Manipulative Design of Scoring Systems

Extended Abstract

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ABSTRACT

Scoring systems give points to candidates according to their positions and are, for example, used in elections and sports tournaments. We study the influence that the design of such systems has on the outcome by introducing two related decision problems. The problem Scoring System Existence asks whether for a given set of profiles there exists a scoring system that makes some distinguished candidate win, whereas Closest Scoring System bounds the choice of an alternative scoring system by some given distance.

KEYWORDS

Scoring System; Manipulation; Distances

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1 INTRODUCTION

Scoring systems are, for example, used in elections to determine the winners. The votes are linear orders over the set of candidates, who get points according to their positions in the votes. Winners are those candidates with a maximum score. This principle is also applicable in other situations, like sports competitions for example. In a Formula 1 race the drivers correspond to the candidates, and their placement in each race represents a vote. The points are given according to their final position in the race. Another example is the Eurovision Song Contest, where the points are given to the candidates through the voting of the single countries.

The design of a scoring system is a crucial point. There are some obvious criteria that should be fulfilled, like a decreasing sequence of points and a reasonable distance between the significant positions. A scoring system used in sports competitions that give the same number of points to the first and second place will hardly be seen as a good rule. The history of the points given in Formula 1 races, however, shows that several slight adaptions of the points have occurred over the years. A natural question is: How much impact has such a change on winner determination? And from the point of view of a manipulator: Can I find a scoring system that makes some desired candidate win? Such questions are particularly important when the outcome of the election involves a lot of money or prestige. In this paper, we give a theoretical background to this problem by introducing the problem Scoring System Existence. As argued above, the existence of only weird scoring systems may not help, as there is no chance to implement them. Therefore, we consider different restrictions on the alternative system and introduce the problem Closest Scoring System, where the aim is to find an alternative scoring system that has a bounded distance to the original one.

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2 PROBLEM DEFINITIONS

An election (C, V) consists of a set C of m candidates and a profile $V = (v_1, \dots, v_n)$ containing linear orders over C. A scoring system for (C, V) is defined through a vector $\vec{\alpha} = (\alpha_1, \dots, \alpha_m) \in \mathbb{R}^m_{\geq 0}$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$. The points a single candidate $c \in C$ gets in this system is the sum of the points according to its position in the votes:

$$\text{score}^{\vec{\alpha}}_{(C,V)}(c) = \sum_{i=1}^{n} \alpha_{\text{pos}(v_i, c)},$$

where $\text{pos}(v, c)$ is the position of candidate c in the linear order v. The winners according to this scoring system are then all candidates with the maximum score. Prominent examples of scoring systems are Borda with the vector $(m-1, m-2, \dots, 0)$ and Plurality with the vector $(1, 0, \dots, 0)$.

Now, we will introduce the problem Scoring System Existence, clarify its importance and compare it to other problems studied before. The completely unconstrained problem is defined as follows.

Scoring System Existence	
Given:	A set of candidates $C = \{c_1, c_2, \dots, c_m\}$, a list of profiles $V_1, V_2, \dots, V_N$ over C, and a candidate $p \in C$.
Question:	Is there a scoring vector $\vec{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_m) \in \mathbb{R}^m_{\geq 0}$ with $\alpha_m = 0$, such that p is the unique winner of election (C, V_j), $1 \leq j \leq N$, with respect to $\vec{\alpha}$?

We consider the unique winner case, since the non-unique winner case is uninteresting insofar as vectors that give the same number of points to almost all candidates are often a trivial solution in the unconstrained case. This definition can be combined with restrictions on the scoring vector (e.g., a certain value at a specific position) in order to get more realistic choices. Note that as a scoring system can always be reshaped to an equivalent scoring system fulfilling $\alpha_m = 0$, we generally assume that $\alpha_m = 0$ holds.

The problem is defined as an election problem, but it is applicable to many different settings. In terms of elections, it models the situation where the chair of the election tries to find a scoring system to guarantee certain outcomes for different possible profiles. This situation could occur, for example, if the chair already knows predictions about the votes before the election and the system itself has been established. Another situation where this problem might occur is (online) surveys or studies in which the participants (i.e., the voters) were previously unaware of the actual system. Here, the originator of the survey or study can influence the outcome by choosing a system that supports its preferred alternative or hypothesis. Another major scope of the problem is competitions, especially sports competitions. In competitions such as the Formula 1 or the Eurovision Song Contest in which scoring systems are used, the races or juries act as voters. Again, the question is whether a prior change of the system, based on suspected placements, or a subsequent change based on the actual placements, can guarantee certain outcomes. This question is particularly interesting if people in charge might have an interest in the success of a certain team because of preferences, arrangements or bets.

In all these cases, one has to keep in mind that arbitrary systems, which follow no idea or intuition, are suspicious and make a targeted modification of the system very obvious. If there exists a scoring system that makes the preferred candidate a unique winner, the manipulative agent may be interested in making her suggestions for changing the system as small as possible. We model this variant by measuring the difference between the original and the new scoring system using distances. Different distances may be used in order to capture specific kinds of changes. Thus, we consider the problem where we ask whether there is an alternative scoring system making the distinguished candidate the unique winner, while the distance between both vectors is bounded by some value. For a distance $\mathcal{D}$ on arbitrary sized vectors, the problem is formally defined as follows.

$\mathcal{D}$-Close Scoring System	
Given:	A set of candidates $C = \{c_1, c_2, \ldots, c_m\}$, a list of profiles $V_1, V_2, \ldots, V_N$ over C, a scoring system $\vec{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_m) \in \mathbb{R}^m_{\geq 0}$ with $\alpha_m = 0$, a distance limit $K \in \mathbb{R}_{\geq 0}$, and a candidate $p \in C$.
Question:	Is there a scoring system $\vec{\alpha}' = (\alpha'_1, \alpha'_2, \ldots, \alpha'_m) \in \mathbb{R}^m_{\geq 0}$ with $\alpha'_m = 0$, such that p is the unique winner of election (C, V_j), $1 \leq j \leq N$, with respect to $\vec{\alpha}'$ and $\mathcal{D}(\vec{\alpha}, \vec{\alpha}') \leq K$?

We will illustrate the above problem definitions with a short example including one profile.

Example 2.1. Assume we are given an election (C, V) with $C = \{a, b, c, d\}$ and $V = (v_1, v_2, v_3)$ as follows:

$$v_1 : a > b > d > c$$
$$v_2 : d > c > b > a$$
$$v_3 : c > b > a > d.$$

If the election (C, V) is evaluated by the scoring system characterized by $\vec{\alpha} = (4, 2, 1, 0)$ we receive $\text{score}^{\vec{\alpha}}_{(C,V)}(a) = 5$, $\text{score}^{\vec{\alpha}}_{(C,V)}(b) = 5$, $\text{score}^{\vec{\alpha}}_{(C,V)}(c) = 6$, and $\text{score}^{\vec{\alpha}}_{(C,V)}(d) = 5$. Thereby, c would be the unique winner of the election.

Suppose we want to check whether there exists a scoring system $\vec{\alpha}' \in \mathbb{N}^4_{\geq 0}$ with $\alpha'_4 = 0$ in which our preferred candidate b is the unique winner of the election. Note that b is the unique winner of the election if and only if $\text{score}^{\vec{\alpha}'}_{(C,V)}(b) > \text{score}^{\vec{\alpha}'}_{(C,V)}(x)$ holds for $x \in \{a, c, d\}$. Therefore, b is the unique winner of the election if and only if $2 \cdot \alpha'_2 > \alpha'_1$ and $\alpha'_2 + \alpha'_3 > \alpha'_1$ hold, with the first condition being contained in the second one due to $\alpha'_2 \geq \alpha'_3$. Thus, $(1, 1, 1, 0)$ and $(2, 2, 1, 0)$ would, for example, be valid solutions for Scoring System Existence. If however we, require that the *Euclidean distance* $\mathcal{D}^2(\vec{\alpha}, \vec{\alpha}') = \sqrt{\sum_{j=1}^4 (\alpha_j - \alpha'_j)^2}$ between $\vec{\alpha}$ and $\vec{\alpha}'$ is at most $\sqrt{2}$, they are not solutions for $\mathcal{D}^2$-Close Scoring System because they deviate too much from $\vec{\alpha} = (4, 2, 1, 0)$, with a respective Euclidean distance of $\sqrt{10}$ and $\sqrt{4}$. On the other hand, $(4, 3, 2, 0)$ and $(3, 2, 2, 0)$ would be possible solutions within a distance of $\sqrt{2}$ to $\vec{\alpha} = (4, 2, 1, 0)$.

Using a linear program formulation, it can be shown that the unrestricted problem can be solved in polynomial time.

Theorem 2.2. *Scoring System Existence is in* P *for* $\vec{\alpha} \in \mathbb{R}^m_{\geq 0}$, $\mathbb{Q}^m_{\geq 0}$, *and* $\mathbb{N}^m_{\geq 0}$.

For the more restricted problem $\mathcal{D}$-Close Scoring System the situation is different. The complexity varies for different domains of the scoring vector and depends on the actual distance. Note that reducing the election(s) to a (integer) linear program is highly relevant to both the theoretical efficient solution (see Theorem 2.2) and the solution of the computational hard problems in practice. Apart from that, the transformation is a useful tool in the construction of hardness reductions.

Related Work. The study of election problems from a computational point of view belongs to the field of Computational Social Choice. A special case of our problem has already been studied by Baumeister et al. [2]. They showed that Scoring System Existence for one profile is NP-complete for $(\alpha_1, ..., \alpha_{m-4}, x_1, x_2, x_3, 0)$ with $x_i = 1$ for at least one $i \in \{1, 2, 3\}$, if we a assume that the votes are given in succinct representation. The technical requirement of succinct representation means that a profile is stored as a list of votes with their corresponding multiplicity instead of listing each vote separately. However, Theorem 2.2 implies that their problem for unconstrained vectors can be solved efficiently. Uncertainty about the used voting rule has also been studied by Elkind and Erdélyi [5]. They considered the problem where a set of possible voting rules is given and a manipulator tries to ensure the victory of some distinguished candidate regardless of the voting rule used. Another related problem is the robustness of elections studied by Shiryaev et al. [7] and Bredereck et al. [3]. They take into account the minimum degree of change necessary regarding the votes to alter the outcome of the election. The relationship between distances, elections, and voting rules has been studied, by Eckert et al. [4], Elkind et al. [6], and Baumeister et al. [1].

3 CONCLUSIONS

We have introduced the Scoring System Existence problem and the $\mathcal{D}$-Close Scoring System problem to provide a theoretical background for the analysis of the influence on the design of scoring systems. In future work, the exact computational complexity of these problems should be considered, as well as an experimental evaluation of real-world data. The next step would be to consider a distribution of profiles, e.g. based on predictions, rather than a set of profiles. Here one would look for a scoring system that maximizes the chances of success.

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