Vote For Me! Election Control via Social Influence in Arbitrary Scoring Rule Voting Systems

Extended Abstract

Federico Corò
Gran Sasso Science Institute – L'Aquila, Italy
federico.coro@gssi.it

Emilio Cruciani
Gran Sasso Science Institute – L'Aquila, Italy
emilio.cruciani@gssi.it

Gianlorenzo D'Angelo
Gran Sasso Science Institute – L'Aquila, Italy
gianlorenzo.dangelo@gssi.it

Stefano Ponziani
Gran Sasso Science Institute – L'Aquila, Italy
stefano.ponziani@gssi.it

ABSTRACT

Online social networks are used to diffuse opinions and ideas among users, enabling a faster communication and a wider audience. The way in which opinions are conditioned by social interactions is usually called social influence. Social influence is extensively used during political campaigns to advertise and support candidates.

We consider the problem of exploiting social influence in a network of voters to change their opinion about a target candidate with the aim of increasing his chance to win or lose the election in a wide range of voting systems. We introduce the *Linear Threshold Ranking*, a natural and powerful extension of the well-established *Linear Threshold Model*, which describes the change of opinions taking into account the amount of exercised influence. We are able to maximize the score of a target candidate up to a factor of $1 - 1/e$ by showing submodularity. We exploit such property to provide a $\frac{1}{3}(1 - 1/e)$-approximation algorithm for the *constructive* election control problem and a $\frac{1}{2}(1 - 1/e)$-approximation algorithm for the *destructive* control problem. The algorithm can be used in *arbitrary scoring rule voting systems*, including *plurality rule* and *borda count*.

KEYWORDS

Election Control; Influence Maximization; Social Networks; Voting

ACM Reference Format:
Federico Corò, Emilio Cruciani, Gianlorenzo D'Angelo, and Stefano Ponziani. 2019. Vote For Me! Election Control via Social Influence in Arbitrary Scoring Rule Voting Systems. In *Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), Montreal, Canada, May 13–17, 2019,* IFAAMAS, 3 pages.

1 INTRODUCTION

Recently, there has been a growing interest on the relationship between social networks and political campaigning. Political campaigns nowadays use social networks to lead elections in their favor; for example, they can target specific voters with fake news [1]. A real-life example of political intervention in this context occurred in the US Congressional elections in 2010, where a set of users were encouraged to vote with a message on Facebook. These messages directly influenced the real-world voting behavior of millions of people [2]. Another example is that of French elections in 2017, where automated accounts in Twitter spread a considerable portion of political content trying to influence the outcome [5].

There exists an extensive literature on manipulating elections without considering the underlying social network structure of the voters; we point the reader to a recent survey [4]. Nevertheless, there are only few studies that exploit opinion diffusion in social networks to change the outcome of elections. Independent Cascade Model (ICM) [6] has been considered as diffusion process in the problem of constructive/destructive election control [8], that consists in changing voters' opinions with the aim of maximizing/minimizing the margin of victory of some target candidate. A variant of the Linear Threshold Model (LTM) [6] with weights on the vertices has been considered on a graph in which each node represents a cluster of voters with a specific list of candidates and there is an edge between two nodes if they differ by the ordering of a single pair of adjacent candidates [3].

In this work we focus on the election control problem via social influence [8]: Given a social network of voters, we want to select a subset of voters that, with their influence, will change the opinion of the network's users about a target candidate, maximizing its chances to win/lose (we remark that we consider the scenario in which only the opinions about a target candidate can be changed). Previous work only studied plurality rule; moreover, in the diffusion model previously considered, an influenced voter shifts up or down the position of the target candidate in its ranking by just one position, regardless of the amount of influence received [8]. We study the election control problem in arbitrary scoring rule voting systems; moreover we consider a different diffusion model, that takes into account the degree of influence that voters exercise on the others and can describe the scenario in which a high influence on someone can radically change its opinion. A full version of our paper can be found at https://arxiv.org/abs/1902.07454.

2 LINEAR THRESHOLD RANKING

We consider the scenario in which a set of candidates are running for the elections and a social network of voters will decide the winner. In particular we focus on the general case of *scoring rules*, in which each voter expresses his preference as a ranking; each

Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), N. Agmon, M. E. Taylor, E. Elkind, M. Veloso (eds.), May 13–17, 2019, Montreal, Canada.

candidate is then assigned a score, computed as a function f of the positions he was ranked.

We represent the underlying social network as a directed graph $G = (V, E)$. We define N_v as the set of incoming neighbors for each $v \in V$. Let $C = \{c_1, \dots, c_m\}$ be the set of m candidates; we refer to our *target candidate*, i.e., the one that we want to make win/lose the elections, as $c_\star$. Each $v \in V$ has a permutation π_v of C, i.e., its list of preferences for the elections; we denote the position of candidate c_i in the preference list of node v as $\pi_v(c_i)$.

Let $B \in \mathbb{N}$ be the initial budget, i.e., the maximum size of the set of active nodes A_0 from which the LTM process starts. The diffusion process unfolds in discrete time steps as follows: A node v becomes active if the sum of the weights of the edges coming from active nodes at time $t-1$ is greater than or equal to its threshold t_v, i.e., $v \in A_t$ if and only if $v \in A_{t-1}$ or $\sum_{u \in A_{t-1}:(u,v)\in E} b_{uv} \geq t_v$. After the LTM process stops, i.e., no more nodes are being activated, the position of $c_\star$ in the preference list of each node changes according to a function of its incoming active neighbors. Let A be the set of active nodes at the end of LTM. The threshold t_v of each node $v \in V$ models its strength in retaining its original opinion about candidate $c_\star$: The higher is the threshold the lower is the probability that v is influenced by its neighbors. The weight on an edge b_{uv} measures the influence that node u has on node v. We define the number of positions that $c_\star$ goes up in π_v as

$$\pi_v^{\uparrow}(c_\star) := \min\left(\pi_v(c_\star) - 1, \left\lfloor \frac{\alpha(\pi_v(c_\star))}{t_v} \sum_{u \in A,\, (u,v) \in E} b_{uv} \right\rfloor\right),$$

where $\alpha : \{1, \dots, m\} \to [0, 1]$ is a function that depends on the position of $c_\star$ in π_v and models the rate at which $c_\star$ shifts up. We call this process the *Linear Threshold Ranking* (*LTR*).

After *LTR*, the new position of $c_\star$ will be $\tilde{\pi}_v(c_\star) := \pi_v(c_\star) - \pi_v^{\uparrow}(c_\star)$; the candidates overtaken by $c_\star$ will shift one position down.

In the problem of *election control* we want to maximize the chances of the target candidate to win the elections under *LTR*. To achieve that, we maximize its expected *Margin of Victory* (MoV) w.r.t. the most voted opponent, akin to that defined in [8].

Let c and $\tilde{c}$ be the candidates, different from $c_\star$, with the highest score before and after *LTR*, respectively. We define the *margin*, i.e., difference in score between the most voted opponent and $c_\star$ before and after *LTR*, as:

$$\mu(\emptyset) := \sum_{v \in V} [f(\pi_v(c)) - f(\pi_v(c_\star))],$$

$$\mu(A_0) := \sum_{v \in V} [f(\tilde{\pi}_v(\tilde{c})) - f(\tilde{\pi}_v(c_\star))],$$

where $f : \{1, \dots, m\} \to \mathbb{N}$ is a *non-increasing scoring function* that assigns a score to each position. Thus, the *election control* problem is formalized as

$$\begin{array}{ll} \max_{A_0} & \mathbf{E}[\mathrm{MoV}(A_0)] := \mathbf{E}[\mu(\emptyset) - \mu(A_0)] \\ \text{s.t.} & |A_0| \leq B. \end{array}$$

3 MAXIMIZING THE MARGIN OF VICTORY

To solve the problem of maximizing the MoV we focus on the score of the target candidate. We first prove that the score of the target candidate is a monotone submodular function w.r.t. the initial set of seed nodes A_0: This allows us to get a $(1 - 1/e)$-approximation of the maximum score through the use of a greedy algorithm (that we denote as Greedy) that iteratively selects the node that maximizes the increment in score [7]. As in influence maximization problems, we define an alternative random process based on live-edge graphs, called *Live-edge Dice Roll* (*LDR*), and show its equivalence to *LTR*; then we use *LDR* to compute the score of $c_\star$.

Definition 3.1. Live-edge Dice Roll process (*LDR*):

(1) Each node $v \in V$ selects edge (u, v) with probability b_{uv}, and no edge is selected with probability $1 - \sum_{u \in N_v} b_{uv}$.
(2) Each node v with $\pi_v(c_\star) > 1$ that is reachable from A_0 rolls a biased $\pi_v(c_\star)$-sided dice and changes the position of $c_\star$ in its list according to the outcome, i.e., picks a random number s_v in $[0, 1]$ and sets $\tilde{\pi}_v(c_\star)$ as follows

$$\tilde{\pi}_v(c_\star) = \begin{cases} 1 & \text{if } s_v \leq \frac{\alpha(\pi_v(c_\star))}{\pi_v(c_\star)-1}, \\ \ell & \text{if } \frac{\alpha(\pi_v(c_\star))}{\pi_v(c_\star)-\ell+1} < s_v \leq \frac{\alpha(\pi_v(c_\star))}{\pi_v(c_\star)-\ell}, \\ & \quad \text{for } \ell = 2, \dots, \pi_v(c_\star) - 1, \\ \pi_v(c_\star) & \text{if } s_v > \alpha(\pi_v(c_\star)). \end{cases}$$

We show that *LTR* and *LDR* have the same distribution.

Theorem 3.2. *Given a set of initially active nodes A_0 and a node $v \in V$, let $\tilde{\pi}_v^{LTR}(c_\star)$ and $\tilde{\pi}_v^{LDR}(c_\star)$ be the position of node v at the end of LTR and LDR, respectively, both starting from A_0. Then, $\mathbf{P}(\tilde{\pi}_v^{LTR}(c_\star) = \ell) = \mathbf{P}(\tilde{\pi}_v^{LDR}(c_\star) = \ell)$, for each $\ell = 1, \dots, \pi_v(c_\star)$.*

Thanks to Theorem 3.2 we are able to write the score of $c_\star$ as a non-negative linear combination of reachability functions in the live-edge graphs; we can also show that these functions are monotone and submodular w.r.t. A_0. Then, we can use Greedy to find a set A_0 that approximates the optimum not worse than $1 - 1/e$.

Moreover, given the equivalence of *LDR* with *LTR*, we can formulate our objective function as the average $\mathrm{MoV}_{G'}$ computed on a sampled live-edge graph G', namely $\mathbf{E}[\mathrm{MoV}(A_0)] = \mathbf{E}[\mathrm{MoV}_{G'}(A_0)]$.

We use these two properties to obtain a constant-factor approximation for the problem of maximizing the MoV. In particular, we use the solution A_0 computed by the approximation algorithm for score maximization and show that we only lose an extra $\frac{1}{3}$ approximation factor. Roughly speaking, we get a factor $\frac{1}{3}$ because we can formulate the MoV as the sum of three terms that are lower-bounded by the score of $c_\star$.

Theorem 3.3. *Greedy gives a $\frac{1}{3}(1 - 1/e)$ approximation to the problem of maximizing MoV in arbitrary scoring rule voting systems.*

Destructive election control. As for the constructive case, we achieve a constant factor approximation also in the *destructive election control* problem through a reduction from the destructive to the constructive case. Given an instance of destructive control, we build an instance of constructive control in which we simply reverse the rankings of each node and complement the scoring function to its maximum value. Roughly speaking, this reduction maintains invariant the absolute value of the change in margin of the score of any candidate between the two cases. The reduction allows us to maximize the score of the target candidate in the constructive case and then to map it back to the destructive case. Here, though, we get a factor $\frac{1}{2}$ because we can reconstruct the optimum in the approximation by only lower bounding two terms.

REFERENCES

[1] Hunt Allcott and Matthew Gentzkow. 2017. *Social Media and Fake News in the 2016 Election*. Working Paper 23089. National Bureau of Economic Research. https://doi.org/10.3386/w23089

[2] Robert M. Bond, Christopher J. Fariss, Jason J. Jones, Adam D. I. Kramer, Cameron Marlow, Jaime E. Settle, and James H. Fowler. 2012. A 61-million-person experiment in social influence and political mobilization. *Nature* 489 (12 Sep 2012), 295. http://dx.doi.org/10.1038/nature11421

[3] Piotr Faliszewski, Rica Gonen, Martin KouteckÃ¡, and Nimrod Talmon. 2018. Opinion Diffusion and Campaigning on Society Graphs. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI-18*. International Joint Conferences on Artificial Intelligence Organization, 219–225. https://doi.org/10.24963/ijcai.2018/30

[4] Piotr Faliszewski, Jörg Rothe, and Hervé Moulin. 2016. *Control and Bribery in Voting*. Cambridge University Press, 146–168. https://doi.org/10.1017/CBO9781107446984.008

[5] Emilio Ferrara. 2017. Disinformation and Social Bot Operations in the Run Up to the 2017 French Presidential Election. *First Monday* 22 (06 2017). https://doi.org/10.5210/fm.v22i8.8005

[6] David Kempe, Jon Kleinberg, and Éva Tardos. 2015. Maximizing the Spread of Influence through a Social Network. *Theory Of Computing* 11, 4 (2015), 105–147.

[7] G. L. Nemhauser, L. A. Wolsey, and M. L. Fisher. 1978. An analysis of approximations for maximizing submodular set functions—I. *Mathematical Programming* 14, 1 (01 Dec 1978), 265–294. https://doi.org/10.1007/BF01588971

[8] Bryan Wilder and Yevgeniy Vorobeychik. 2018. Controlling Elections Through Social Influence. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS '18)*. International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 265–273. http://dl.acm.org/citation.cfm?id=3237383.3237428