Proportional Representation in Elections: STV vs PAV

Extended Abstract

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ABSTRACT

We consider proportionality in multiwinner elections and observe that PAV and STV are fundamentally different. We argue that the former is proportional and the latter is degressively proportional.

KEYWORDS

multiwinner voting; committee scoring rules; proportionality; Single Transferable Vote; Proportional Approval Voting

ACM Reference Format:
Piotr Faliszewski, Piotr Skowron, Stanisław Szufa, and Nimrod Talmon. 2019. Proportional Representation in Elections: STV vs PAV. In *Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), Montreal, Canada, May 13–17, 2019,* IFAAMAS, 3 pages.

1 INTRODUCTION

We consider the problem of selecting a committee that represents the voters proportionally. One way of finding such a committee is to apply the *Single Transferable Vote (STV)* rule, which is considered to give proportional results, while another one is to use *Proportional Approval Voting* (PAV) [8, 15]; it has recently been shown that PAV satisfies a number of appealing properties pertaining to proportionality [1–3, 11, 12]. Yet, even though both these rules aim at achieving proportional representation, they might produce fundamentally different results. For example, let us consider elections (in two dimensional Euclidean space) where the ideal points of the candidates are drawn uniformly at random from a disc in the centre (dark grey points in the picture on the left of Figure 1) while the ideal points of the voters are drawn uniformly at random from the ring that surrounds that disc (light grey points in Figure 1). Each voter ranks the candidates from best to worst by sorting their ideal points with respect to the Euclidean distance from the voter's points [4, 5].

We generated 100 elections of this form (each with 100 voters and 20 candidates) and for each of them we computed PAV and STV winning committees of size 10; we present the ideal points of the members of these winning committees in the pictures in the centre (PAV) and on the right (STV) of Figure 1, as black points; both pictures contain the points of the winning committee members from all the elections. We see that STV is better at mimicking the distribution of the voters, whereas PAV is better at selecting more central candidates, that represent all the voters.

We conclude that there is no single notion of proportionality. We refer to the kind of proportionality represented by STV as *individual proportionality* and to the kind represented by PAV as *group proportionality*. We seek to understand these notions better by expressing STV in the same language as PAV (i.e., as an OWA-based rule). Specifically, we find that STV is degressively proportional.

Proc. of the 18th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2019), N. Agmon, M. E. Taylor, E. Elkind, M. Veloso (eds.), May 13–17, 2019, Montreal, Canada.

2 PRELIMINARIES

For an integer t, we denote the set $\{1, \ldots, t\}$ by $[t]$. By an election $E = (C, V)$, we mean a pair that consists of a set of candidates $C = \{c_1, \ldots, c_m\}$ and a set of voters $V = \{v_1, \ldots, v_n\}$, where each voter $v_i \in V$ has a linear preference order over all the candidates from C. A multiwinner voting rule $\mathcal{R}$ is a function that given an election $E = (C, V)$ and a committee size k, outputs a family of size-k subsets of C, the committees that tie as election winners.

Single Transferable Vote (STV). We consider the following variant of the STV rule (see, e.g., [10]). Let $E = (C, V)$ be the input election with n voters. We use *Droop quota* $q = \lfloor \frac{n}{k+1} + 1 \rfloor$. To compute a winning committee of size k, start with an empty committee and execute the following steps until the committee has k candidates: (1) If some candidate is ranked first by at least q voters, then find a candidate c that is ranked first by the largest number of voters and remove him from the election, together with some q voters that rank him first. (2) If no candidate is ranked first by

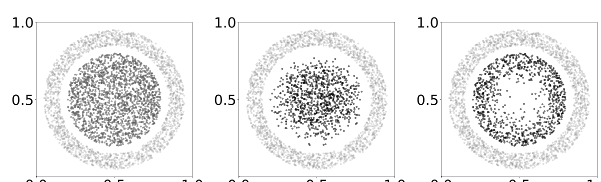

Figure 1: 2D scatter plots for the simulation in the introduction. Light grey ring of points in each picture represent the voters. Central grey points in the left picture represent the candidates. Black points in the middle and right pictures represent the winners under PAV and STV rule respectively.

at least q voters, then find a candidate d that is ranked first by the fewest voters and remove d from the election.

OWA-Based Rules. A single-winner scoring function for the case of m candidates is a non-increasing function $\gamma\colon [m] \to \mathbb{R}$ that associates each position in a preference order with a non-negative score. We focus on the Borda scoring function: $\beta_m(i) = m - i$; the k-Approval scoring function: $\alpha_k(i) = [i \leq k]$; and the k-Truncated Borda scoring function: $\beta_m^k(i) = (k - i) \cdot [i \leq k]$ (for a logical expression F, by $[F]$ we mean 1 if F is true and 0 otherwise).

An OWA-based multiwinner rule $\mathcal{R}_f$, for m candidates and committees of size k, is defined through a single-winner scoring function γ and an OWA vector $\Lambda = (\lambda_1, \ldots, \lambda_k)$ (OWA stands for *ordered weighted average* [16]). If some voter v ranks members of a given size-k committee W on positions $i_1 \leq \ldots \leq i_k$, then v assigns W a score of $f_{m,k}(i_1, \ldots, i_k) = \lambda_1\gamma(i_1) + \ldots + \lambda_k\gamma(i_k)$. The $\mathcal{R}_f$-winning committees are exactly those for which the sum of the scores given by the voters is the highest. PAV uses the k-Approval scoring function and OWA vectors of the form $(1, 1/2, \ldots, 1/k)$. OWA-based rules are due to Skowron et al. [13]; see also [6, 14].

Euclidean Elections. In 1D Euclidean elections, each voter and candidate has an ideal point $p \in \mathbb{R}$. Each voter forms his preference order by sorting the candidates with respect to the increasing distance of their ideal points from his. We focus on 1D Euclidean elections where the ideal points are drawn from the $[0, 1]$ interval. In the Uniform model, the ideal points of the candidates and voters are drawn uniformly at random; in the Uniform/Gaussian model, the ideal points of the candidates are drawn uniformly at random, whereas the ideal points of the voters are drawn from the Gaussian distribution with mean 0.5 and std. deviation 0.15; in the Asymmetric Gaussian, model the ideal points of the candidates and voters are drawn with probability 30% from a Gaussian distribution with mean 0.25, and with probability 70% from a Gaussian distribution with mean 0.75 (both Gaussians have std. deviation 0.1).

3 RESULTS

Our goal is to find an OWA-based rule that approximates STV as well as possible. We present our methodology and then the results.

Methodology. To evaluate how similar two committees from an Euclidean election are, we use the following distance: Consider some election E and committees $U = \{u_1, \ldots, u_k\}$, $W = \{w_1, \ldots, w_k\}$. For each u_j and w_j, we write $p(u_j)$ and $p(w_j)$, respectively, to denote their ideal points. Denoting the set of all permutations of $[k]$ by S_k, we define the distance between W and U as $\mathrm{dist}(W, U) = \min_{\sigma \in S_k} \frac{1}{k} \sum_{i=1}^{k} \left(p(w_i) - p(u_{\sigma(i)})\right)^2$. In other words, we find a matching between the members of W and U that minimizes the sum of the squares of the distances between the matched committee members.

To find an OWA-based rule that approximates STV, we generate elections $E_1 = (C_1, V_1), \ldots, E_t = (C_t, V_t)$ and for each election E_i we compute an STV committee W_i (of a given size k). Let $\mathcal{R}_{\gamma,\Lambda}$ be an OWA-based rule, defined through a scoring function γ and an OWA vector Λ. For each election E_i, we compute one committee $U_i \in \mathcal{R}_{\gamma,\Lambda}(E_i, k)$. We define the distance between $\mathcal{R}_{\gamma,\Lambda}$ and $W_1, \ldots, W_t$ to be $\mathrm{dist}(\mathcal{R}_{\gamma,\Lambda}; W_1, \ldots, W_t) = \frac{1}{t} \sum_{i=1}^{t} \mathrm{dist}(W_i, U_i)$. We find an OWA vector that minimizes this value (using algorithm from [7]).

STV as an OWA-Based Rule. Let γ be one of our three single-winner scoring functions and let $\mathcal{E}$ be a model of generating elections, $\mathcal{E} \in$ {Uniform, Uniform/Gaussian, Asymmetric Gaussian}. We generated $t = 50$ elections $E_1, \ldots, E_t$ from $\mathcal{E}$ and, using our methodology, we computed the OWA vector $\Lambda = (\lambda_1, \ldots, \lambda_k)$ that minimized the distance $\mathrm{dist}(\mathcal{R}_{\gamma,\Lambda}; W_1, \ldots, W_t)$ (see Figure 2). To better understand the computed OWA vectors, we sought further vectors that would be as close to the computed ones as possible, but that would be expressed using a simple formula. We found that vectors of the form $h(x, y) = \left(1, \frac{1/2^x + y}{1+y}, \frac{1/3^x + y}{1+y}, \ldots, \frac{1/n^x + y}{1+y}\right)$ suffice.

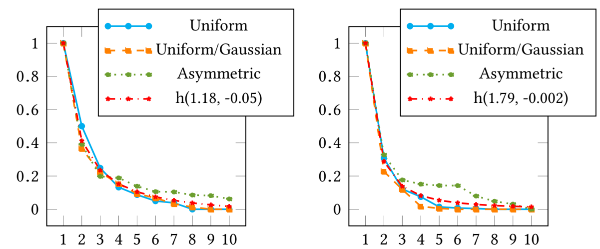

Figure 2: OWA vectors for approximating STV using 10-Approval (left) and 10-Tr.-Borda (right) scoring functions.

Degressive Proportionality. In the related model of apportionment, where the task is to divide parliamentary seats among political parties, proportionality has a natural interpretation—the number of seats obtained by each party should be proportional to the number of votes the party received. In contrast, a voting rule is proportional in a *degressive* way [9] if it assigns disproportionately many seats to parties with smaller support. Interestingly, there is a strong correlation between the slope of the OWA vector and the type of proportionality the rule guarantees [11].

Intuitively, this relation can be explained as follows. Consider an OWA vector $\Lambda = (1/z_1, 1/z_2, 1/z_3, \ldots, 1/z_n)$, which is normalized so that $z_1 = 1$. We define the vector $cost = (z_1, \ldots, z_n)$. Then, the i-th element of this vector can be interpreted as the relative cost of obtaining the i-th seat. Indeed, when $\mathcal{R}$ decides which of the two parties, P_i or P_j, should get the next seat, $\mathcal{R}$ compares the numbers of voters supporting each of these parties against the costs of obtaining the additional seats, and chooses the party with the higher ratio. Thus, we have the following observation. Let R be an OWA rule with k-Approval scoring function and a normalized OWA vector $\Lambda = (1/z_1, 1/z_2, 1/z_3, \ldots, 1/z_n)$. Let *cost* be as defined above. Then, the following hold:

(1) If *cost* is linear, then R is linearly proportional.
(2) If *cost* is convex, then R is degressive proportional.
(3) If *cost* is concave, then R is progressive proportional.

Our results suggest that STV is degressively proportional.

Conclusions. We conclude that there are several types of proportionality for multiwinner electons that can be classified based on the OWA vectors used to define or approximate voting rules.

ACKNOWLEDGMENTS

Piotr Faliszewski was supported by the National Science Centre, Poland, under project 2016/21/B/ST6/01509.

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