# Local Distance Restricted Bribery in Voting

Extended Abstract

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## ABSTRACT

Studying complexity of various bribery problems has been one of the main research focus in computational social choice. In all the models of bribery studied so far, the briber has to pay every voter some amount of money depending on what the briber wants the voter to report and the briber has some budget at her disposal. Although these models successfully capture many real world applications, in many other scenarios, the voters may be unwilling to deviate too much from their true preferences. In this paper, we study the computational complexity of the problem of finding a preference profile which is as close to the true preference profile as possible and still achieves the briber's goal subject to budget constraints. We call this problem LOCAL DISTANCE RESTRICTED $BRIBERY. We consider three important measures of distances, namely, swap distance, footrule distance, and maximum displacement distance, and resolve the complexity of the local distance restricted bribery problem for many common voting rules.

## KEYWORDS

Computational social choice; bribery; algorithms; complexity

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## Introduction

Any election scenario is susceptible to control attacks of various kinds – internal or external agents may try to influence the election system in someone's favor. One such attack which has been studied extensively in computational social choice is *bribery*. In every model of bribery studied so far (see [29]), we have the preferences of a set of voters, an external agent called briber with some budget, a bribing model which dictates how much one has to bribe any voter to persuade her to cast a vote of briber's choice, and the computational problem is to check whether it is possible to bribe the voters subject to the budget constraint so that some alternative of briber's choice becomes the winner. This models not only serve as a true theoretical abstraction of various real world scenarios but also generalizes many other important control attacks, for example, coalitional manipulation [1, 8]. In this paper, we study a refinement of the above bribery model motivated by the following important observation made by Obraztsova and Elkind [36, 37]

> *"...if voting is public (or if there is a risk of information leakage), and a voter's preference is at least somewhat known to her friends and colleagues, she may be worried that voting non-truthfully can harm her reputation yet hope that she will not be caught if her vote is sufficiently similar to her true ranking. Alternatively, a voter who is uncomfortable about manipulating an election for ethical reasons may find a lie more palatable if it does not require her to re-order more than a few candidates."*

Indeed, in the context of bribery, there can be situations where a voter may be bribed to report some preference which "resembles" her true preference but a voter is simply unwilling to report any preference which is far from her true preference. We remark that existing models of bribery do not capture the above constraint since, intuitively speaking, the budget feasibility constraint in these models restricts the total money spent (which is a global constraint) whereas the situations above demand (local) constraints per voter. For example, let us think of a voter $v$ with preference $a \succ b \succ c$. Suppose the voter $v$ can be persuaded to make at most two swaps and the cost of persuading her does not depend on the number of swaps she performs in her preference. This could be the situation when she is happy to change her preference as briber advises (simply because she trusts the briber that her change will finally ensure a better social outcome) but does not wish to deviate from her own preference too much to avoid social embarrassment. One can see that the classical model of bribery (SWAP BRIBERY for example) fails to capture the intricacies of this situation (for example, making the cost per swap to be 0 fails because the voter $v$ is not willing to cast $c \succ b \succ a$). In this paper, we fill this research gap by proposing a bribery model which directly addresses these scenarios.

More specifically, we study the computational complexity of the following problem which we call LOCAL DISTANCE RESTRICTED $BRIBERY. Given preferences $\mathcal{P} = (\succ_i)_{i\in[n]}$ of a set of agents, non-negative integers $(\delta_i)_{i\in[n]}$ denoting the distance change allowed for corresponding agents, non-negative integers $(p_i)_{i\in[n]}$ denoting the prices of every preference, a non-negative integer budget $\mathcal{B}$, and an alternative $c$, compute if the preferences can be changed subject to the "price, distance, and budget constraints" so that $c$ is a winner in the resulting election for some voting rule. We also study an interesting special case of the LOCAL DISTANCE RESTRICTED $BRIBERY problem where $\delta_i = \delta$ for some non-negative integer $\delta$ and $p_i = 0$ for every $i$ and $\mathcal{B} = 0$; we call the latter problem LOCAL DISTANCE RESTRICTED BRIBERY. In this paper, we study the following commonly used distance functions on the set of all possible preferences (permutations on the set of alternatives): (i) swap distance [32], (ii) footrule distance [40], and (iii) maximum displacement distance [36, 37]. The swap distance (aka Kendall Tau distance, bubble sort distance, etc.) between two preferences is the

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| Voting rule | Distance Metric | | |
|---|---|---|---|
| | Swap | Footrule | Maximum displacement |
| Plurality | $^\star P$ | | |
| Veto | | | |
| $k$-approval | $^\star P$ for $\delta = 1$<br>NP-complete for $\delta = 2$ | $^\star P$ for $\delta \leqslant 3$<br>NP-complete for $\delta = 4$ | $P$, $^\star P$ for $\delta_i = 1, \forall i$<br>$^\star$NP-complete for $\delta_i = 2, \forall i$ |
| Borda | NP-complete for $\delta = 1$ | NP-complete for $\delta = 2$ | NP-complete for $\delta = 1$ |
| Maximin | NP-complete for $\delta = 1$ | NP-complete for $\delta = 2$ | NP-complete for $\delta = 1$ |
| Copeland$^\alpha$, $\alpha \in [0, 1]$ | NP-complete for $\delta = 1$ | NP-complete for $\delta = 2$ | NP-complete for $\delta = 1$ |
| Simplified Bucklin | $^\star P$ for $\delta = 1$<br>NP-complete for $\delta = 2$ | $^\star P$ for $\delta \leqslant 3$<br>NP-complete for $\delta = 4$ | $P$, $^\star P$ for $\delta_i = 1, \forall i$<br>$^\star$NP-complete for $\delta_i = 2, \forall i$ |
| Bucklin | NP-complete for $\delta = 1$ | NP-complete for $\delta = 2$ | NP-complete for $\delta = 1$ |

**Table 1: The results marked ⋆ hold for the Local Distance Restricted $Bribery problem; others hold for the Local Distance Restricted Bribery problem.**

number of pairs of alternatives which are ranked in different order in these two preferences. Whereas the footrule distance (maximum displacement distance respectively) between two preferences is the sum (maximum respectively) of the absolute value of the differences of the positions of every alternative in two preferences.

*Contribution.* We study the computational complexity of the Local Distance Restricted $Bribery and Local Distance Restricted Bribery problems for the plurality, veto, $k$-approval, a class of scoring rules which includes the Borda voting rule, maximin, Copeland$^\alpha$ for any $\alpha \in [0, 1]$, Bucklin, and simplified Bucklin voting rules for the swap, footrule, and maximum displacement distance. We summarize our results in Table 1.

*Related Work.* Faliszewski et al. [25] propose the first bribery problem where the briber's goal is to change a minimum number of preferences to make some candidates win the election. Then they extend their basic model to more sophisticated models of Shift bribery and $Bribery [26, 27]. Elkind et al. [22] extend this model further and study the Swap bribery problem where there is a cost associated with every swap of alternatives. Dey et al. [16] show that the bribery problem remains intractable for many common voting rules for an interesting special case which they call Frugal bribery. The bribery problem has also been studied in various other preference models, for example, truncated ballots [2], soft constraints [38], approval ballots [39], campaigning in societies [24], CP-nets [19], combinatorial domains [34], iterative elections [35], committee selection [6], probabilistic lobbying [3], etc. Erdelyi et al. [23] study the bribery problem under voting rule uncertainty. Faliszewski et al. [28] study bribery for the simplified Bucklin and the Fallback voting rules. Xia [41], and Kaczmarczyk and Faliszewski [30] study the destructive variant of bribery. Dorn and Schlotter [20] and Bredereck et al. [5] explore parameterized complexity of various bribery problems. Chen et al. [7] provide novel mechanisms to protect elections from bribery. Knop et al. [33] provide a uniform framework for various control problems. Although most of the bribery problems are intractable, few of them, Shift bribery for example, have polynomial time approximation algorithms [21, 31]. Manipulation, a specialization of bribery, is another fundamental attack on election [9]. In the manipulation problem, a set of voters (called manipulators) wants to cast their preferences in such a way that (when tallied with the preferences of other preferences) makes some alternative win the election. Obraztsova and Elkind [36, 37] initiate the study of optimal manipulation in that context.

There is also a related line of work in computational social choice on broader control problems [4, 10, 12–15, 17, 18].

## Preliminaries

We will consider the following distance functions in this paper. *Swap distance:* $d_{\text{swap}}(\succ_1, \succ_2) = |\{\{a, b\} \subset \mathcal{A} : a \succ_1 b, b \succ_2 a\}|$, *Footrule distance:* $d_{\text{footrule}}(\succ_1, \succ_2) = \sum_{a \in \mathcal{A}} |pos(a, \succ_1) - pos(a, \succ_2)|$, *Maximum displacement distance:* $d_{\max \text{dis}}(\succ_1, \succ_2) = \max_{a \in \mathcal{A}} |pos(a, \succ_1) - pos(a, \succ_2)|$.

*Definition 0.1 (Local Distance Restricted Bribery).* Given a set $\mathcal{A}$ of alternatives, a profile $\succ = (\succ_i)_{i \in [n]} \in \mathcal{L}(\mathcal{A})^n$ of $n$ preferences, a positive integer $\delta$, and an alternative $c \in \mathcal{A}$, compute if there exists a profile $\succ' = (\succ'_i)_{i \in [n]} \in \mathcal{L}(\mathcal{A})^n$ such that

(i) $d(\succ_i, \succ'_i) \leqslant \delta$ for every $i \in [n]$
(ii) $r(\succ') = \{c\}$

We denote any arbitrary instance of Local Distance Restricted Bribery by $(\mathcal{A}, \mathcal{P}, c, \delta)$.

*Definition 0.2 (Local Distance Restricted $Bribery).* Given a set $\mathcal{A}$ of alternatives, a profile $\succ = (\succ_i)_{i \in [n]} \in \mathcal{L}(\mathcal{A})^n$ of $n$ preferences, positive integers $(\delta_i)_{i \in [n]}$ denoting distances allowed for every preference, non-negative integers $(p_i)_{i \in [n]}$ denoting the prices of every preference, a non-negative integer $\mathcal{B}$ denoting the budget of the Briber, and an alternative $c \in \mathcal{A}$, compute if there exists a subset $J \subseteq [n]$ and a profile $\succ' = (\succ'_i)_{i \in J} \in \mathcal{L}(\mathcal{A})^{|J|}$ such that

(i) $\sum_{i \in J} p_i \leqslant \mathcal{B}$
(ii) $d(\succ_i, \succ'_i) \leqslant \delta_i$ for every $i \in J$
(iii) $r\left((\succ'_i)_{i \in J}, (\succ_i)_{i \in [n] \setminus J}\right) = \{c\}$

We denote any arbitrary instance of Local Distance Restricted $Bribery by $(\mathcal{A}, \mathcal{P}, c, (\delta_i)_{i \in [n]}, (p_i)_{i \in [n]})$.

We remark that the optimal bribery problem, as described in Definition 0.1, demands the alternative $c$ to win uniquely. It is equally motivating to demand that $c$ is a co-winner. As far as the optimal bribery problem is concerned, we can easily verify that all our results, both algorithmic and hardness, extend easily to the co-winner case. However, we note that it need not always be the case in general (see Section 1.1 in [42] for example). The proofs can be found in the full version [11].

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