Simple Contrapositive Assumption-Based Frameworks

Extended Abstract

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ABSTRACT

We study the Dung semantics for extended forms of assumption-based argumentation frameworks (ABFs), based on *any* contrapositive propositional logic, and whose defeasible rules are expressed by *arbitrary formulas* in that logic. New results on the well-founded semantics for such ABFs are reported, the redundancy of the closure condition is shown, and the use of disjunctive attacks is investigated. Useful properties of the generalized frameworks are also considered.

KEYWORDS

ABA frameworks; Dung semantics; Defeasible reasoning.

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1 PRELIMINARIES

Assumption-based argumentation frameworks (ABFs), thoroughly described in [1], were introduced in the 1990s as a computational structure to capture and generalize several formalisms for defeasible reasoning. Among other, ABFs have been used to model reasoning in multi-agent systems (see, e.g., [5, 6, 8]). In this paper, which is a companion of [7], we study a large family of ABFs, called *simple contrapositive*. For this, we first recall some basic notions.

Definition 1.1. A (propositional) *logic* for a language $\mathcal{L}$ is a pair $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$, where $\vdash$ is binary relation between sets of formulas and formulas in $\mathcal{L}$, which is reflexive (if $\psi \in \Gamma$ then $\Gamma \vdash \psi$), monotonic (if $\Gamma \vdash \psi$ and $\Gamma \subseteq \Gamma'$, then $\Gamma' \vdash \psi$), and transitive (if $\Gamma \vdash \psi$ and $\Gamma', \psi \vdash \phi$, then $\Gamma, \Gamma' \vdash \phi$). We assume that $\vdash$ is *non-trivial* (there are Γ, ψ for which $\Gamma \nvdash \psi$), *structural* (closed under substitutions: for every substitution θ, if $\Gamma \vdash \psi$ then $\{\theta(\gamma) \mid \gamma \in \Gamma\} \vdash \theta(\psi)$), and *finitary* (if $\Gamma \vdash \psi$ then there is a finite $\Gamma' \subseteq \Gamma$ s.t. $\Gamma' \vdash \psi$).

We assume that the language $\mathcal{L}$ contains at least the connectives $\{\neg, \wedge, \vee, \supset\}$ and the propositional constant F for falsity, with their usual definitions (see [7, Definition 2]). The $\vdash$-transitive closure of a set Γ of $\mathcal{L}$-formulas is denoted $Cn_{\vdash}(\Gamma) = \{\psi \mid \Gamma \vdash \psi\}$. We shall denote $\neg\Gamma = \{\neg\gamma \mid \gamma \in \Gamma\}$, and for a finite Γ we denote by $\bigwedge\Gamma$ (respectively, by $\bigvee\Gamma$) the conjunction (respectively, the disjunction) of all the formulas in Γ.

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Definition 1.2. A logic $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ is *explosive*, if for $\mathcal{L}$-formula ψ, the set $\{\psi, \neg\psi\}$ is $\vdash$-inconsistent, that is, $\psi, \neg\psi \vdash \mathsf{F}$. We say that $\mathfrak{L}$ is *contrapositive*, if for every Γ and ψ it holds that $\Gamma \vdash \neg\psi$ iff either $\psi = \mathsf{F}$, or for every $\phi \in \Gamma$ we have that $\Gamma \setminus \{\phi\}, \psi \vdash \neg\phi$.

Classical logic, intuitionistic logic, and standard modal logics, are all specific cases of explosive and contrapositive logics.

Next , we generalize the definition of ABFs (in [1]):

Definition 1.3. An *assumption-based framework* (ABF) is a tuple $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, where: $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ is a propositional Tarskian logic; Γ (the *strict assumptions*) and Ab (the *candidate or defeasible assumptions*) are distinct (countable) sets of $\mathcal{L}$-formulas, where the former is assumed to be $\vdash$-consistent and the latter is assumed to be nonempty; and $\sim : Ab \rightarrow \wp(\mathcal{L})$ is a *contrariness operator*, assigning a finite set of $\mathcal{L}$-formulas to every defeasible assumption in Ab, such that for every $\psi \in Ab$ where $\psi \nvdash \mathsf{F}$ it holds that $\psi \nvdash \bigwedge \sim\psi$ and $\bigwedge \sim\psi \nvdash \psi$.

A *simple contrapositive* ABF is an assumption-based framework $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, where $\mathfrak{L}$ is an explosive and contrapositive logic, and $\sim\psi = \{\neg\psi\}$.

Note that, unlike the setting of Bondarenko et al. [1], an ABF may be based on *any* Tarskian logic $\mathfrak{L}$. Also, the strict as well as the candidate assumptions are formulas that may not be just atomic.

Defeasible assertions in an ABF may be attacked in the presence of a counterargument. This is described in the next definition.

Definition 1.4. Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be an assumption-based framework, $\Delta, \Theta \subseteq Ab$, and $\psi \in Ab$. We say that Δ *attacks* ψ iff $\Gamma, \Delta \vdash \phi$ for some $\phi \in \sim\psi$. Δ attacks Θ if Δ attacks some $\psi \in \Theta$.

The last definition gives rise to the following adaptation to ABFs of the usual semantics for abstract argumentation frameworks [4].

Definition 1.5. ([1]) Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be an assumption-based framework, and let $\Delta \subseteq Ab$. Below, maximum and minimum are taken with respect to set inclusion. Then: Δ is *closed* if $\Delta = Ab \cap Cn_{\vdash}(\Gamma \cup \Delta)$. Δ is *conflict-free* iff there is no $\Delta' \subseteq \Delta$ that attacks some $\psi \in \Delta$. Δ is *naive* iff it is closed and maximally conflict-free. Δ *defends* a set $\Delta' \subseteq Ab$ iff for every closed set Θ that attacks Δ' there is $\Delta'' \subseteq \Delta$ that attacks Θ. Δ is *admissible* iff it is closed, conflict-free, and defends every $\Delta' \subseteq \Delta$. Δ is *complete* iff it is admissible and contains every $\Delta' \subseteq Ab$ that it defends. Δ is *grounded* iff it is minimally complete. Δ is *preferred* iff it is maximally admissible. Δ is *stable* iff it is closed, conflict-free, and attacks every $\psi \in Ab \setminus \Delta$. Δ is *well-founded* iff $\Delta = \bigcap\{\Theta \subseteq Ab \mid \Theta \text{ is complete}\}$.

The set of naive (respectively, complete, preferred, stable, grounded, well-founded) extensions of $\mathbf{ABF}$ is denoted $\mathsf{Naive}(\mathbf{ABF})$ (respectively, $\mathsf{Com}(\mathbf{ABF})$, $\mathsf{Prf}(\mathbf{ABF})$, $\mathsf{Stb}(\mathbf{ABF})$, $\mathsf{Grd}(\mathbf{ABF})$, $\mathsf{WF}(\mathbf{ABF})$). It is clear that a well-founded extension of an ABF is unique.

Definition 1.6. Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$. A set $\Delta \subseteq Ab$ is *maximally consistent* in **ABF**, if (a) $\Gamma, \Delta \nvdash \mathsf{F}$ and (b) $\Gamma, \Delta' \vdash \mathsf{F}$ for every $\Delta \subsetneq \Delta' \subseteq Ab$. The set of the maximally consistent sets in **ABF** is denoted $\mathsf{MCS}(\mathbf{ABF})$.

Proposition 1.7. [7] *Let* $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF. Then:*

- $\mathsf{Naive}(\mathbf{ABF}) = \mathsf{Prf}(\mathbf{ABF}) = \mathsf{Stb}(\mathbf{ABF}) = \mathsf{MCS}(\mathbf{ABF})$,
- *if* $\mathsf{F} \in Ab$ *then* $\mathsf{Grd}(\mathbf{ABF}) = \bigcap \mathsf{MCS}(\mathbf{ABF})$.

2 SOME GENERALIZATIONS

In this section we give a series of new results concerning Dung's semantics for ABFs and some of its useful enhancements.

2.1 The Well-Founded Extension

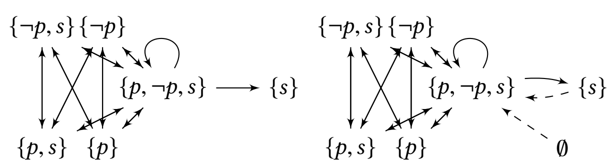
First, we consider the well-founded semantics for ABFs (recall Definition 1.5). This semantics has not been considered in [7], and it is useful when there is no unique minimal complete extension.

The existence of a well-founded extension for any simple contrapositive ABF follows from the following claim:

Proposition 2.1. *Any simple contrapositive ABF has a complete extension.*

The next example shows that, as in the case of the grounded semantics, the well-founded extension of an assumption-based framework **ABF** does not always coincide with $\bigcap \mathsf{MCS}(\mathbf{ABF})$.

Example 2.2. Let $\mathfrak{L}$ be classical logic (CL), $\Gamma = \emptyset$, and $Ab = \{p, \neg p, s\}$. A corresponding attack diagram is shown in Figure 1a.



(a) Attack diagram for Ex. 2.2 **(b) Attack diagram for Ex. 2.7.**

In this case, we have that $\mathsf{Com}(\mathbf{ABF}) = \{\emptyset, \{p, s\}, \{\neg p, s\}\}$, thus $\mathsf{WF}(\mathbf{ABF}) = \emptyset$. However, $\bigcap \mathsf{MCS}(\mathbf{ABF}) = \{s\}$.

Again (see the second item of Proposition 1.7), the situation in Example 2.2 can be avoided by requiring that $\mathsf{F} \in Ab$.

Proposition 2.3. *Let* $\mathbf{ABF} = \langle \mathcal{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF. If* $\mathsf{F} \in Ab$ *then* $\mathsf{WF}(\mathbf{ABF}) = \bigcap \mathsf{MCS}(\mathbf{ABF}) = \mathsf{Grd}(\mathbf{ABF})$.

2.2 Lifting the Closure Requirement

According to Definition 1.5, extensions of an ABF are required to be closed. This is a standard requirement for ABFs (see, e.g., [1, 3, 9]). In this section we show that the closure condition is not necessary for simple contrapositive ABFs.

Definition 2.4. Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be an ABF. A subset $\Delta \subseteq Ab$ is *weakly admissible* (in **ABF**) iff it is conflict-free, and defends every $\Delta' \subseteq \Delta$. We say that Δ is *weakly complete* (in **ABF**) iff it is weakly admissible and contains every $\Delta' \subseteq Ab$ that it defends.

Weak admissibility (weak completeness) is thus admissibility (completeness) without the closure requirement. The next proposition shows the redundancy of the closure requirement:

Proposition 2.5. *Let* $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositrive ABF. Then a set* $\Delta \subseteq Ab$ *is: (a) stable iff it is conflict-free and attacks every* $\psi \in Ab \setminus \Delta$*, (b) naive iff it is maximally conflict-free, and (c) preferred iff it is maximal weakly admissible. Moreover, if* $\mathsf{F} \in Ab$*, then* Δ *is grounded iff it is minimal weakly complete.*

2.3 Using Disjunctive Attacks

The next generalization concerns disjunctive attacks rather than pointed attacks (Definition 1.4).

Definition 2.6. Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ be a simple contrapositive ABF. We say that a set $\Delta \subseteq Ab$ *attacks* a set $\Theta \subseteq Ab$ if there is a finite subset $\Theta' \subseteq \Theta$ such that $\Gamma, \Delta \vdash \bigvee \neg \Theta'$.

Example 2.7. Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, and $Ab = \{p, \neg p, s\}$. A corresponding attack diagram is shown in Figure 1b, where strict lines represent standard attacks (Definition 1.4), and dashed lines represent attacks that are applicable only according to the disjunctive version of attacks (Definition 2.6).

Note that the 'contaminating' set $\{p, \neg p, s\}$ attacks the set $\{s\}$. However, when disjunctive attacks are allowed the attacking set $\{p, \neg p, s\}$ is counter-attacked by ($\{s\}$ itself and) the emptyset (since $\emptyset \vdash \neg p \vee \neg\neg p$), thus $\{s\}$ is defended by $\emptyset$ (which is not the case when only 'standard' attacks are allowed).

In what follows we further assume that the base logic $\mathfrak{L}$ respects the following de Morgan rules:

$$\textit{de Morgan I: } \bigvee \neg \Delta \vdash \neg \bigwedge \Delta, \quad \textit{de Morgan II: } \neg \bigwedge \Delta \vdash \bigvee \neg \Delta. \quad (1)$$

One clear benefit of using disjunctive attacks in this setting is that the inconsistency problems of argumentation-based extensions, first discussed in [2], are avoided:

Proposition 2.8. *Let* $\mathfrak{L}$ *be a logic in which de Morgan's rules in (1) are satisfied, and let* $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF with disjunctive attacks. If* $\Delta \subseteq Ab$ *is conflict-free then there are no* $\phi_1, \ldots, \phi_n \in \Delta$ *such that* $\Gamma, \Delta \vdash \neg \bigwedge_{i=1}^{n} \phi_i$.

When using disjunctive attacks, we still have that: (a) preferred and stable semantics are reducible to naive semantics, and (b) the correspondence to reasoning with maximally consistent subsets is preserved. Moreover, for disjunctive attacks, (c) the grounded extension is well-behaved, even without requiring that $\mathsf{F} \in Ab$ (cf. the second item in Proposition 1.7).

Theorem 2.9. *Let* $\mathfrak{L}$ *be a logic in which de Morgan's rules in (1) hold, and let* $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF with disjunctive attacks. Then* $\mathsf{Naive}(\mathbf{ABF}) = \mathsf{Prf}(\mathbf{ABF}) = \mathsf{Stb}(\mathbf{ABF}) = \mathsf{MCS}(\mathbf{ABF})$. *Moreover,* $\mathsf{Grd}(\mathbf{ABF}) = \bigcap \mathsf{MCS}(\mathbf{ABF})$.

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