Student-Project-Resource Matching-Allocation Problems: Two-Sided Matching Meets Resource Allocation

Extended Abstract

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ABSTRACT

We consider a student-project-resource matching-allocation problem, in which *students* (resp. *resources*) are matched (resp. allocated) to *projects*. A project's capacity for students is endogenously determined by the resources allocated to it. Traditionally, (1) resources are allocated to projects based on some expectations, and then (2) students are matched with projects based on the capacities determined by (1). Although resource allocation and two-sided matching are well-understood, unless the expectations used in the first problem are correct, we obtain a suboptimal outcome. Thus, it is desirable to solve this problem as a whole without dividing it.

In this paper, we show that finding a nonwasteful matching is $\mathrm{FP^{NP}}$[log]-hard, and that deciding whether a stable matching (i.e. nonwasteful and fair) exists is $\mathrm{NP^{NP}}$-complete. We also show that no strategyproof mechanism can satisfy fairness and very weak efficiency requirements. Then, we develop a new strategyproof mechanism, called Sample and Vote Deferred Acceptance (SVDA), that strikes a good balance between fairness and efficiency.

KEYWORDS

Matching; Resource Allocation; Computational Complexity

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1 INTRODUCTION

In this paper, we introduce a simple but fundamental model, which we call Student-Project-Resource matching-allocation (SPR). On one hand, this problem is a two-sided, many-to-one matching problem [24], where students are matched with projects based on their preferences. On the other hand, it also contains a resource allocation problem [20], since resources are allocated to projects. A common practice is to determine the resource allocation part, based on some expectations or past data and fix the capacities of projects. Then, the actual matching of students to projects is determined by a matching mechanism. If the expectations used in the first problem are incorrect, the outcome is suboptimal: excess demand of seats and excess supply may coexist in the same matching-allocation. One real-life instance where this practice is applied is the nursery-school waiting list problem[1] [22]. Unlike that standard two-sided matching setting where the capacity of each project is exogenously fixed, we assume capacities are endogenously determined by the resource allocation.

Related works. Two-sided matching has been attracting considerable attention [3, 12, 13]. Real-world matching markets are subject to a variety of distributional constraints [19], including regional maximum quotas [17], minimum quotas [8, 25, 26] and diversity constraints [6, 10, 18]. Other works examine the computational complexity for finding a matching [4, 7, 11]. A similar model was recently considered [16], but with a compact representation scheme. Besides, in three-sided matching problems [2, 14, 21] three types of players/agents are matched, e.g., males, females, and pets, or students, projects and lecturers [1, 5, 19, 23].

Our model. We introduce necessary definitions and notations:

Definition 1.1. An *SPR instance* is a tuple $(S, P, R, \succ_S, \succ_P, T_R, q_R)$.

- $S = \{s_1, \ldots, s_{|S|}\}$ is a set of students.
- $P = \{p_1, \ldots, p_{|P|}\}$ is a set of projects.
- $R = \{r_1, \ldots, r_{|R|}\}$ is a set of resources.
- $\succ_S = (\succ_s)_{s \in S}$ are the students' preferences over set $P \cup \{\emptyset\}$.
- $\succ_P = (\succ_p)_{p \in P}$ are the projects' preferences over set $S \cup \{\emptyset\}$.
- Resource r has capacity $q_r \in \mathbb{N}_{>0}$, and $q_R = (q_r)_{r \in R}$.
- Resource r is compatible with $T_r \subseteq P$, and $T_R = (T_r)_{r \in R}$.

Contract $(s, p) \in S \times P$ means student s is matched to project p. Contract (s, p) is acceptable for student s (resp. project p) if $p \succ_s \emptyset$ holds (resp. $s \succ_p \emptyset$). W.l.o.g., we define set of contracts $X \subseteq S \times P$ by $(s, p) \in X$ if and only if it is acceptable for p.

Definition 1.2 (Matching). A matching is a subset $Y \subseteq X$, where for every student $s \in S$, subset $Y_s = \{(s, p) \in Y \mid p \in P\}$ satisfies $|Y_s| \leq 1$, and either $Y_s = \emptyset$, or $Y_s = \{(s, p)\}$ and $p \succ_s \emptyset$, holds. For a matching Y, let $Y(s) \in P \cup \{\emptyset\}$ denote the project s is matched, and $Y(p) \subseteq S$ denote the set of students assigned to project p.

Definition 1.3 (Allocation). An allocation $\mu : R \to P$ maps each resource r to a project $\mu(r) \in T_r$. Let $q_\mu(p) = \sum_{r \in \mu^{-1}(p)} q_r$.[2]

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[1] In April 2018, the number of children waiting for nursery school places in Japan reached a record-breaking 55,433 toddlers.

[2] For $\mu^{-1}(p) = \emptyset$, we assume that an empty sum equals zero.

Definition 1.4 (Feasibility). A feasible matching (Y, μ) is matching-allocation couple where $|Y(p)| \leq q_\mu(p)$ holds for every $p \in P$.

Traditionally (e.g. with fixed quotas), for feasible matching (Y, μ) and $(s, p) \in X \setminus Y$, we say student s *claims an empty seat* of p if $p \succ_s Y(s)$ and matching $Y \setminus \{(s, Y(s))\} \cup \{(s, p)\}$ is feasible with *same* allocation μ. However, in our setting, since the distributional constraint is endogenous and as flexible as allocations are, the definition of nonwastefulness uses this flexibility, as follows.

Definition 1.5 (Nonwastefulness). Given feasible matching (Y, μ), a contract $(s, p) \in X \setminus Y$ is a claiming pair if and only if:

- student s has preference $p \succ_s Y(s)$, and
- $Y \setminus \{(s, Y(s))\} \cup \{(s, p)\}$ is feasible with some (new) allocation.

A feasible matching (Y, μ) is nonwasteful if it has no claiming pair.

Definition 1.6 (Fairness). Given feasible matching (Y, μ), contract $(s, p) \in X \setminus Y$ is an envious pair if and only if:

- student s has preference $p \succ_s Y(s)$, and
- there exists student $s' \in Y(p)$ such that p prefers $s \succ_p s'$.[3]

A feasible matching (Y, μ) is fair if it has no envious pair.

Definition 1.7 (Stability). A feasible matching (Y, μ) is stable if it is nonwasteful and fair (no claiming/envious pair).

Definition 1.8 (Pareto Efficiency). Matching Y is Pareto dominated by Y' if all students weakly prefer Y' over Y and at least one student strictly prefers Y'. A feasible matching is Pareto efficient if no feasible matching Pareto dominates it.[4]

Definition 1.9. Given any SPR instance, a *mechanism* outputs a feasible matching (Y, μ). If a mechanism always obtains a feasible matching that satisfies property A (e.g., fairness), we say this mechanism is A (e.g., fair). A mechanism is strategyproof if no student gains by reporting a preference different from her true one.

While mechanism Serial Dictatorship (SD) obtains a Pareto efficient (thus also nonwasteful) matching in time P^{NP}, mechanism Artificial Caps Deferred Acceptance (ACDA) obtains a fair matching in polynomial-time [9].

Example 1.10. Nonwastefulness and fairness are incompatible since there exists an instance with no stable matching. Let us show a simple example with two students s_a, s_b, two projects p_a, p_b, and a unitary resource compatible with both. Students' preferences are $p_a \succ_{s_a} p_b$ and $p_b \succ_{s_b} p_a$. Projects' are $s_b \succ_{p_a} s_a$ and $s_a \succ_{p_b} s_b$. By symmetry, assume the resource is allocated to p_a. From fairness, s_b must be allocated to p_a. Then (s_b, p_b) becomes a claiming pair.

2 THE COMPLEXITY OF SPR

We found several complexity results, fully detailed [15].

THEOREM 2.1. *Given an SPR instance and a matching Y, deciding whether an allocation μ exists such that (Y, μ) is a feasible matching, is NP-complete.*

THEOREM 2.2. *Given an SPR instance and a feasible matching (Y, μ), deciding whether it is nonwasteful is coNP-complete.*

THEOREM 2.3. *Given an SPR instance, finding a nonwasteful matching (Y, μ) is $FP^{NP}[log]$-hard.*

THEOREM 2.4. *Given an SPR instance, deciding whether a stable matching exists is NP^{NP}-complete.*

3 MECHANISM SVDA

We discuss how to develop a strategyproof mechanism that can strike a good balance between fairness and efficiency. A full description of the contents in this section including proofs, as well as additional contents, can be found in http://mpra.ub.uni-muenchen.de/92720. We introduce two conditions that related to efficiency. The first one is called *weak nonwastefulness.*

Definition 3.1 (Weak Nonwastefulness). For feasible matching (Y, μ), student s is a *strongly claiming student* if $Y(s) = \emptyset$, and for any feasible matching (Y, μ'), s claims an empty seat of some project p (p can be different for each μ'). A feasible matching is weakly nonwasteful if it has no strongly claiming student.

To define another concept called *resource efficiency*, we first define *unanimous preferences.*

Definition 3.2 (Unanimous Preference). Students unanimously prefer p over p' if for every $s \in S$, $(s, p) \in X$ and $p \succ_s p'$ hold.

Definition 3.3 (Resource Efficiency). Resource allocation μ is resource efficient if any resource r, such that $p', p \in T_r$ and students unanimously prefer p over p', is not allocated to p'. A mechanism is resource efficient if it always returns a resource efficient allocation.

Now we are ready to introduce our impossibility theorem.

THEOREM 3.4. *No mechanism exists that is fair, weakly nonwasteful, resource efficient, and strategyproof.*

Mechanism 1 (Sample and Vote Deferred Acceptance (SVDA))**.**

Step 1: Select $S' \subseteq S$, which we call the sampled students. We call $S \setminus S'$ the regular students. Then run SD and find (partial) matching $Y_{S'}$ for S'.
Step 2: Allocate $R' \subseteq R$ to projects such that $Y_{S'}$ is feasible and R' is minimal: no $R'' \subsetneq R'$ makes $Y_{S'}$ feasible.
Step 3: Allocate $R \setminus R'$ based on the preferences of S'. Then run DA for $S \setminus S'$. The capacity of p is $q_\mu(p) - |Y_{S'}(p)|$, where μ is the current resource allocation.

To decide allocation $R \setminus R'$ based on the preferences of S', we use the following simple method. For each r, each $s \in S'$ (hypothetically) votes for candidates T_r based on $\succ_s$, where each project obtains the Borda score based on $\succ_s$. Then r is allocated to the winner.

THEOREM 3.5. *SVDA is strategyproof, resource efficient, weakly nonwasteful and fair among $S \setminus S'$, i.e., no regular student has justified envy toward another regular student.*

Prospects. Future works include theoretically identifying the optimal sample size, finding tractable cases and dealing with various constraints on the allocation of resources.

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[3] $(Y \setminus \{(s, Y(s)), (s', Y(s'))\}) \cup \{(s, p)\}$ is feasible with same allocation μ.
[4] Pareto efficiency implies nonwastefulness (not vice versa).

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