sharp_\ell$ for short respectively.

Assume that $\exists\ 1 \leq s \leq k$ satisfying $b^\sharp_s > \frac{\gamma B}{s}$, then

$$\frac{b^+_k}{m^+_k w^+_k} \leq \frac{\gamma B}{\sum_{\ell \leq k} m^+_\ell w^+_\ell} = \frac{\gamma B}{\sum_{\ell \leq k} m^\sharp_\ell w^\sharp_\ell}$$
$$\leq \frac{\gamma B}{\sum_{\ell \leq s} m^\sharp_\ell w^\sharp_\ell} \leq \frac{\gamma B}{s \cdot m^\sharp_s w^\sharp_s} \tag{12}$$

Moreover, by the assumption we have $\gamma B < s \cdot b^\sharp_s$ and so

$$\frac{\gamma B}{s \cdot m^\sharp_s w^\sharp_s} < \frac{b^\sharp_s}{m^\sharp_s w^\sharp_s} \tag{13}$$

Combining (7) and (8), we obtain

$$\frac{b^+_k}{m^+_k w^+_k} < \frac{b^\sharp_s}{m^\sharp_s w^\sharp_s} \tag{14}$$

Now, we have arrived at a contradiction, since $(i_k, j_k) \prec (i^\sharp_s, j^\sharp_s)$ in $\mathcal{L}$, and thus can't be picked up. □

Given the above result, it's still unclear whether i can bid $b' > \frac{\gamma B}{s_{ij}}$, and obtain a new rank s'_{ij} of marginal value, and let (i, j) still be selected. We can show that actually this is impossible.

Lemma 8. *The threshold bid for a unit (i, j) in the winning set, denoted as θ_{ij}, is upper bounded by $\frac{\gamma B}{s_{ij}}$.*

Proof. Assume $\exists (i^\sharp_s, j^\sharp_s), s \leq k$ and $\theta_{i^\sharp_s j^\sharp_s} > \frac{\gamma B}{s}$. So, agent $i^\sharp_s$ can raise its bid to b' satisfying $\frac{\gamma B}{s} < b' < \theta_{i^\sharp_s j^\sharp_s}$ and still let $(i^\sharp_s, j^\sharp_s)$ be picked up. Now suppose agent $i^\sharp_s$ bid b' and let m' be the marginal value of unit $(i^\sharp_s, j^\sharp_s)$ in this case. Trivially, we have $m' \leq m^\sharp_s$, and

$$\frac{b'}{m' w^\sharp_s} \geq \frac{b'}{m^\sharp_s w^\sharp_s} > \frac{\gamma B}{s \cdot m^\sharp_s w^\sharp_s} \geq \frac{b^+_k}{m^+_k w^+_k} \tag{15}$$

For any $(i^\sharp_\ell, j^\sharp_\ell)$ where $\ell < s$ (the items used to be ranked before item $(i^\sharp_s, j^\sharp_s)$ in the weighted marginal value sort), we denote its new marginal value as m'_ℓ and new cost as b'_ℓ. Consider the relation between the two items $(i^\sharp_\ell, j^\sharp_\ell)$ and $(i^\sharp_s, j^\sharp_s)$, there are 3 cases as follows:

Case 1 (they belong to the same agent): By the tie breaking rule, we still have $(i^\sharp_\ell, j^\sharp_\ell) \prec (i^\sharp_s, j^\sharp_s)$ in $\mathcal{L}$, and thus $m'_\ell > m'$.

Case 2 (they belong to different agents in the same region): Since $m^\sharp_\ell \geq m^\sharp_s$, $b^\sharp_\ell \leq b^\sharp_s$, and therefore

$$b'_\ell = b^\sharp_\ell \leq b^\sharp_s \leq b'. \tag{16}$$

Case 3 (they belong to different regions): Trivially, we have $m'_\ell = m^\sharp_\ell$ and $b'_\ell = b^\sharp_\ell$, and therefore

$$\frac{b'_\ell}{m'_\ell w^\sharp_\ell} = \frac{b^\sharp_\ell}{m^\sharp_\ell w^\sharp_\ell} \leq \frac{b^+_k}{m^+_k w^+_k} < \frac{b'}{m' w^\sharp_s} \tag{17}$$

Moreover, we have $m'_\ell = m^\sharp_\ell \geq m^\sharp_s \geq m'$.

It is easy to find out in all the above 3 cases unit $(i^\sharp_\ell, j^\sharp_\ell)$ will be picked up and be ranked before $(i^\sharp_s, j^\sharp_s)$ in $\mathcal{L}$. Let s' be the unit $(i^\sharp_s, j^\sharp_s)$'s new rank of marginal value. We have $s' \geq s$ and therefore

$$b' > \frac{\gamma B}{s} \geq \frac{\gamma B}{s'}. \tag{18}$$

So, by proposition 7, $(i^\sharp_s, j^\sharp_s)$ can't be picked up, and this is a contradiction. □

Based on lemma 8, an upper bound for total threshold payment of all the selected units follows directly.

Lemma 9. *The threshold payment required by* Greedy-LDR(S, B, γ) *is upper bounded by $(1 + lnN)\gamma B$.*

Proof. By lemma 8, the threshold payment is

$$\sum_{i=1}^{n} \sum_{j=1}^{a_i} \theta_{ij} = \sum_{s=1}^{k} \theta_{i^\sharp_s j^\sharp_s} \leq \sum_{s=1}^{k} \frac{\gamma B}{s}$$
$$\leq \sum_{s=1}^{N} \frac{\gamma B}{s} = \gamma B \sum_{s=1}^{N} \frac{1}{s} \leq (1 + \ln N)\gamma B \tag{19}$$

That is, it is upper bounded by $(1 + lnN)\gamma B$. □

Let $opt(S, B)$ be the value of the optimal solution in the public information case. We can relate it to the greedy solution in the strategical case, by the following lemma.

LEMMA 10. *Let $\delta^* w^*$ be the highest marginal value of all the units and $0 < \gamma \le 1$, then we have*

$$\frac{1+\gamma}{2+\gamma} v(\text{GREEDY-LDR}(S, B, \gamma)) + \frac{\delta^* w^*}{2+\gamma} > \frac{\gamma}{2+\gamma} opt(S, B) \quad (20)$$

PROOF. Let $\hat{k}$ be the maximal index for which $\sum_{\ell=1}^{\hat{k}} b_\ell^+ \le B$, $b'_{\hat{k}+1} = B - \sum_{\ell=1}^{\hat{k}} b_\ell^+$ and $m'_{\hat{k}+1} = m_{\hat{k}+1}^+ \cdot \frac{b'_{\hat{k}+1}}{b_{\hat{k}+1}}$. So, we can define the *fractional greedy solution* as

$$f(S, B) = \sum_{\ell=1}^{\hat{k}} m_\ell^+ w_\ell^+ + m'_{\hat{k}+1} w_{\hat{k}+1}^+ \quad (21)$$

Let $T = \{(i_1, j_1), \ldots, (i_k, j_k)\}$ be the subset returned by GREEDY-LDR(S, B, γ). So, $\forall h \in \{k+1, ..., \hat{k}\}$, we have

$$\frac{b_h^+}{m_h^+ w_h^+} \ge \frac{b_{k+1}^+}{m_{k+1}^+ w_{k+1}^+} > \frac{\gamma B}{\sum_{\ell=1}^{k+1} m_\ell^+ w_\ell^+} \quad (22)$$

where the last inequality follows from the fact that the greedy strategy stops at item $k+1$. Hence

$$b_h^+ > \frac{\gamma B \cdot m_h^+ w_h^+}{\sum_{\ell=1}^{k+1} m_\ell^+ w_\ell^+} \quad (23)$$

Similarly, we can obtain

$$b'_{\hat{k}+1} > \frac{\gamma B \cdot m'_{\hat{k}+1} w_{\hat{k}+1}^+}{\sum_{\ell=1}^{k+1} m_\ell^+ w_\ell^+} \quad (24)$$

Adding inequality (19) for each h and inequality (20) together:

$$\gamma B \frac{\sum_{h=k+1}^{\hat{k}} m_h^+ w_h^+ + m'_{\hat{k}+1} w_{\hat{k}+1}^+}{\sum_{\ell=1}^{k+1} m_\ell^+ w_\ell^+} < \sum_{h=k+1}^{\hat{k}} b_h^+ + b'_{\hat{k}+1} \le B \quad (25)$$

which implies

$$\sum_{h=k+2}^{\hat{k}} m_h^+ w_h^+ + m'_{\hat{k}+1} w_{\hat{k}+1}^+ < \frac{1-\gamma}{\gamma} m_{k+1}^+ w_{k+1}^+ + \frac{1}{\gamma} \sum_{\ell=1}^{k} m_\ell^+ w_\ell^+ \quad (26)$$

It is trivial that $opt(S, B) \le f(S, B)$, and moreover

$$f(S, B) = \sum_{\ell=1}^{k+1} m_\ell^+ w_\ell^+ + \sum_{\ell=k+2}^{\hat{k}} m_\ell^+ w_\ell^+ + m'_{\hat{k}+1} w_{\hat{k}+1}^+$$

$$< \frac{1+\gamma}{\gamma} \sum_{\ell=1}^{k} m_\ell^+ w_\ell^+ + \frac{1}{\gamma} m_{k+1}^+ w_{k+1}^+ \quad (27)$$

$$\le \frac{1+\gamma}{\gamma} v(\text{GREEDY-LDR}(S, B, \gamma)) + \frac{1}{\gamma} \delta^* w^*$$

Therefore, this lemma holds. □

Now, we have prepared all the necessary parts for building a truthful budget-feasible mechanism with performance guarantee for the setting considered in this paper.

4.4 A budget-feasible randomized mechanism

Lemma 9 implies to bound the total payment of the greedy solution by B, at most we can set $\gamma = \frac{1}{1+lnN}$. Together with lemma 10, we can propose a random mechanism as follows.

RANDOM-MECHANISM-LDR

1. Let $S' = \{i \in S \mid b_i \le B\}$,
   $R = \{j \in [m] \mid \exists i \in S' \ s.t. \ r(i) = j\}$,
   $j^* \in \arg\max_{j \in R} w_j \delta_1^j$, $i^* \in J_{j^*}$
2. with probability $\frac{1+\ln N}{3+2\ln N}$ return (e^{i^*}, B)
3. with probability $\frac{2+\ln N}{3+2\ln N}$
   – $S = (a_1, ..., a_n) \leftarrow$ GREEDY-LDR$(S, B, \frac{1}{1+\ln N})$
   – $P = (\sum_{j=1}^{a_1} \text{TP}(1, j), ..., \sum_{j=1}^{a_n} \text{TP}(n, j))$
   – return (S, P)

Basically, the above mechanism first of all delete all units with cost bid higher than B, and then with probability $\frac{1+\ln N}{3+2\ln N}$ it selects the unit with the highest value and pay all the budget to the corresponding agent; with probability $\frac{2+\ln N}{3+2\ln N}$ it selects the greedy solution and pay the corresponding agents according corollary 2. The above probabilities is actually set up based on lemma 10 to achieve a relatively good performance guarantee.

THEOREM 11. RANDOM-MECHANISM-LDR *is tractable, universally truthful,* IR *and budget-feasible.*

PROOF. *1) Tractability*: Tractability of this mechanism is trivial, since it is obvious that the greedy allocation and the threshold payments can be computed in polynomial time.

2) Universally truthfulness: the sub-mechanism in step 2 picks up a unit with the highest weighted marginal value and pays all the budget to the corresponding agent and is trivially truthful. By lemma 5 and proposition 6, GREEDY-LDR is monotone, and the payment transfered to each agent is the total threshold payment of all her units. Hence, by corollary 2, this sub-mechanism is truthful. Therefore, RANDOM-MECHANISM-LDR is universally truthful.

3) IR: notice that all the agents that bid a unit cost higher than B will be deleted directly. Hence, the sub-mechanism in step 2 is trivially individually rational. And so does the sub-mechanism in step 3, since the payment to each agent is the sum of the threshold bid of al her picked up units, which is no less than her bid unit cost.

4) Budget feasibility: by lemma 9, the threshold payment required by GREEDY-LDR$(S, B, \frac{1}{1+\ln N})$ is upper bounded by $(1+\ln N)\frac{1}{1+\ln N}B = B$. Moreover, according to proposition 6 the payment spend is actually the threshold payment. Hence, budget feasibility is satisfied. □

Finally, we can establish a theoretical performance lower bound for the proposed randomized mechanism.

THEOREM 12. RANDOM-MECHANISM-LDR *is* $(3 + 2\ln N)$ *- approximation.*

PROOF. Let $\gamma = \frac{1}{1+\ln N}$, we can obtain $\frac{1+\gamma}{2+\gamma} = \frac{2+\ln N}{3+2\ln N}$, and $\frac{1}{2+\gamma} = \frac{1+\ln N}{3+2\ln N}$. Hence, according to lemma 10, the expected value obtained by this randomized mechanism is $\frac{1+\gamma}{2+\gamma}v(\text{GREEDY-LDR}(S, B, \gamma)) + \frac{\delta^* w^*}{2+\gamma} > \frac{\gamma}{2+\gamma}opt(S, B) = \frac{1}{3+2\ln N}opt(S, B)$. □

Note that, the approximation ratio obtained by our mechanism isn't a constant. However, we can conclude that, our mechanism is already optimal within a constant factor.

PROPOSITION 13. *No universally truthful and budget-feasible mechanism can do better than* $\ln N$*-approximation for* LDR.

PROOF. Since no universally truthful and budget-feasible mechanism can do better than $\ln N$-approximation for bounded knapsack valuation functions [6]. And we have shown that LDR is a superset of bounded knapsack. □

Intuitively, the above proposition implies as $N \to \infty$, our setting turns into a continuous setting, and the approximation ratio is unbounded. This result is actually consistent with that of [21], which implies no assumption is put on the bidder dominance α. It is possible that $\alpha = 0$, and therefore the approximation ratio is unbounded.

5 A DETERMINISTIC MECHANISM

We can also propose a deterministic mechanism for our setting, based on the proposed greedy allocation function. According to lemma 10 mentioned above, it is clear that bounded performance lower bound guarantee can be achieved by simply return $\arg\max_{T\in\{\text{GREEDY-LDR}(S,B,\gamma),\{(i^*,1)\}\}} v(T)$. But unfortunately, it is trivial to show this comparison will damage monotonous of the allocation function and thus damage truthfulness. Motivated by [7], we can circumvent this issue by comparing $(i^*, 1)$ and optimal fractional solution for set $S \setminus \{(i^*, 1)\}$.

MECHANISM-LDR

1. Let $S' = \{i \in S \mid b_i \le B\}$,
   $R = \{j \in [m] \mid \exists i \in S' \ s.t. \ r(i) = j\}$,
   $j^* \in \arg\max_{j\in R} w_j \delta_1^j$, $i^* \in J_{j^*}$
2. If $\delta_1^{i^*} w_{i^*} \ge \frac{f(S\setminus\{(i^*,1)\},B)}{1+\ln N+\sqrt{2+3\ln N+\ln^2 N}}$ return (e^{i^*}, B)
   otherwise
   – $S = (a_1, ..., a_n) \leftarrow \text{GREEDY-LDR}(S, B, \frac{1}{1+\ln N})$
   – $P = (\sum_{j=1}^{a_1} \text{TP}(1, j), ..., \sum_{j=1}^{a_n} \text{TP}(n, j))$
   – return (S, P)

THEOREM 14. MECHANISM-LDR *is tractable, truthful,* IR *and budget-feasible.*

PROOF. Tractability is trivial. Truthfulness is also trivial, since the bid of agent i^* is independent to the value of $f(S \setminus \{(i^*, 1)\}, B)$, and the two sub-mechanisms as have shown in the proof of theorem 11 are truthful. Moreover, IR and budget feasibility can also be shown similarly. □

The performance guarantee lower bound of this deterministic mechanism can be established as follows.

THEOREM 15. *The deterministic mechanism* MECHANISM-LDR *is* $2 + \ln N + \sqrt{2 + 3\ln N + \ln^2 N}$ *- approximation.*

PROOF. We denote as $\beta = 1 + \ln N + \sqrt{2 + 3\ln N + \ln^2 N}$ $= (1 + \ln N)(1 + \sqrt{\frac{2+\ln N}{1+\ln N}})$. With the obtained allocation $\mathcal{A}$, there are the following 2 cases:

Case 1 ($\mathcal{A} = e^{i^*}$): We have $\beta \cdot \delta^* w^* \ge f(S \setminus (i^*, 1), B)$, and therefore

$$opt(S, B) \le f(S \setminus (i^*, 1), B) + \delta^* w^* \le (1 + \beta)\delta^* w^* \quad (28)$$

Case 2 ($\mathcal{A} = \text{GREEDY-LDR}(S, B, \gamma)$, where $\gamma = \frac{1}{1+\ln N}$):

$$\text{By (27) We have } \beta \cdot \delta^* w^* < f(S \setminus (i^*, 1), B) \le f(S, B) < \frac{1+\gamma}{\gamma}v(\text{GREEDY-LDR}(S, B, \gamma) + \frac{1}{\gamma}\delta^* w^* \quad (29)$$

Hence, $\delta^* w^* < \frac{1+\gamma}{\beta\gamma-1}v(\text{GREEDY-LDR}(S, B, \gamma))$, and by lemma 10, we can obtain

$$\begin{aligned} opt(S, B) &< \frac{1+\gamma}{\gamma}v(\text{GREEDY-LDR}(S, B, \gamma)) + \frac{\delta^* w^*}{\gamma} \\ &< (\frac{1+\gamma}{\gamma} + \frac{1}{\gamma} \cdot \frac{1+\gamma}{\beta\gamma - 1})v(\text{GREEDY-LDR}(S, B, \gamma)) \quad (30) \\ &= (1 + \beta)v(\text{GREEDY-LDR}(S, B, \gamma)) \end{aligned}$$

Hence, the mechanism is always $(1 + \beta)$-approximation. □

6 CONCLUSION

Cellular traffic offloading is nowadays an important problem in mobile networking. We focus on the theoretical aspect of this problem, and aim to propose a truthful mechanism that optimally mitigates the overloading issue of a macrocell base station with a given fixed budget. We proposed to use the class of local diminishing return (LDR) demand valuation functions to evaluate the offloading resources obtained from the agent, and then our problem can be formalized as a multi-unit budget feasible mechanism problem for LDR demand valuation functions. We then proposed a greedy-based randomized mechanism for this setting, and proved it is budget-feasible, truthful, individual rational and $(3 + 2\ln N)$ - approximation. We also proposed a deterministic mechanism which is budget-feasible, truthful, individual rational and $(2 + \ln N + \sqrt{2 + 3\ln N + \ln^2 N})$ - approximation. Our work has successfully extended the multi-unit budget-feasible mechanism class, and proposed a novel mechanism for solving the cellular traffic offloading problem.

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